An Analysis of Dynamic Pseudo Relative Permeability Methods

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ABSTRACT
The properties and limitations of 6 widely used dynamic pseudo relative permeability methods are analysed. The example proposed by Stone is used to illustrate our findings. The analytical results are confirmed by numerical simulation.

INTRODUCTION
Dynamic pseudo relative permeabilities are often used in reservoir simulation in an attempt to capture the effects on multi-phase fluid flow of heterogeneities not represented on the (coarse) simulation grid, and to compensate for numerical dispersion effects. By ‘dynamic’, we mean pseudo relative permeabilities generated from the results of a ‘fine grid’ simulation model, as opposed to those calculated under an assumption of capillary or gravity equilibrium.

Numerous dynamic pseudo relative permeability methods have been published. The purpose of this paper is simply to analyse the properties of 6 of the most widely used of these methods. The difficulty of using any dynamic pseudo relative permeability method reliably is discussed at length in another paper [1].

We focus our analysis on the displacement of oil by water, and neglect fluid and rock compressibilities, but many of our conclusions apply to more general situations.

THE METHODS
Kyte and Berry
In the well-known Kyte and Berry method [2], average pressures for each coarse grid block and total flow rates of each phase between each pair of adjacent coarse grid blocks are calculated from the fine grid simulation results. These values are substituted in the coarse grid Darcy equations to infer the coarse grid (pseudo) relative permeability values that would be required to reproduce the fine grid flows. For the water phase and z-direction flow in an xz cross section of unit width:

\[ k_{rw} = -\frac{\mu_w \bar{q}_w \Delta z}{\Delta z k_z \Delta \Phi_w} \]  

where the \( \bar{q}_w \) is the appropriate sum of the fine grid flows and the coarse grid potential difference is estimated from:

\[ \Delta \Phi_w = \Delta \bar{P}_w - g \rho_w \Delta D \]  

The average water phase pressure in each coarse grid block is:

\[ \bar{P}_w = \frac{\sum_{j=J_1}^{J_2} [k_j k_{rwj} \delta z_j (P_{wj} - g \rho_w (D_j - D))]_{i=(I_1+I_2)/2}}{\sum_{j=J_1}^{J_2} [k_j k_{rwj} \delta z_j]_{i=(I_1+I_2)/2}} \]  

and the corresponding pseudo capillary pressure is:

\[ \bar{P}_c = \bar{P}_w - \bar{P}_o \]  

From the way the pseudos are constructed, it is clear that they should in principle allow the fine grid solution to be exactly reproduced on a coarse grid (provided that a different set of pseudos is generated and used for each coarse grid block and each flow direction; and see also [3]).

However, if \( \Delta \Phi_w \) has the same sign as \( \bar{q}_w \), a negative pseudo relative permeability arises, and if \( \Delta \Phi_w \) is zero, the pseudo relative permeability is infinite: both these situations can occur, and this is the root of most problems specific to the Kyte and Berry method [4]. Also, use of a different weighting factor for each phase.

Symbols are defined at the end of the text.
in equation 3 means that non-zero pseudo capillary pressure can arise when there is no fine grid capillary pressure, which is an unnecessary complication, while averaging only over the central plane of fine grid cells implies different capillary pressures for different flow directions, which is a more serious difficulty.

**Pore Volume Weighted**

The Pore Volume Weighted method [5] differs from Kyte and Berry only in the definition of average pressure, for which a pore volume weighted average over the entire coarse grid block is used:

$$
\bar{P}_w = \frac{\sum_{j=J_1}^{J_2} \sum_{i=I_1}^{I_2} \delta x_i \delta y_j \phi_{ij} (P_{wj} - g \rho_w (D_{ij} - D))}{\sum_{j=J_1}^{J_2} \sum_{i=I_1}^{I_2} \delta x_i \delta y_j \phi_{ij}}
$$

(5)

With this definition, the pseudo capillary pressure is the same for all flow directions and is zero if there is no fine grid capillary pressure. Thus, it is a clear improvement over the original Kyte and Berry method.

This method should also allow the fine grid solution to be exactly reproduced on a coarse grid, but the problem of negative or infinite pseudo relative permeabilities remains.

