BUILDING COMPLEX VELOCITY MODELS USING AN ADAPTIVE TETRAHEDRAL MESH

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We use an irregular tetrahedral mesh to represent 3-D velocity models. The mesh is generated by individually inserting data points into the model with refinement of the mesh to include the newly added point. When isolated data points are to be interpolated, a Delauney tessellation is often a good choice since it partitions the model into tetrahedra connecting nearest-neighbor data points by its empty-circumsphere criterion. We use Delauney tessellation to import isolated velocity picks to form a 3-D velocity field and to import isolated surface picks to form a simply connected 3-D surface (Wiggins et. al, 1993). Velocity data comes to the explorationist in small parts: first, velocity picks for a shallow layer; then, a shallow horizon; then a deeper layer velocities and surfaces. A modeling system must represent these parts **and** combine them into a single model.

When velocity fields are combined with and truncated by surfaces, and when closely approaching or intersecting surfaces are to be combined, then simple introduction of points by Delauney tessellation does not succeed. Some features of the original velocity field are considered essential; i.e., they must be preserved during the combination of the individual model parts. For example, tetrahedral faces that define discontinuities in the velocity or its first derivative cannot be changed without changing the represented velocity. Similarly, faces defining a surface cannot be removed without creating a hole or deformation in the surface.

Figure 1(a) shows a vertical slice through a velocity model constructed from rms-velocity and time picks. The time-velocity picks were converted to interval-velocity and depth points using Dix's formula. The resulting model has vertically block-constant velocities but contains lateral gradients as required to interpolate horizontally. Figure 1(b) shows a surface occupying the same space. Then, the surface points were added to the velocity model with each surface point being assigned the velocity at that point in the original velocity model to produce the model in Figure 2(a). Even though none of the original points has been modified and all the added surface points have their velocity assigned from the original model, the velocity has changed at other places in the model because of rearrangement of the tetrahedral faces of the model that define the interpoint velocity interpolation.

One approach to this problem that is created when merging models is to keep the parts of the model in separate data structures with some indexing system to indicate how they relate. Alternately, we find that forming a coherent single model with integral topological information is a considerable advantage when operating on the model in editing it, tracing rays, and in interrogating the model for use in other applications. Therefore, we had to find methods for merging parts of models to form a more complex model while preserving all the essential parts of each parent model.

Typically, all the vertices of each model are essential; if not, they can be deleted before the merge begins. All tetrahedral faces lying on a surface are essential to represent the surface. Some, but not all, of the tetrahedral faces embedded in a smoothly varying velocity field are essential. These existing parts are not sufficient, however, for the construction of a merged tetrahedral model. Where the tetrahedral edges of one source model intersect the faces of the other,

new vertices are required in the merged tessellation. After these intersection vertices are added, the essential faces of the original models can be represented in the merged model by a collection of smaller faces connecting the original vertices and the new vertices created at the face intersection points.

The composite model does not require any vertices other than the data points and the intersection points. Any desired face necessarily will connect neighboring points. However, it may be easier to arrive at the desired model by adding additional points. For example, a composite model can be built by Delauney tessellation of all the data points and intersection points of both parent models and then it can be searched for the presence of all essential faces. If a given essential face does not exist, we can try to force its appearance in the model by adding a new control point at its center. In practice, this procedure may converge slowly.

In adding a point to an already tessellated model using the Delauney criterion, it is necessary to form a cavity in the model by removing all the faces lying between the new point and any other three points to which it should connect to form the new tessellation. If any of these faces is essential, the insertion creates an undesirable change in the model. This problem can be avoided by using constrained Delauney tessellation. Constrained Delauney tessellation has been studied and found useful in other areas of computer-aided geometric design (Baker, 1989; Cline and Renka, 1990). Using the constrained tessellation when adding a point, all the faces are removed according to the Delauney criterion except the essential ones and the resulting cavity is redivided to create circumspheres that contain only the added point and three of the cavity wall points.

Figure 2(b) shows a merge of the models in Figures 1(a) and 1(b) with proper addition of the intersection points. Now the essential velocity-defining faces are preserved and the velocity field is unchanged.

Figures 3 and 4 show two surfaces of a salt structure in the Gulf of Mexico. The surfaces were interpreted from 3-D poststack depth migrations. Each surface is embedded in a 3-D tetrahedral mesh and is made up of the data points and the tetrahedral faces that connect them along the desired surface. The top-of-salt surface (Figure 3) was interpreted from a first depth migration and has a vertical skirt added to define a region of salt velocity that we used in a second depth migration. The bottom-of-salt surface (Figure 4) was interpreted from the second depth migration. Figure 5 shows the model after the surfaces have been merged. The region inside this composite surface was set to salt velocity and the exterior was set to a sediment velocity field. The resulting model was used in a third depth migration to visualize the sediments below the salt. Although neither surface had vertices where it intersected the other, the merge algorithm calculated the intersection points and created vertices there.

The necessity for merging algorithms was created by our insistence on having a single topologically coherent model. Though the merging operations require repeated analysis of the connectivity of the model parts, the compute time is approximately linear with model size because the model carries complete topologic information. Thus, the inclusion of adequate topologic information in the model is required to enable the efficient merging just as it is required to enable efficient geophysical operations on the model.

References

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(a)

(b)

Figure 1. (a) A vertical slice through a 3-D velocity model. The velocity is rendered on the vertical panel as a grey scale. Since the model was constructed from V_{rms} -time datapoints using Dix's formula, the regions of the model contain vertically block-constant velocity. (b) A surface occupying the same space as the velocity field, but represented in a separate model.

(a)

(b)

Figure 2. (a) The surface of 1(b) inserted into velocity model 1(a) without care to preserve the faces present in the original velocity model. Although the model is the same at each original velocity point and has been kept fixed at the original values at the surface insertion points, the interpolation has been altered in between the data points. The original faces of model 1(a) are essential to preserve the original interpolation. (b) A model formed by merging models 1(a) and 1(b) with preservation of the faces of 1(a). Now the velocity is exactly the same as in 1(a).

Figure 3. The top of a salt mass from the Gulf of Mexico.

Figure 4. The bottom of the salt mass. This surface and the one in Figure 3 intersect at many places around their edges and extend past the intersection line.

Figure 5. The surfaces of Figures 3 and 4 after they have been merged by a constrained Delaunay tessellation and the overlapping parts of the surfaces have been trimmed off with an interactive editing program.