CAN K-ρ RELATIONS BE ESTIMATED THROUGH GROUND WATER FLOW MODEL INVERSION?

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1 INTRODUCTION

In connection with construction of ground water models an extensive geophysical survey of the area is often made. In Denmark the model area is usually mapped by means of dense geoelectrical surface measurements. The geophysical surface measurements give an area coverage and are relatively inexpensive. If these geoelectrical data could be used not only to determine the geometrical structure of the aquifer, but also to give an estimate of the variation of the hydraulic conductivity within the aquifer, this would be a major advantage. Relations between hydraulic conductivity and electrical resistivity (K-ρ relation) often suggested is either a power relation or a simple linear relation like for example:

\[ \ln(K) = a \cdot \ln(\rho) + b \]  
(Mazâc et al. 1985), or

\[ K = a \cdot \rho + b \]  
(Kelly 1977).

Such a relation is site specific (Mazâc et al. 1985) and must be estimated every time a new area is investigated. The eventual relation can be estimated by direct measurements of both the electrical resistivity and the hydraulic conductivity, in situ or in the laboratory, but this is expensive.

We try to estimate the parameters (a and b) of this K-ρ relation by inverse ground water modelling, i.e. the calibration is done by non-linear regression using the ground water flow inversion code MODFLOWP (Hill 1992) to make the model response fit the observed hydraulic heads. The minimized objective function is:

\[ S(a, b) = \sum_{i=1}^{N} (h_i^* - h_i(a, b))^2 \]  

where a, b are the model parameters to be estimated, \( h^* \) is the vector of measured hydraulic heads and \( h(a, b) \) is the corresponding vector of predicted (modelled) heads at the measured wells using the parameters a and b.

To analyse under which conditions this relation can be estimated through inversion, a simple two-dimensional steady state synthetic model has been constructed, in which the true conditions and character of the errors are completely known. We analyse how the K-ρ relation, the errors in ρ-data, and the errors in hydraulic head data influence the efficiency of the proposed estimation method. First, models with no noise on ρ-data are examined. Gradually normally distributed, non-correlated and correlated noise is added to ρ-data.
2 CONSTRUCTION OF THE MODEL

The relation between K and \( \rho \) considered here is a power relation like equation 1. The parameters, describing the relation has been chosen to: \( a = 1 \) and \( b = \ln(1e^{-4}) = -9.210 \) (figure 1.a).

In the resistivity model considered here the resistivity in the aquifer varies gradually between \( 1 \cdot 10^{-1} \) and \( 1 \cdot 10^{5} \) \( \Omega \)m, i.e. \( K \) varies between \( 1 \cdot 10^{-5} \) and \( 1 \cdot 10^{-1} \) m/s. This wide range was chosen in order to get as good conditions as possible for the determination of the parameters describing the K-\( \rho \) relation. The generated error free resistivity field is shown in figure 2.a.

In all the examples shown here we assume that the hydraulic head is observed at 21 grid points as shown in figure 1.b, and in all examples uniform distributed random noise, with a standard deviation of 0.05 meter, has been added to the hydraulic head data.

![Considered power relation between K and \( \rho \), \( a = 1 \) and \( b = \ln(1e^{-4}) \).](image1a)

(a) Considered power relation between \( K \) and \( \rho \), \( a = 1 \) and \( b = \ln(1e^{-4}) \).

![Points of observed hydraulic head. + are points of observation. Constant head boundaries are shown by the grey shading.](image1b)

(b) Points of observed hydraulic head. + are points of observation. Constant head boundaries are shown by the grey shading.

Figure 1: Input data

The aquifer boundaries to the east and the west have been chosen to be impermeable boundaries, while the boundaries to the north and south have been chosen to be constant head boundaries, shown by the grey shading in 1.b. The recharge is assumed to be 50 mm per year and constant over the hole area.

Four examples are shown here. Firstly the noise free true resistivity field in figure 2.a was used. Secondly uniform distributed random noise with a standard deviation of 0.1 and 1.0 respectively were added to the true resistivity model (figure 2.b-c). Thirdly uniform distributed correlated noise with a variogram sill of 0.12 and a range of 350 meter was added to the reference resistivity model (figure 2.d).

3 RESULTS

The results of the four examples are shown in figure 3. The estimated parameters (i.e. \( a \) and \( b \) in equation 1) resulting from the inversion on noise-free resistivity data are precise to an accuracy of 4 significant numbers.
Figure 2: The considered resistivity models. a) True noise-free resistivity model. b) Uniform distributed random noise added to the resistivity model, std=0.1. c) Uniform distributed random noise added to the resistivity model, std=1.0. d) Uniform distributed correlated noise added to the resistivity model, variogram sill=0.1² and range=350 meter.

The estimated parameters resulting from the inversion on resistivity data after adding uniform distributed random noise, with a standard deviation of 0.10 is precise to an accuracy of 2 significant numbers. The estimated power relation is shown in figure 3 together with the true power relation. The difference can hardly be noticed.

Even when adding noise with a standard deviation of 1.0 (figure 2.c) the power relation can be estimated. The estimated parameters are now only accurate to 1 significant number and it is possible to distinguish between the estimated relation and the true relation on figure 3, but the estimate is still good.

Finally correlated noise was added to the resistivity data (figure 2.d) and also in this case it was possible to estimate the K-ρ relation (figure 3). Again the estimated parameters are only accurate to 1 significant number, but the estimate is still good.

4 CONCLUSION AND FUTURE WORK

We managed to estimate the K-ρ relation in all the examples we tested even when adding noise with a standard deviation of 1.0 and even when adding correlated noise. One reason for the success is probably the large resistivity/conductivity range used in these examples. More narrow
ranges of $p$ (and/or $K$) and different choices of the coefficients $a$ and $b$ should therefore be tested. Furthermore the effect of choosing a wrong relation should be tested (e.g. choosing a simple linear relation while the true relation is actually a power function). The minimum number of observed hydraulic head data necessary in order to estimate the $K$-$p$ relation together with the maximum allowable noise on resistivity data should be tested more thoroughly. And of course the method should be tested on real data.

5 REFERENCES

A computer program (MODFLOWP) for estimating parameters of a transient, three-dimensional ground-water flow model using nonlinear regression.

Geoelectric sounding for estimation aquifer hydraulic conductivity.
Ground Water 15(6), 420–425.

A hydrogeophysical model for relations between electrical and hydraulic properties of aquifers.