**Stone**

Stone [4] was the first to use total mobility as a way of avoiding the problems associated with estimating the coarse grid average pressures. However, his formulae for obtaining the pseudo relative permeabilities from the average fractional flow and average total mobility:

$$
k'_{rw} = \mu_w f_w \bar{\lambda}_t \quad k'_{wo} = \mu_o (1 - f_w) \bar{\lambda}_t
$$

(6)

where the average fractional flow of water is:

$$
f_w = \frac{q_w}{q_w + q_o}
$$

(7)

neglect the effect of gravity on the coarse grid. Also, his averaging of total mobility only over the outlet face of the coarse grid block:

$$
\bar{\lambda}_t = \frac{\sum_{j=J_1}^{J_2} [T_{xj} \lambda_{ij}]_{i=I_2}}{\sum_{j=J_1}^{J_2} [T_{xj}]_{i=I_2}}
$$

(8)

is inadequate when there are significant variations in total mobility [6].

Thus, this method will not reproduce the fine grid results on the coarse grid if there are significant gravity (or capillary pressure) effects on the coarse grid, or if there are significant variations in total mobility. Also, while avoiding possible division by zero, it will give a negative pseudo relative permeability if the net flows of the two phases are in opposite directions ($f_w < 0$ or $f_w > 1$).

**Total Mobility**

Methods similar to Stone's but which use better definitions of the average total mobility have been proposed by several authors [6, 7, 8]. In the method of Christie et al [8], which we will refer to as the Total Mobility method, coarse grid gravity effects are also allowed for. More specifically, the average total mobility is found from:

$$
\bar{\lambda}_t = \frac{k \bar{\lambda}_t}{k}
$$

(9)

where $k_\lambda$ and $k$ represent upscaled values obtained by solution of a Laplace equation (see Appendix A). The pseudo relative permeabilities are then given by:

$$
k'_{rw} = \mu_w \bar{\lambda}_w \quad k'_{wo} = \mu_o (\bar{\lambda}_t - \bar{\lambda}_w)
$$

(10)

where $\bar{\lambda}_w$ is obtained from solution of the quadratic equation:

$$
G \bar{\lambda}_w^2 - (\bar{u}_t + G \bar{\lambda}_t)\bar{\lambda}_w + \bar{f}_w \bar{u}_t \bar{\lambda}_t = 0
$$

(11)

where $G = kg(p_w - \rho_o)$. Here $f_w$ is obtained from equation 7 and $\bar{u}_t$ from the coarse grid simulation results. See Appendix B for a derivation of this equation, a discussion of the choice of root, and a demonstration that negative pseudo relative permeability can occur if the net phase flows are in opposite directions ($f_w < 0$ or $f_w > 1$), as with the Stone method.

Equation 11 represents an exact solution of the coarse grid Darcy equations in the absence of capillary pressure. However, in solving for $s_\lambda$ in equation 9, a gravity term is neglected and local boundary conditions are necessarily imposed (see Appendix A). These two approximations, while generally not severe, mean that the fine grid solution will not be exactly reproduced on the coarse grid.

Note that there is no point in trying to 'improve' the method by eliminating these two approximations, since this would lead back to a 'rigorous' method of the Kyte and Berry or Pore Volume Weighted type. The attraction of the Total Mobility method is that it may be useful in some cases where the rigorous pseudos are unuseable. While it guarantees neither reproduction of the fine grid results nor useable pseudos, it makes a minimum of approximation and is thus a compromise between rigorous methods and methods such as Stone, Quasi-Steady State and Weighted Relative Permeability which make much more severe approximations.
Quasi-Steady State

This method, in which the permeability ($kk_r$) of each phase is upscaled (by solution of a Laplace equation, for example), has appeared in several guises [9,10,11]. The pseudo relative permeabilities are obtained from:

$$k'_{rw} = \frac{kk_{rw}}{k} \quad k'_{ro} = \frac{kk_{ro}}{k}$$  \hspace{1cm} (12)

The method generally produces pseudos that are smooth curves with values between zero and one, so there are no problems in actually using them in the coarse grid simulation, but this does not justify its use outside its range of validity.

