Finite-Difference Ground-Penetrating Radar Modeling in Frequency-Dependent Media

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SUMMARY

Realistic modeling of electromagnetic wave propagation in the radar frequency band requires a full solution of Maxwell's equations as well as a complete description of the material properties. We present a two-dimensional (2-D) finite-difference time-domain solution of Maxwell's equations that allows to account for the frequency dependence of the dielectric permittivity and electrical conductivity typical of many near-surface materials. In close analogy with the viscoacoustic case, the governing equations are obtained by assuming dielectric and conducting relaxation functions. The finite-difference solution is second-order accurate in time and fourth-order accurate in space, conditionally stable, and computationally only marginally more expensive than its standard equivalent without frequency-dependent material properties. The algorithm is applied to a realistic problem illustrating the respective damping effects of conductivity and frequency-dependent dielectricity.

INTRODUCTION

Time-domain electromagnetic methods in geophysics can be divided into relatively low frequency transient (TEM; < 10 kHz) and ground-penetrating radar methods (GPR; > 10 MHz). For the solution of general time-dependent electromagnetic problems, numerical techniques are required (Kunz and Luebbers, 1993; Goodman, 1994; Bergmann et al., 1996). Such modeling techniques have commonly been implemented with frequency-independent electromagnetic parameters. This is adequate for ideal dielectric materials, but may be inappropriate for many geophysical materials, where electromagnetic fields are commonly characterized by frequency-dependent velocities and attenuation properties (Turner and Siggins, 1994).

Attenuation in the radar frequency domain depends on the dielectric permittivity and the conductivity of the medium, which may be described as complex frequency-dependent quantities. Important attenuating processes include ionic conductivity, the Maxwell-Wagner effect and bound and free water relaxation processes (Hasted, 1973). These processes can be modeled by single Debye relaxation peaks. In many geological materials, the combined effects of the various attenuation mechanisms in the radar frequency band can be approximated by a linear function of frequency (Jonscher, 1977; Turner and Siggins, 1994), corresponding to a constant quality factor Q . A realistic description of these combined attenuation processes can be obtained through a generalized Debye or Zener model, which comprises a large number of Debye peaks. In close analogy with the viscoacoustic case, this behavior can be described by several Debye-type relaxation peaks (Robertsson et al., 1994). The response of free charges at high frequencies may produce an out-of-phase component of σ , which will contribute to the effective permittivity (Turner and Siggins, 1994). This component is related to an electric current that is out-of-phase with respect to the electric field. Such a relaxation function can be described by a Kelvin-Voigt type model.

Carcione (1996) presented a pseudospectral method that includes frequency dependence of dielectric permittivity and electrical conductivity. Here, we present a finite-difference (FD) method, which uses a similar approach to describing the frequency-dependence of the constitutive parameters to that of Carcione (1996). However, we chose a time rather than a frequency domain approach because of its conceptual simplicity and high portability. Moreover, compared to the pseudospectral method, complicated structures and sharp material boundaries are generally easier to implement in FD solutions. A similar FD method was recently presented by Xu and McMechan (1997). However, they neither considered frequency dependence of the electrical conductivity, nor did they perform a detailed analysis of the numerical properties of their method.

We apply this algorithm to simulate an amplitude versus offset (AVO) application of ground-penetrating radar, illustrating the separate effects of conductive and dielectric attenuation of electromagnetic waves.

MAXWELL'S VECTOR AND TRANSVERSE ELECTRIC EQUATIONS

Maxwell's equations for an isotropic heterogeneous medium are:

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}, \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad (1)$$

where $\vec{H}$ is the magnetic field, $\vec{D}$ the dipole moment density, $\vec{J}$ the conduction current density, $\vec{E}$ the electric field and $\vec{B}$ the magnetic flux. Attenuation and dispersion processes in the radar frequency domain depend on the dielectric permittivity ε and electrical conductivity σ , whereas the magnetic permeability μ is essentially frequency-independent (e.g., Turner and Siggins, 1994). To include the frequency-dependence of permittivity and conductivity, the following generalized constitutive relations may be used (Carcione, 1996; Bergmann et al., 1997):

$$\vec{D} = \frac{\partial \varepsilon}{\partial t} * \vec{E}, \quad \vec{B} = \mu \vec{H}, \quad \vec{J} = \frac{\partial \sigma}{\partial t} * \vec{E}. \quad (2)$$

Here, the multiplications in the constitutive relations for frequency-independent materials have been replaced by convolutions in time and $\varepsilon = \varepsilon(t)$ and $\sigma = \sigma(t)$ have become time- and thus frequency-dependent. The convolutions describe the memory effect of the relaxation functions, whereas the partial derivatives of ε and σ describe the time-dependent behavior. Note that the time derivatives are necessary to compensate for the integral effect of the convolutions.

