V015 INITIAL ANALYSIS OF STABILITY OF PRESTACK VELOCITY INVERSION OF SEISMIC DATA

TIANXI WANG

Hangzhou Institute of Petroleum Geology, CNPC, 166-1 Xixi Road, Hangzhou 310023, China

A method of prestack velocity inversion of seismic data using plane wavefield downward continuation equation has been proposed by WANG [1990,1993]. Some inversion velocity profiles obtained from prestack marine data have been published in the same paper [WANG, 1990,1993]. However, stability, a key problem with the inversion method (recurrent, stripping), has not been discussed. In this note, an initial analysis of the stability problem is introduced.

In brief, a theoretical background of the inversion method is a formulation for downward continuation of plane wavefield in layered medium as the following:

$$\begin{bmatrix} U_{n+1}(\omega, p) \\ D_{n+1}(\omega, p) \end{bmatrix} = \frac{1}{\bar{t}(n, p)} \begin{bmatrix} e^{i\omega \Delta t(n, p)} & -r(n, p)e^{-i\omega \Delta t(n, p)} \\ -r(n, p)e^{i\omega \Delta t(n, p)} & e^{-i\omega \Delta t(n, p)} \end{bmatrix} \times \begin{bmatrix} U_n(\omega, p) \\ D_n(\omega, p) \end{bmatrix} \quad (1)$$

where ω is a circular frequency, $i = \sqrt{-1}$, $p = \sin(j(z))/v(z)$, is a Snell character of plane wavefield, which is constant during propagation in the layered medium, $j(z)$ is an incident angle, $v(z)$ is a velocity function depending on depth, which will be obtained using the inversion method, and

$$r(n, p) = (I_{n+1}(p) - I_n(p)) / (I_{n+1}(p) + I_n(p))$$

$$\bar{t}(n, p) = 2I_{n+1}(p) / (I_{n+1}(p) + I_n(p))$$

$$I_n(p) = v_n \rho_n / \sqrt{1 - p^2 v_n^2}$$

$$\Delta t(n, p) = d_n \sqrt{1 - p^2 v_n^2} / v_n$$

ρ_n is the density of the n -th layer, d_n is the thickness of the n -th layer, $U_n(\omega, p)$ is the up-coming wavefield at top of the layer, $D_n(\omega, p)$ is the down-going wavefield at top of the layer. If denote $U'_n(\omega, p)$ and $D'_n(\omega, p)$ as wavefield at bottom of the layer, according to Claerbort principle, we obtain

$$r(n, p) = \text{Re} \frac{U'_n(\omega, p)}{D'_n(\omega, p)} \quad (2)$$

Since computational steps of the velocity inversion method based on Equ (1) and (2) have been described by Wang [1990,1993], we do not repeat those steps in this note. That algorithm is complex, so the stability analysis is complex too. The initial analysis is just focused on the Equ (1). For simplicify, Equ (1) can be rewritten as

$$X_{n+1} = A_n X_n \quad (3)$$

where

$$X_n = \begin{bmatrix} U_n(\omega, p) \\ D_n(\omega, p) \end{bmatrix} \quad A_n = \begin{bmatrix} e^{i\omega \Delta t(n, p)} & -r(n, p)e^{-i\omega \Delta t(n, p)} \\ -r(n, p)e^{i\omega \Delta t(n, p)} & e^{-i\omega \Delta t(n, p)} \end{bmatrix}$$

Based on matrix theory, the stability analysis of Equ (1) or (3) is to discuss the error ΔX_n after using n steps of Equ (1) or (3) when the x_1 has error Δx_1 . Easily obtained:

$$\Delta x_{n+1} = A_n \cdot A_{n-1} \cdot \dots \cdot A_1 \Delta x_1$$

so that

$$\|\Delta x_{n+1}\| \leq \|A_n\| \|A_{n-1}\| \dots \|A_1\| \|\Delta x_1\|$$

and

$$\|A_n\| \leq \max(1, \lambda_{n,1}^{-1}, 1, \lambda_{n,2}^{-1}),$$

is a norm of the matrix A_n , $\|\Delta x_n\|$ is a norm of vector Δx_n . where $\lambda_{n,1,2}^{-1}$ are two eigenvalues of the matrix A_n . After simple operation, obtain

$$\lambda_{n,1,2}^{-1} = \cos \omega \Delta t(n,p) \pm \sqrt{\cos^2 \omega \Delta t(n,p) - (1-r^2(n,p))} \quad (4)$$

and then proceed to obtain an estimation of the norm of the matrix A_n :

$$\|A_n\| < 1 \quad \text{when } |\cos \omega \Delta t(n,p)| < (1-r^2(n,p))/2 \quad (5)$$

Otherwise,

$$1 \leq \|A_n\| \leq \sqrt{I_{n+1}(p)/I_n(p)} \quad \text{when } I_{n+1}(p) > I_n(p)$$

$$1 \leq \|A_n\| \leq \sqrt{I_n(p)/I_{n+1}(p)} \quad \text{when } I_n(p) > I_{n+1}(p)$$

Therefore, a conclusion of the stability analysis of Equ (1) or (3) can be shown as:

Definition: if the impedance function $I(p)$ of medium to plane wavefield with Snell character p has K local maximum values $I_u(k)$ ($k=1,2,\dots,K$) and L local minimum values $I_v(l)$ ($l=1,2,\dots,L$), then the complex of structure of impedance function can be defined as

$$L_c(p) = C \cdot \prod_{k=1}^K I_u(k) / \prod_{l=1}^L I_v(l) \quad |K-L| \leq 1 \quad (6)$$

where C depends on two boundary values of impedance. Hence, we have

Theorem: If the complex of structure of the impedance function is finite, then for any n , there are $\|\Delta x_n\| \leq B \|\Delta x_1\|$, where $B \leq L_c(p)$.

Obviously, if Equ (5) is satisfied for all of n , the $\|\Delta x_n\| < \|\Delta x_1\|$, it may be said that Equ (1) or (3) is strictly stable; if Equ (5) is not satisfied for any n , the $B = L_c(p)$, $\|\Delta x_n\|$ is finite too, it may be said that Equ (1) or (3) is weakly stable. When $I(p)$ or $L_c(p)$ is infinite, Equ (1) or (3) may be not stable. At this time, $pv(z)=1$, or, $j(z)=\pi/2$. Therefore, this velocity inversion of prestack seismic data can be operated only on precritical data.

In addition, this inversion method just needs two plane wavefield data sets with same frequency and different Snell characters on the theory: When apply the inversion method to real data, the multi-frequency and multi-Snell-character plane wavefield data are needed to obtain more accurate velocity.

REFERENCE

- [1] Tianxi WANG (1990), Prestack inversion of seismic data with plane-layer medium and point source by wavefield downward continuation, SEG 60th Annual Meeting, SP20.
- [2] Tianxi WANG (1993), Prestack velocity inversion of seismic data using plane wavefield downward continuation, Chinese Journal of Geophysics, vol 36, No 3, pp 405-413, by Allerton Press, INC.