FAST TRANSIENT ELECTROMAGNETICS:
REAL DEVICE MODELING AND INFORMATION ANALYSIS

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INTRODUCTION

The use of methods of alternating electromagnetic field for studying of the near-borehole space in dielectric logging and subsurface layers in shallow depth soundings deals with account of joint influence of conductivity and displacement currents. From physical point of view, in heterogeneous conductive medium there is a propagation of electromagnetic wave including scattering and absorbing effects. In comparison with harmonic process the transient one has been investigated episodically and, as a rule, for estimations in standard (Kaufman, 1972; Mahmoud, 1979; Wait, 1981; Anderson, 1989; Kabanikhin, 1994). By the late 90s great interest to fast transient geoelectrics was created by development of registration technology for very early time interval (10 nsec – 1 mcsec) and 2-5 nsec step. Unfortunately, the results of appropriate researches have been published in the brief abstracts and reports of limited access (Pellerin, 1994, 1995; Kim, 1996).

The theoretical substantiation of this method use is generally based on numerical analysis in one-dimensional media. Stable convolution solution was obtained for two homogeneous halfspace (Goldman, 1996). The solution for piecewise uniform layered model and slow alternating waveforms has been developed by use of spectral method (Kim, Lee, 1996). We employed hybrid numerical-analytical algorithm to compute response for the layered-continuous halfspace and surface distributed magnetic antenna. By regular perturbation method the sensitivity functions were derived. Kinematic and dynamic characteristics of profile signals were considered.

FORWARD MODELING

Let the non-magnetic and non-dispersive halfspace have one-dimension distribution of conductivity and dielectric permeability throughout the depth. Functions σ(z) and ε(z) are determined on interval (-Z, Z) and may include finite jumps (or jumps of 1st derivatives) in internal points. Magnetic current \( j_m^{inc} \) with effective moment \( M(r,z,t) \) is a source of incident field, which has radial structure and vertical orientation. Electromagnetic field with due regard of displacement currents satisfies to full Maxwell equation system. There is magnetic mode only at geometric conditions of medium and incident source.

Magnetic current is defined as the product of normalized functions depended separately on spatial and time domain variables

\[
j^r(0,0,j^\infty) = \mu_0 M \cdot \frac{d}{dt} q(t),
\]

where \( M \) – amplitude of current density, \( p(r) \) – radial distribution of current in interval \([0,r_0]\), under condition \( \int_0^{r_0} p(r) r dr = 1 \), \( \delta(z) \) – Dirac function, \( q(t) \) – function of alternating in...
time interval $(0,t_0)$, defines “switch on” regime $(q(0) = 0, \ q(t_0) = 1)$. The value of $t_0$ can be equal infinity.

Field equation is hyperbolic one (telegraph equation). If time domain is interval $(0, T)$ boundaries of spatial domain can be determined exactly

$$Z = T / \sqrt{\mu_0 \varepsilon_0 \varepsilon_{\text{min}}} , \quad R = r_0 + T / \sqrt{\mu_0 \varepsilon_0 \varepsilon_{\text{min}}}$$

Axial problem is reduced to series of initial value boundary problems for Fourier-Bessel transform image. Type of measured signal (induction or magnetic sensor) defines the right part of differential

$$\left\{ \begin{array}{l}
(\varepsilon \varepsilon_0 - \sigma \varepsilon_0 \mu_0 - \frac{\lambda^2}{\mu_0}) u = \delta(z) \int_0^\infty p(r) J_0(\lambda r) r dr \frac{\partial}{\partial t} q(t) \\
u_{|z=\pm Z} = 0 \\
u_{|t=0} = 0, \quad \frac{\partial}{\partial t} u_{|t=0} = 0, \quad z \in [-Z, Z], \quad t \in (0, T)
\end{array} \right.$$  

To solve initial value boundary problem for fixed spatial frequency we employ the finite difference method of the 2nd order of approximation. The use of time integral transform deals with serious difficulties at transition from frequency domain. Convolution algorithms like FFT are unstable and have great computational errors.

Spatial distribution function and time regime of magnetic current are defined near to dependences for real equipment. The type and smoothness properties limit bounds of parameter domains and stability of numerical scheme. We use exponential “cap”-function for spatial source distribution

$$p(r, r_0) = \begin{cases} 
\frac{C}{r_0^2} \frac{1}{e^{1-(r/r_0)^2}}, & 0 \leq r < r_0, \\
0, & r_0 \leq r
\end{cases}, \quad C = \frac{1}{\pi} (e^{-1} - E_1(1))^{-1} = 2.143565,$$

and two dependences for time impulse rule

a) Limited interval

$$q(t) = \begin{cases} 
0 & t \leq 0, \\
0.5 \sin \pi (t/t_0 - 0.5) & t \in (0, t_0), \\
1 & t \geq t_0.
\end{cases}$$

b) Unlimited interval

$$q(t) = \begin{cases} 
0 & t \leq 0, \\
1 - e^{-t/a^2} & t \geq 0.
\end{cases}$$

As a sample, the calculation of signals is considered for subsurface layer of finite thickness.