The components of an electromagnetic wavefield, propagating in a 2-D (x, z) medium, can be extracted from Maxwell's equations (1) and (2), so that both a transverse magnetic (TM) solution (field components E_x , E_z and H_y) and a the transverse electric (TE) solution (field components E_y , H_x and H_z), which are decoupled from each other, are obtained. The isotropic TE equations are:

$$\frac{\partial \varepsilon}{\partial t} * \frac{\partial E_y}{\partial t} = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \frac{\partial \sigma}{\partial t} * E_y, \quad (3)$$

$$\frac{\partial H_x}{\partial z} = \frac{1}{\mu} \left(\frac{\partial E_y}{\partial z} \right), \quad \frac{\partial H_z}{\partial x} = \frac{1}{\mu} \left(-\frac{\partial E_y}{\partial x} \right). \quad (4)$$

Analogous to viscoacoustic theory, a description of the dielectric relaxation processes can be obtained by means of a generalized Debye-type model. The dielectric permittivity function may therefore be modeled through an array of L relaxation mechanisms, whereas the relaxation function of the conductivity may be described by a Kelvin-Voigt type model (Carcione, 1996; Bergmann et al., 1997):

$$\varepsilon(t) = \varepsilon_s \left[1 - \frac{1}{L} \sum_{l=1}^L \left(1 - \frac{\tau_{El}}{\tau_{Dl}} \right) e^{-t/\tau_{Dl}} \right] \mathcal{H}(t), \quad \sigma(t) = \sigma_s [\mathcal{H}(t) + \tau_\sigma \delta(t)], \quad (5)$$

where ε_s is the static permittivity, τ_{El} and τ_{Dl} are $\vec{E}$ - and $\vec{D}$ -field relaxation times ($\tau_{El} \leq \tau_{Dl}$ for the l th mechanism), respectively, $\mathcal{H}(t)$ is the Heaviside function, σ_s is the static conductivity, τ_σ is a relaxation time and $\delta(t)$ is the Dirac function.

In analogy with the viscoacoustic equivalent (e.g., Robertsson et al., 1994), it is possible to eliminate the computationally cumbersome convolution in equation (3) by introducing so-called memory variables m , so that equation (3) is replaced by a set of two new differential equations. For clarity of illustration, we only show the case of one dielectric relaxation mechanism ($L = 1$):

$$\varepsilon_\infty^e \frac{\partial E_y}{\partial t} = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma_\infty^e E_y - \varepsilon_s m, \quad \frac{\partial m}{\partial t} = -\frac{1}{\tau_D} m - \frac{1}{\tau_D^2} \left(1 - \frac{\tau_E}{\tau_D} \right) E_y \quad (6)$$

with

$$\varepsilon_\infty^e = \varepsilon_s \frac{\tau_E}{\tau_D} + \sigma_s \tau_\sigma \quad \text{and} \quad \sigma_\infty^e = \sigma_s + \frac{\varepsilon_s}{\tau_D} \left(1 - \frac{\tau_E}{\tau_D} \right) \quad (7)$$

as the effective optical permittivity and effective optical conductivity, respectively.

Equations (4) and (6) are solved using an $O(2,4)$ accurate FD leap-frog scheme with centered difference approximations in space (Bergmann et al., 1997). Bergmann et al. (1997) detailed explored and documented the numerical properties of this FD scheme, such as stability, numerical dispersion and convergence and compared these properties with alternative FD approximations. The proposed scheme

	medium	$\epsilon_s (\epsilon_0)$	$\mu (\mu_0)$	$\sigma_s (\text{mS/m})$	$\tau_D (\text{ns})$	$\tau_E (\text{ns})$
case 1	dry sand	5	1	1.6	0	0
	saturated sand	24	1	50.0	0	0
case 2	dry sand	5	1	0.3	0.200	0.126
	saturated sand	28	1	0.5	0.633	0.040

Table 1: Material parameters for relatively dry sand and water saturated sand for the two different cases of wave attenuation, examined in Figure 1.

was found to have very favourable numerical properties while it is only less more expensive than a corresponding FD scheme for frequency-independent constitutive parameters.