The assumption of quasi-steady state flow implicit in this method can be seen as follows. In upsampling the phase permeability of (say) water by the Laplace method, a pressure field $P_w$ is obtained from solution of the equation:

$$\nabla \cdot (kk_{rw} \nabla P_w) = 0$$  \hspace{1cm} (13)

whereas (neglecting compressibility, capillary pressure and gravity) Darcy’s law for the water phase is:

$$\phi \frac{\partial S_w}{\partial t} - \frac{1}{\mu_w} \nabla (kk_{rw} \nabla P) = 0$$  \hspace{1cm} (14)

Thus, even if the boundary conditions applied to equation 13 are adequate, the pressure field $P_w$ will be similar to the true pressure field encountered during waterflooding only when the time derivative of $S_w$ is negligible, i.e. in steady or quasi-steady state flows. At a saturation front, this time derivative is in fact infinite (in the absence of capillary pressure). The method is thus inapplicable in viscous dominated cases, which generally involve displacements of a frontal nature. Only when the saturation fronts are smoothed out by capillary forces is the method reliable.

This can be illustrated by considering a piston-like displacement in a homogeneous or mildly heterogeneous reservoir. When the water front is mid-way through the coarse grid cell, $kk_{rw} = 0$ ahead of the flood front and $kk_{ro} = 0$ behind it (Figure 1a). If no-flow or periodic boundary conditions are used on the lateral boundaries, $kk_{rw}$ and $kk_{ro}$ would both be zero, and hence both pseudo relative permeabilities would also be zero, at all times before water breakthrough. If ‘linear in pressure’ lateral boundary conditions are used, the pseudos will not be zero but the flow patterns obtained during upsampling of $kk_{rw}$ and $kk_{ro}$ will have nothing to do with the actual flow occurring during the waterflood (Figure 1b). Thus, the values obtained for the pseudo relative permeabilities before water breakthrough are unlikely to be in any sense correct. In fact,

$$Note that under the assumptions just mentioned, the waterflood pressure is actually a solution of the equation $\nabla \cdot (kk_{rw} \nabla P) = 0$, which is why methods based on total mobility have a stronger basis in non-steady state cases.
sets of pseudos to use in the history-matching is difficult, and the pseudos will almost certainly not be unique. Use of these pseudos for predictive purposes is even more questionable than the use of pseudos obtained from the constructive methods described above.

Recent publications by Heriot-Watt University [16] concern the application of pseudo relative permeabilities, using methods similar to those we analyse, rather than new pseudo relative permeability methods. In renormalisation-based methods [8], the key idea is to perform upscaling in a series of steps rather than in a single step, but any of the methods already mentioned can be used at each step.

SIMPLIFIED STONE'S EXAMPLE

The example suggested by Stone [4] is illustrated in Figure 2. The reservoir model is a dipping vertical cross-section consisting of two non-communicating layers of differing permeability. Water is injected into each layer at a rate proportional to its permeability. Fluid and rock compressibilities are neglected. Each layer is treated as one-dimensional, so the solution can be obtained analytically.

We first consider a simplified version of this example in which we assume piston-like displacement of oil by water in each layer, as well as equal porosity and end-point relative permeabilities in each layer. Only 3 average saturations then occur, the 2 end-point saturations and 1 intermediate saturation when one layer is at $S_w = S_{wc}$ and the other at $S_w = 1 - S_{ro}$. For each of the 6 methods, we calculate the ‘effective’ relative permeabilities (and capillary pressure) at this intermediate saturation for an equivalent one-layer model, and compare the implied fractional flow and pressure gradient with the actual values. We use the term ‘effective’ rather than ‘pseudo’ as there is no gridding involved and hence no numerical dispersion. The details of this analysis are presented in Appendix C; the following paragraphs summarise the main results.

Kyte and Berry

The Kyte and Berry effective relative permeabilities are:

$$k'_w = \frac{k_2 k_0}{2k}$$

$$k'_o = \frac{k_1 k_0}{2k}$$

These are always positive and less than the end-point values, and are independent of the flow rate. However, the effective capillary pressure at the intermediate saturation varies both with position and with flow rate, and it is necessary to use this capillary pressure to obtain the correct fractional flow in the equivalent one-layer problem. If this is done, the oil and water phase pressure gradients in the one-layer model are then equal to the pressures in the oil- and water-filled layers of the two-layer model respectively.

Pore Volume Weighted

The expression for the Pore Volume Weighted effective relative permeabilities is not as simple as equation 16: they depend on rate and $k'_{rw}$ may become negative at low flow rates. However, there is no effective capillary pressure, and the correct fractional flow is obtained in the equivalent one-layer problem (provided the effective relative permeabilities are used as calculated, even if they are negative). The pressure in the one-layer model is then equal to the average pressure in the two-layer model.