Model: $\varepsilon_1 = 5, \ p_1 = 500$ Ohm.m, $h_1 = 1.5$ m, $\varepsilon_2 = 3, \ p_2 = 1000$ Ohm.m
Geoelectrical model is generated under assumption of low-conductive cross-section like “dry earth” model without water-saturated soil layer. Source is placed at domain \((r,z) \in [0,0.25] \times [-0.1,0.1] \) (m). Time duration of impulse is equal to 3 nsec. Response is measured by a set of vertical induction receivers distanced from source antenna on 5-15 m with 0.5 m step. There are a few waveforms. Waveform \(A\) is a diverging package of waves in air and layer. Waveform \(B\) is generated by wave reflected from the bottom of layer and propagates with velocity of subsurface layer. Waveform \(C\) corresponds to the initially reflected response and propagates as refracted wave. Besides this, there are multiple reflections of decreased amplitudes (waveforms \(D\)).

**INFORMATION ANALYSIS**

Sensitivity functions on medium and equipment configuration parameters are means for optimal design of sounding array. Gradient of residual function between observed and synthetic data is a necessary evaluation for linearized inversion (McGillivray, Oldenburg; 1990).

Let \(\bar{u}(z,t)\) be a solution of initial value boundary problem, which conjugated to general one of LaGrange’s identity. Any linear transformation of response \(u(z,t)\) can be represented as functional

\[
J(z,t) = \int_{\Omega} \int_{-0}^{2} p(z',t';z,t) u(z',t') \, dz' \, dt',
\]

where \(p\) is the right part of conjugate equation \(L \bar{u} = p\), having sense of device characteristic or measurement. It is valid follow formulae

\[
J_f = (f, \bar{u}) = J_p = (p,u).
\]

Source variation in general problem defines functional variation

\[
\delta J_f = (\delta f, \bar{u}),
\]

where \(\bar{u}\) is weight function (or information value function). If there are small regular perturbations of electromagnetic properties (Born approximation condition), functional variation is determined by expression

\[
\delta J_f = -\mu_0(\varepsilon, \frac{\partial^2}{\partial t^2} \varepsilon + \frac{\partial}{\partial t} \delta \sigma, \bar{u}).
\]

To evaluate sensitivity functions, functional \(J\) is signal \(u\). This statement formulates conjugate value boundary problem

\[
\begin{cases}
(\varepsilon \varepsilon_0 \frac{\partial^2}{\partial t^2} \varepsilon - \delta \frac{\partial}{\partial t} \varepsilon - \frac{\delta^2}{\partial z^2} + \frac{\lambda^2}{\mu_0}) \bar{u} = \delta(z' - z) \delta(t' - t), \\
\bar{u}_{z=\pm Z} = 0, \\
\bar{u}_{t=0} = 0, \\
\bar{u}_{t=T} = 0,
\end{cases}
\]

\(z', z \in [-Z,Z], \ t', t \in [0,T].\)

As this statement takes place, \(\bar{u}\) is expressed by Green function

\[
\bar{u}(z', t'; z, t) = G(z, T-t; z', T-t'),
\]

which is a fundamental solution of general problem.

In a case of piecewise uniform distributions on conductivity and dielectric permeability the sensitivity functions are gradient-vectors, which components are determined by

\[
\frac{\partial u(z,t)}{\partial \varepsilon_j} = -\mu_0 \varepsilon_0 \left( \frac{\partial^2}{\partial z^2} u, \bar{u} \right)_{(0,T)\times(Z_j,Z_{j+1})}, \quad \frac{\partial u(z,t)}{\partial \sigma_j} = -\mu_0 \left( \frac{\partial}{\partial t} u, \bar{u} \right)_{(0,T)\times(Z_j,Z_{j+1})}.
\]

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\[
\frac{\partial u(z,t)}{\partial z_j} = \mu_0 \varepsilon_0 (\varepsilon_{j+1} - \varepsilon_j) \left( \frac{\partial^2}{\partial t^2} u(z_j), \bar{u}(z_j) \right)_{(0,T)}, \quad \frac{\partial u(z,t)}{\partial z_i} = -\mu_0 \left( \frac{\partial}{\partial t} u(z_i), \bar{u}(z_i) \right)_{(0,T)}.
\]

If boundaries of dielectric and conductivity layers coincide, the boundaries of appropriate boundaries are combined. Integral representation of sensitivity functions is correct for calculation with finite precision and it’s effective in comparison with difference method.

**CONCLUSIONS**

The one-dimension problem of fast transient electromagnetics with account of displacement currents, spatial geometry of vertical magnetic antenna and impulse form has been considered. Stable hybrid algorithm has been developed. It is used for computing of signal directly in time domain including propagation, intermediate and diffusive stages. Information functionals of responses are reduced for equipment design and inversion.

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