NUMERICAL EXAMPLE

Attenuation of GPR waves could be caused either by high static conductivity or by significant dielectric frequency dependence. A set of simulations is concerned with potential applications of amplitude versus offset measurements (AVO) in GPR investigations. We consider a plane two-layered medium, consisting of a relatively dry sandy layer overlying a semi-infinite water-saturated sand medium (parameters described in Table 1). This model has obvious applications in ground-water studies. The AVO measurements are based on common shot gather simulations. The 2-D line source signal is a Ricker wavelet with a center frequency of 100 MHz. Two different wave attenuation processes for the medium is considered: For case 1, the attenuation of the layers is governed by pure electrical conductivity, whereas for case 2, the attenuation is mostly governed by a dielectric relaxation process. The moderate conductivities could be caused by moderate levels of salinity or pollutants in the water. Therefore, these simulations are relevant to studies in coastal (ground-water exploration) and agricultural regions.

For electromagnetic waves in conducting media (case 1) the quality factor can be expressed as:

$$\frac{1}{Q(\omega)} = \frac{\sigma_s}{\omega \epsilon_s}, \quad (8)$$

whereas for attenuation caused by conductivity and a single Debye peak (case 2) the quality factor is given by (Bergmann et al., 1997):

$$Q(\omega) = \frac{1 + \omega^2 \tau_D \tau_E}{\omega (\tau_D - \tau_E) + \frac{\sigma_s}{\omega \epsilon_s} (1 + \omega^2 \tau_D^2)} \quad (9)$$

The material properties for the two types of attenuation (i.e. cases 1 and 2) are shown in Table 1. They are chosen so that the phase velocities and quality factors are the same for the corresponding pairs of media at the center frequency of 100 MHz: $c_p \approx 0.447 c_0$ and $Q \approx 17.5$ for the relatively dry sand and $c_p \approx 0.200 c_0$ and $Q \approx 2.7$ for the saturated sand.

The computational results for the two different attenuation processes are shown in Figure 1, in which the reflection coefficients (absolute values) as functions of angle of incidence are plotted for three different frequencies (50, 100, and 150 MHz). The reflection coefficient is defined as $R = E_r/E_i$, where E_r is the amplitude of the reflected E field and E_i the amplitude of the incident E field. The curves were obtained by correcting each trace for the effects of cylindrical divergence as well as the effects of attenuation. Therefore, the reflection coefficients in Figures 1a and b should represent the effects of pure dispersion. All reflection coefficients at normal incidence have been verified by an analytical solution.

Although Q has the same value for the center frequency in both cases, considerable variations in the curves can be observed. These differences reflect the differing intrinsic dispersions caused by the different attenuation processes [see equations (8) and (9)]. Despite their simplicity, these examples illustrate both the potential and the limitations of AVO-type measurements for constraining the physical properties of the shallow subsurface using GPR data. Therefore, AVO measurements in GPR may be useful for detecting and describing different frequency behavior of materials.

CONCLUSIONS

We have presented a finite-difference time-domain technique for modeling electromagnetic propagation in frequency-dependent dielectric and and conducting media using a convolutional formulation of Maxwell's equations. By analogy with viscoacoustic theory, time-dependent dielectric and conducting relaxation functions were expressed as an array of Zener elements and a single Kelvin-Voigt model,

Figure 1: Reflection coefficients (absolute values) of a relatively dry sand/saturated sand interface for different frequencies. (a) attenuation governed by pure electrical conductivity (Case 1 in Table 1); (b) attenuation mostly governed by a dielectric relaxation process (Case 2 in Table 1). Solid: $f = 50$ MHz, dashed: $f = 100$ Hz, dash-dotted: $f = 150$ MHz.

respectively. The computationally cumbersome convolutions were replaced by memory variables. The finite-difference solution has been implemented using a staggered leap-frog scheme of second-order accuracy in time and fourth-order accuracy in space.

Simulation of two-dimensional subsurface AVO models illustrated how both electrical conductivity and dielectric relaxation processes introduce frequency-dependent attenuation and dispersion of the electromagnetic wavefield. To demonstrate the respective effects of electrical conductivity and dielectric relaxation, we compared the results of simulations in which the same Q at center frequency was obtained through the two different attenuation processes. We found that the different dispersion mechanisms cause significant differences, which may be relevant to AVO, tomographic or stratigraphic investigations using ground-penetrating radar.

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