A method of prestack velocity inversion of seismic data using plane wave field downward continuation equation has been proposed by Wang [1990, 1993]. Some inversion velocity profiles obtained from prestack marine data have been published in the same paper [Wang, 1990, 1993]. However, stability, a key problem with the inversion method (recurrent, stripping), has not been discussed. In this note, an initial analysis of the stability problem is introduced.

In brief, a theoretical background of the inversion method is a formulation for downward continuation of plane wavefield in layered medium as the following:

\[
\begin{pmatrix}
U_{n+1}(\omega, p) \\
D_{n+1}(\omega, p)
\end{pmatrix}
= \frac{1}{t(n, p)} \begin{pmatrix}
1 & -r(n, p) e^{-i\omega \Delta t(n, p)} \\
-r(n, p) e^{i\omega \Delta t(n, p)} & e^{-i\omega \Delta t(n, p)}
\end{pmatrix}
\begin{pmatrix}
U_n(\omega, p) \\
D_n(\omega, p)
\end{pmatrix}
\]

(1)

where \(\omega\) is a circular frequency, \(i = \sqrt{-1}\), \(p = \sin(j(z))/v(z)\), is a Snell character of plane wavefield, which is constant during propagation in the layered medium, \(j(z)\) is an incident angle, \(v(z)\) is a velocity function depending on depth, which will be obtained using the inversion method, and

\[
\begin{align*}
    r(n, p) &= \frac{(I_{n-1}(p) - I_n(p))}{(I_{n-1}(p) + I_n(p))} \\
    t(n, p) &= 2(I_{n-1}(p) + I_n(p)) \\
    I_n(p) &= \rho_n \frac{d_n}{\rho_n} \\
    \Delta t(n, p) &= \frac{\rho_n}{\rho_n} (\frac{d_n}{\rho_n} + \frac{1}{v(z)})
\end{align*}
\]

\(\rho_n\) is the density of the \(n\)-th layer, \(d_n\) is the thickness of the \(n\)-th layer, \(U_n(\omega, p)\) is the up-coming wavefield at top of the layer, \(D_n(\omega, p)\) is the down-going wavefield at top of the layer. If denote \(U_n'(\omega, p)\) and \(D_n'(\omega, p)\) as wavefield at bottom of the layer, according to Claerbort principle, we obtain

\[
\begin{align*}
    U_n'(\omega, p) &= \frac{U_n(\omega, p)}{D_n(\omega, p)} \\
    D_n'(\omega, p) &= \frac{D_n(\omega, p)}{r(n, p) e^{-i\omega \Delta t(n, p)}}
\end{align*}
\]

(2)

Since computational steps of the velocity inversion method based on Equ (1) and (2) have been described by Wang [1990, 1993], we do not repeat those steps in this note. That algorithm is complex, so the stability analysis is complex too. The initial analysis is just focused on the Equ (1). For simplicity, Equ (1) can be rewritten as

\[
x_{n+1} = A_n x_n
\]

(3)

where

\[
x_n = \begin{pmatrix}
U_n(\omega, p) \\
D_n(\omega, p)
\end{pmatrix}, \quad A_n = \begin{pmatrix}
e^{i\omega \Delta t(n, p)} & -r(n, p) e^{-i\omega \Delta t(n, p)} \\
-r(n, p) e^{i\omega \Delta t(n, p)} & e^{-i\omega \Delta t(n, p)}
\end{pmatrix}
\]

Based on matrix theory, the stability analysis of Equ (1) or (3) is to discuss the error \(\Delta x_n\) after using \(n\) steps of Equ (1) or (3) when the \(x_1\) has error \(\Delta x_1\). Easily obtained.
\[ \Delta x_{n+1} = A_n x_n \]
so that
\[ \| \Delta x_{n+1} \| \leq \| A_n \| \| x_n \| \]
and
\[ \| x_n \| \leq \max(1, \lambda, \lambda^2, \lambda^3, \ldots) \]
is a norm of the matrix \( A_n \), \( \| x_n \| \) is a norm of vector \( x_n \). where \( \lambda \) is a two eigenvalues of the matrix \( A_n \). After simple operation, obtain
\[ \lambda^2 = \cos \omega \Delta t(n,p) \pm \sqrt{\sin^2 \omega \Delta t(n,p) - (1-r^2(n,p))} \]
and then proceed to obtain an estimation of the norm of the matrix \( A_n \):
\[ \| A_n \| < 1 \]
when
\[ \| \cos \omega \Delta t(n,p) \| < 1 - r^2(n,p)/2 \]
Otherwise,
\[ \| A_n \| < \sqrt{I_{n-1}(p)/I_n(p)} \text{ when } I_{n-1}(p) > I_n(p) \]
\[ \| A_n \| < \sqrt{I_n(p)/I_{n-1}(p)} \text{ when } I_n(p) > I_{n-1}(p) \]
Therefore, a conclusion of the stability analysis of Eq. (1) or (3) can be shown as:

Definition: if the impedance function \( I(p) \) of medium to plane wavefield with Snell character \( p \) has \( K \) local maximum values \( I_u(k) \) \((k=1,2,\ldots,K)\) and \( L \) local minimum values \( I_v(l) \) \((l=1,2,\ldots,L)\), then the complex of structure of impedance function can be defined as
\[ L_c(p) = \frac{\prod I_u(k)}{\prod I_v(l)} \quad \text{and} \quad \| \Delta x_n \| < B \| \Delta x_1 \| \]
where \( C \) depends on two boundary values of impedance. Hence, we have

Theorem: If the complex of structure of the impedance function is finite, then for any \( n \), there are
\[ \| \Delta x_n \| < B \| \Delta x_1 \| \]
where \( B < L_c(p) \).

Obviously, if Eq. (5) is satisfied for all of \( n \), the \( \| \Delta x_n \| < \| \Delta x_1 \| \) it may be said that Eq. (1) or (3) is strictly stable; if Eq. (5) is not satisfied for any \( n \), the \( B > L_c(p) \), \( \| \Delta x_n \| \) is finite too, it may be said that Eq. (1) or (3) is weakly stable. When \( I(p) \) or \( L_c(p) \) is infinite, Eq. (1) or (3) may not be stable. At this time, \( \nu(z) = 1 \) or \( j(z) = \infty \). Therefore, this velocity inversion of prestack seismic data can be operated only on precritical data.

In addition, this inversion method just needs two plane wavefield data sets with same frequency and different Snell characters on the theory; When apply the inversion method to real data, the multi-frequency and multi-Snell-character plane wavefield data are needed to obtain more accurate velocity.

REFERENCE