Stone Method

Stone's method gives effective relative permeabilities that are positive, but which may be greater than the end-point values if the viscosity ratio is large. The correct fractional flow is obtained in the equivalent one-layer problem only if there is no gravity effect (i.e. if the reservoir is non-dipping or if the oil and water densities are equal). Even if there is no gravity effect, the one-layer model pressure is not equal to the average pressure from the two-layer model.

Quasi-Steady State/Weighted Rel Perm

The Quasi-Steady State and Weighted Relative Permeability effective relative permeabilities are both identical with the Kyte and Berry ones, given by equation 16, for this problem. They are thus positive and less than the end-point values. However, since there is no effective capillary pressure in these methods, the correct fractional flow is not obtained in the equivalent one-layer problem, even if there is no gravity effect.

Total Mobility

The total mobility method is rather difficult to analyse. However, it can be shown that the effective relative permeabilities are positive (but may be greater than the end-point values) and the correct fractional flow is obtained in the equivalent one-layer problem. The pressure in the one-layer model does not follow the average pressure in the two-layer model.
Discussion

These results confirm the statements made above about the properties of the various methods:

1. the Kyte and Berry and Pore-Volume Weighted methods reproduce the correct fractional flow and pressure gradient in the equivalent one-layer problem;

2. neither the Kyte and Berry nor the Pore-Volume Weighted method is ideal, because the former requires a position-dependent effective capillary pressure while the latter can give negative effective relative permeabilities;

3. the Total Mobility method reproduces the correct fractional flow but not the average pressure gradient; the other methods reproduce neither quantity.

STONE'S EXAMPLE

Introduction

The example suggested by Stone differs from the simplified version just considered in that the displacement is not piston-like, and that specific values are assigned to all the parameters (Table 1). In each layer, there is a Buckley-Leverett type of solution with a shock front, behind which the water saturation gradually increases to \( S_{w} = 1 - S_{o} \). The solution can be obtained analytically and is shown in Figure 3. Also shown there is the solution obtained by numerical simulation with a grid of 50 x 2 blocks.

Stone proposed this example to show that the Kyte and Berry method can lead to negative pseudo relative permeabilities. He obtained good results for this example using his own ‘Stone’ method.

Guzman et al [13] also present an analysis of this example. They point out that it is not a realistic example because the flow rate is so low (it is nearly 3000 years before water breakthrough) and because the specification of fixed injection rates for each layer is unusual.

Despite its unrealistic nature, this example is useful for illustrating the properties of the various pseudo relative permeability methods because the solution can be obtained analytically.

Effective Relative Permeabilities

We again calculate the ‘effective’ relative permeabilities which would be applicable if the two-layer model were replaced by a homogeneous one-layer model. These can be calculated in a semi-analytic manner. The solution of the one-layer problem with the resulting effective relative permeabilities can be obtained in similar fashion. The details are given in Appendix D.

Figure 4 shows the Pore Volume Weighted, Stone and Total Mobility effective relative permeabilities. The Pore Volume Weighted effective relative permeability for water is negative in a saturation interval near \( S_{w} = 0.4 \) corresponding to the region between the water fronts in the two layers. The Stone effective relative permeabilities are all positive but that for oil is greater than one in the intermediate saturation interval. The Total Mobility effective relative permeabilities show no undesirable behaviour.

Figure 4 also shows effective relative permeabilities obtained numerically. They were generated for a region consisting of blocks (40,1) and (40,2) in the 50 x 2 grid. Very similar results were obtained for a region consisting of blocks (20,1) and (20,2). These numerical results confirm the analytic results, any differences between the two being attributable to numerical dispersion in the 50 x 2 grid simulation which causes different average saturation values to be encountered (note that the post-processing software outputs a zero value whenever it calculates a negative value).

For the Kyte and Berry method, the analysis reveals

\[ k_{rw} = \frac{S_{w}-S_{w,e}}{1-S_{w}-S_{o}} \]

\[ k_{ro} = \frac{1-S_{w}-S_{o}}{1-S_{w}-S_{o}} \]

because the porosities in the two layers are not equal, there is a non-zero effective capillary pressure in the Pore Volume Weighted method, but this is a constant independent of saturation and position, so may be neglected.
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Figure 4. Effective Rel Perms

Figure 5. Kyte and Berry Effective Rel Perms

Figure 6. Kyte and Berry Effective Capillary Pressure

One-Layer Solution

Water saturations and pressures at $8 \times 10^5$ days for the one-layer model are compared with the average saturations from the two-layer model in Figure 7.

