Finite-Difference Modeling of Ground-Penetrating Radar Data: The Zero-Offset Approximation

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SUMMARY
Most ground-penetrating radar surveys are acquired in quasi zero-offset mode, i.e. source-receiver offsets are small compared to the target depth. The correct way to realistically model such an experiment would be to compute a corresponding sequence of common-offset radargrams. In finite-difference time domain (FDTD) modeling this approach would be excessively expensive and hence is substituted by initiating a downward travelling plane wave at the earth's surface. However, this method of zero-offset modeling is only realistic for horizontally layered, one-dimensional media, and thus is unsatisfactory in most realistic situations and defeats the very purpose of finite-difference modeling. Therefore, we propose an alternative approach based on the “exploding reflector” concept. This approximation is more realistic, even in highly complex media, and is only marginally more expensive than the plane wave method.

PLANE WAVE VERSUS EXPLODING RELECTOR APPROACH
In the recent past, forward modeling of ground-penetrating radar (GPR) has seen tremendous theoretical and methodological progress (e.g., Goodman, 1994; Wang and Tripp, 1996; Carcione, 1996; Xu and McMechan, 1997; Bergmann et al., 1997). To date, however, there have been few, if any, successful attempts to model actual GPR surveys. One major reason for this is that the most common mode of GPR data acquisition is a common-offset, “echo-sounding” fashion (Annan, 1992). Due to the small source-receiver offset with respect to target depth, this allows to approximate such radargrams as zero-offset sections. Particularly for FDTD modeling, a one-to-one adaption of this experimental configuration is impractical, because for each source-receiver configuration the full electromagnetic wavefield would have to be evaluated throughout the entire subsurface model. Currently, the most common computational shortcut to model zero-offset GPR data is to initiate a downward propagating plane wave at the earth’s surface (e.g., Carcione, 1996; Xu and McMechan, 1997). This approach faithfully models specular reflections from horizontally layered, i.e. one-dimensional, subsurface, but does not correctly predict the travel time and amplitude response of more complex, two- or three-dimensional, structures (Table 1). Given that a major motivation for FDTD modeling is its ability to properly account for complex diffraction, scattering and interference phenomena, the plane wave approach must be considered as unsatisfactory.

A possible remedy is the exploding reflector concept introduced in the 1970s to model and migrate CMP-stacked, i.e. zero-offset, reflection seismic data (e.g., Claerbout and Doherty, 1972; Loewenthal et al., 1976; Claerbout, 1985). The exploding reflector concept places a point source at each grid point of the discretised subsurface model where a
Table 1: Comparison of plane wave and exploding reflector concepts.

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<th>Plane wave approximation</th>
<th>Exploding reflector concept</th>
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<tr>
<td>pro</td>
<td>• easy to implement</td>
<td>• predicts relative amplitudes and travel times of primary reflections and diffractions</td>
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<td>• predicts phases of multiple reflections/diffractions</td>
<td>• source directivity can be accounted for</td>
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<tr>
<td>contra</td>
<td>• not valid for two- and three-dimensional structures due to invalid reflection/diffraction travel time and amplitude modeling</td>
<td>• amplitudes and travel times of multiple reflections/diffractions are incorrect</td>
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<td>• does not account for geometric wave front spreading</td>
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change in the material properties occurs. All sources are scaled by the impedance contrast prevailing at their location and are then initiated simultaneously. The zero-offset section is obtained by recording the resulting one way wavefield at the surface and multiplying its travel time by a factor of two, or, alternatively, using half the actual velocities. This approach correctly predicts the travel times and relative amplitudes of primary reflections, diffractions and interference phenomena (Table 1). In contrast to the true two way travel near-offset section, reflection/transmission losses for deeper parts of the model are systematically underestimated. Therefore, the exploding reflector method is not a dynamic, but kinematic equivalent of a zero-offset section. Another obvious shortcoming of this method is that amplitudes, phases and travel times of multiple reflections and diffractions are not modeled correctly (Claerbout, 1985). Still, we consider the exploding reflector method to be a much more attractive zero-offset approximation for GPR modeling than the commonly used plane wave approximation. In the following, we calculate zero-offset radargrams for a simple two-dimensional subsurface model using both plane wave and the exploding reflector method, and compare and discuss the respective results.

MODELING ZERO-OFFSET GPR DATA

To compare the plane wave and the exploding reflector methods as zero-offset approximations, we calculated corresponding FDTD radargrams for a two-dimensional subsurface model consisting of a simple wedge structure (Figure 1). The material properties are $\varepsilon = 4\varepsilon_0$ and $\mu = \mu_0$ (limestone) and $\varepsilon = 12\varepsilon_0$ and $\mu = \mu_0$ (gravel sand), with $\varepsilon_0$ and $\mu_0$ being the dielectric permittivity and magnetic permeability in vacuum, respectively. Ray paths for both zero-offset approximations are schematically illustrated in Figure 1. It is obvious that the plane wave approximation fails to correctly model the travel times of the diffracted part of the zero-offset wavefield.

The resulting zero-offset radargrams ($E_y$ component) are shown in Figure 2. The FDTD algorithm used is based on a $O(2,4)$-accurate staggered grid approximation of Maxwell’s equations (Bergmann et al., 1996). The source signal was a Ricker wavelet with a center frequency of 200 MHz. We did not model the air-ground interface in
Figure 1: 2-D sub-surface model with a limestone block in gravel sand. Material properties: $\varepsilon = 4\varepsilon_0$ and $\mu = \mu_0$ for the limestone and $\varepsilon = 12\varepsilon_0$ and $\mu = \mu_0$ for the gravel sand. Dashed arrows denote ray paths: (a) plane wave approximation, and (b) exploding reflector concept.

In order to avoid unnecessary complications to the synthetic sections due to the direct air wave and multiple reflections. In both cases, reflection and diffraction events for the wedge structure are clearly visible. Additionally, the plane wave radargram contains the direct ground wave at $t = 0$, which is not accounted for by using the exploding reflector calculation. Both radargrams are normalized and displayed with a constant gain. As mentioned above, the exploding reflector solution gives a one way propagation solution. The travel times of the exploding reflector response were thus multiplied by a factor of 2 to emulate a “two way travel” radargram for a comparison with the plane wave approximation (Figure 2). This conversion to two way travel time results in the wavelet being stretched by a factor of 2 compared to the plane wave solution. As predicted by the ray paths, the plane wave solution models the events from the horizontal reflector correctly. However, significant amplitude and travel time differences for the diffraction from the wedge are evident. The diffraction events of the plane wave calculation have only about half the moveout predicted by the zero-offset concept, and hence would not collapse when migrated with the proper velocity.

Figure 2: Radar sections ($E_y$ component) using the model shown in Figure 1 for the plane wave (left) and exploding reflector (right) calculations. Trace spacing: 0.06 m. The one way travel time radargram of the exploding reflector model was multiplied by a factor of 2 to get a “two way time” radargram.
CONCLUSIONS

We have compared two zero-offset approximations for FDTD ground-penetrating radar forward modeling: the plane wave and exploding reflector approaches. The commonly used plane wave approximation leads to incorrect results, as soon as the model is no longer one-dimensional. In contrast, the exploding reflector method reasonably models amplitudes, phases and travel times of primary reflections and diffractions, even in complex media. The computational effort is similar for both methods. Moreover, the exploding reflector approach also offers the perspective of incorporating source directivity characteristics into the zero-offset FDTD modeling of ground-penetrating radar data.

REFERENCES