With the Total Mobility effective relative permeabilities, an excellent match to the two-layer model is obtained for water saturation (Figure 7a), though the pressure is almost equal to the pressure in the higher permeability layer (layer 2) rather than to the average pressure (Figure 7b). The analysis of the simplified Stone problem (Appendix C) indicated that the average pressure distribution would not be reproduced using the Total Mobility method. This is confirmed by these results. The close agreement with the pressure in layer 2 is probably just a coincidence.

The Kyte and Berry effective relative permeabilities have been verified numerically, as shown in Figure 5. Any differences between the analytic and numerical results are again attributable to numerical dispersion and suppression of negative values.
The Stone effective relative permeabilities result in a piston-like displacement, a completely wrong result (Figure 7a). To use the Pore Volume Weighted effective relative permeabilities, we have set the negative values to zero: the resulting water saturation, labelled ,PVOL+,' in Figure 7a, is in very poor agreement with the two-layer model. We do not bother to show the pressures for these two methods.

We have not attempted to calculate the one-layer solution using the Kyte and Berry effective relative permeabilities because the presence of a capillary pressure and the dependence on position makes this difficult. Note that if we had retained the negative values, the Pore Volume Weighted effective relative permeabilities would have given the correct fractional flow curve for the one-layer model over the relevant saturation ranges, as shown by the symbols in Figure 8. This would appear to suggest that the two-layer model results could be reproduced using this method if negative relative permeabilities could be accepted. However, the 'gaps' in the fractional flow curve should not be filled in by linear interpolation, but by calculating the fractional flow from linear interpolation of the relative permeability values, because this is usually what happens in a simulation. This leads to a fractional flow curve with sign changes and infinite values (labelled 'interpolated' in Figure 8). The usual tangent construction cannot be applied to such a curve, and it is not clear to us what solution is implied.

Finally, we have verified the results numerically by running simulations on a 50 x 1 grid using the numerically-generated effective relative permeabilities. The Kyte and Berry results were again not used because of their shape and their dependence on position. The negative Pore Volume Weighted values were set to zero. The Stone values greater than one were set equal to one. Figure 9 shows the results. Water cut and injection well pressure are shown rather than water saturation and pressure distributions, but it is clear that they do indeed confirm the analytic results.

Pseudo relative permeabilities

We have also generated pseudo relative permeabilities for use in a coarse grid one-layer model comprising just 5 x 1 blocks. This has been done numerically because it is difficult to do analytically. In Figure 10, we compare the numerically-generated pseudo relative...
permeabilities with both analytically and numerically generated effective relative permeabilities (which were already shown in Figures 4 and 5). The pseudos are generated for the 4th block in the coarse grid: for all the methods we would expect the pseudos to be different for each block in the coarse grid, unlike the effective relative permeabilities which were independent of position for all methods except Kyte and Berry.

For all 4 methods, the main difference between the effective and pseudo relative permeabilities is that the pseudos are shifted to the right on the saturation axis. This is the classical result when compensating for numerical dispersion. Some non-monotonicity has crept into the Pore Volume Weighted pseudos at high water saturation.

Figure 11 shows the results obtained on the coarse grid when these pseudos are used: the peaks in injection well pressure for the Pore Volume Weighted pseudos arise from the very low total mobility occurring at water saturations just less than 0.7, while in the other cases the pressure is essentially hydrostatic and the 'steps' result from the switch from the oil phase gradient to the water phase gradient as the water becomes mobile in each coarse grid block.

Qualitatively these results are in line with our previous conclusions: the Total Mobility method gives a good result whereas the Stone and Pore Volume Weighted methods (with values constrained to lie between 0 and 1) do not. The performance of the Total Mobility method is not quite as good as before, but this is to be expected when only one set of pseudos is used for all the coarse grid blocks.

**Discussion**

This example has highlighted the possible difficulties with the Kyte and Berry and Pore Volume Weighted methods, as was intended by Stone when he proposed it. However, it should be remembered that it is an unrealistic example. Our experience is that in more realistic one and two dimensional examples, these methods generally work well. Difficulties may be encountered more frequently in three-dimensional models with complex geometry, but at present it is the Pore Volume Weighted method that we recommend trying first.

A surprising aspect of our results is that Stone's method gives a very poor result for Stone's problem! This means that the good results presented in Stone's paper [4] were not obtained with the method described in that paper. Specifically, some treatment of gravity must have been made to obtain the results presented, which are of similar quality to the results we obtained with the Total Mobility method. Guzman et al [13] describe a method of treating gravity (and capillary pressure) which they attribute to Stone. Using this method, they get results similar to those presented by Stone. Thus, this may be what Stone actually did. However, Guzman et al also demonstrate an inconsistency in this treatment of gravity (the oil and water phases are not treated in the same way).
Figure 11. Numerical Solution with Pseudo Rel Perms

In their analysis of Stone's problem, Guzman et al are themselves inconsistent in presenting analytic solutions for the equivalent one-layer model using pseudos generated for a coarse grid model. It is inappropriate to use these pseudos, which compensate for numerical dispersion, in an analytic solution which is already free from numerical dispersion. They also neglect the pseudo capillary pressure, which we have shown to be essential to the consistency of the Kyte and Berry method in this example.

As already noted, the Total Mobility method makes two approximations which prevent the fine grid solution from being exactly reproduced on the coarse grid. In Stone's example, the discrepancy shows up in the pressure rather than the flow rates, because the total flow rate is imposed and the flow vectors of the two phases are parallel: under these circumstances the fractional flow is guaranteed to be matched. In other examples, particularly multi-dimensional ones, errors may occur in pressure, total flow rate and fractional flow. However, we have yet to find an example where these errors are significant.

**CONCLUSIONS**

The arguments and results presented in this paper have shown that:

1. the Pore-Volume Weighted method in principle allows a fine grid solution to be reproduced on a coarse grid (under appropriate assumptions), but in practice problems can arise in the form of negative values or values much greater than one;

2. the Kyte and Berry method has the same properties as the Pore-Volume Weighted method but additional problems arise in two- and three-dimensional cases because of the directional nature of the pseudo capillary pressure; use of a pseudo capillary pressure when there is no capillary pressure on the fine grid is also an unnecessary complication;

3. the Total Mobility method makes a minimum of approximation but does not guarantee reproduction of the fine grid solution on the coarse grid; negative values can occur, but only if the phases are flowing in opposite directions;

4. the Stone, Quasi-Steady State and Weighted Relative Permeabilities methods make more severe approximations; the Quasi-Steady State and Weighted Relative Permeability methods produce 'well-behaved' pseudos, but none of the methods reproduces the fine grid solution on the coarse grid except under very restricted circumstances.

There is thus no problem-free method applicable in all cases, though we have found the Pore Volume Weighted and Total Mobility methods to be reasonably reliable for simple cases. However, even if there were such a method, serious difficulties with the reliable use of dynamic pseudo relative permeabilities would still remain. These are discussed at length in another paper [1].

**NOMENCLATURE**

- \( D \) depth
- \( f \) fractional flow
- \( g \) acceleration due to gravity
- \( h \) layer thickness
- \( J_1, J_2, J_3, J_4 \) limiting fine grid indices for coarse grid block
- \( k \) absolute permeability
- \( k_r \) relative permeability
- \( L \) reservoir length
- \( P \) pressure
- \( P_c \) capillary pressure
- \( q \) volumetric flow rate
- \( S \) saturation
- \( S_{oi}, S_{or} \) initial/residual oil saturation
- \( S_{wc} \) connate water saturation
- \( t \) time
- \( T \) transmissibility
- \( u \) Darcy velocity
- \( W \) reservoir width (y-direction)
- \( \Delta x, \Delta z \) coarse grid block size
- \( \delta x, \delta z \) fine grid block size
- \( \theta \) dip angle
- \( \lambda \) mobility
- \( \mu \) viscosity
- \( \rho \) density
- \( \phi \) porosity
- \( \Phi \) potential
Subscripts:
- \( i, j \) in grid block \( i \) or \( j \)
- \( o \) oil phase
- \( w \) water phase
- \( z \) in \( z \)-direction
- \( 1, 2 \) in layer 1 or layer 2

Superscripts:
- \( o \) end-point value
- \( \bar{\ } \) average or coarse grid value
- \( \prime \) pseudo value

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