Direct Conditioning of Gaussian Random Fields to Dynamic Production Data

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Paper presented at the 5th European Conference on the Mathematics of Oil Recovery, Leoben, Austria, 3-6 Sept, 1996

ABSTRACT

In this paper, we address the conditioning of Gaussian random fields to time-dependent production data, e.g. pressure or production rate. The goal of obtaining a set of conditioned permeability fields is to make a prediction of the reservoir performance which includes a quantified uncertainty.

Instead of the brute force approach of generating many permeability fields and keeping those that satisfy the production data within a give measure, we shall use a method that gradually modifies an initially generated permeability field towards a match with the production data, without destroying its local spatial structure.

INTRODUCTION

In the previous ECMOR conference, we reported on the Bayesian inversion method for conditioning reservoir models to dynamic production data (Floris & Bos, 1994). This way of conditioning may be called indirect, because the conditioning is on the parameters of the reservoir model, e.g. average permeability, standard deviation of permeability or correlation length. On the other hand, direct conditioning methods are methods in which the reservoir model itself is conditioned to the dynamic production data.

In this paper, we shall apply such a direct method for the conditioning of a Gaussian Random Field to dynamic production data.

The method we use, has initially been developed by Gomez-Hernandez for the conditioning of Gaussian Random Fields (GRFs) to hydrostatic head data in groundwater modelling (Gomez-Hernandez et al., 1995). It is called the Self-Calibrated Method (SCM). The

calculation of hydrostatic head data involves the solution of a time-independent potential problem for a given permeability field. In a recent report, the method was applied to a solute transport problem (Wen et al., 1996). We have applied the method to a multi-phase flow problem, typical for oil recovery.

Several other techniques have been introduced already by more or less independent communities. The so-called pilot point method was proposed by de Marsily for characterising permeability from interference tests (de Marsily et al., 1984 & Certes, 1991). Oliver (1994) has developed a method for the conditioning of permeability fields to well test data. Recently, also Thibaut (1996) proposed a method for direct conditioning of permeability fields to production data. All the above methods for conditioning to reservoir production data are based on the same methodology, but differ in the selection of the specific ingredients in the methodology. The methodology has been developed mainly within the groundwater community, and has not yet filtrated through to the oil industry, which is one of the reasons for presenting this paper.

METHOD

Our goal is to generate permeability fields that are conditioned to the following data : 1) permeability measured at wells (static well data), 2) spatial statistics of the field, i.e. a semi-variogram, and 3) production data, e.g. pressures, flow rates (dynamic well data).

For this purpose, we perform the following steps

1. Generate an initial permeability field conditioned to the static well data and the spatial statistics;

2. Simulate the production behaviour for the current permeability field;
3. Stop if the production behaviour matches the observed dynamic well data;
4. Determine a correction field that optimally modifies the permeability field towards a history match, without destroying the conditioning to static well data and spatial statistics;
5. Add the correction field to the current permeability field;
6. Return to step 2.

Let us look closer into these steps.

Step 1 has been studied extensively in the literature. For Gaussian Random Fields, methods such as turning bands, sequential Gaussian simulation and simulated annealing are available through the public domain geostatistical software library GSLIB (Deutsch and Journel, 1992). This step results in a permeability grid, K_{ijk} , $i=1..n_x$, $j=1..n_y$, $k=1..n_z$.

Step 2 requires the simulation of multi-phase flow through the permeability field for a given well production strategy. This step is the most time-consuming in the algorithm. For an automated algorithm such as SCM, it is required that the reservoir simulator can be run in batch, i.e. without any interactive user input.

Step 3 requires a history match criterion for judging whether the simulated output agrees with the observed production data. A typical criterion is the sum of squares of the mismatch between the simulated and observed production data at a set of timesteps, $\|P_{w,obs} - P_{w,sim}\|^2$ where $P_{w,obs}$ and $P_{w,sim}$ are the observed and simulated production vectors at subsequent timesteps. Through the use of a Bayesian likelihood function (Floris and Bos, 1994), prior knowledge on the permeability field can also be incorporated in the match criterion.

Step 4 is the most involved step and we shall continue the discussion with this step, after observing that steps 5 and 6 are formalities. In step 4, our goal is to define a correction field to the current permeability field that leads to a successful history match without destroying the conditioning to static well data and the spatial statistics. Maintaining the conditioning to static well data is important, because the permeability field close to the wells is important for the production behaviour at the wells. Maintaining the spatial structure is important, because inversion of grid block permeability without taking into account spatial structure tends to result in smooth permeability fields. This may in turn lead to an underestimation of the forecasting uncertainty, because of the tendency to filter out possible extremes.

In step 4, we define a grid of master points which are regularly spaced on a grid that is coarser than the grid of the GRF. The wells are also defined as master points. Each master point controls the permeability in its

neighbourhood. At each master point, we define a correction term, denoted by ΔK_{IJK} , $I=1..m_x$, $J=1..m_y$, $K=1..m_z$, where capital indices indicate numbering on the coarse master point grid. In the wells, the correction terms are zero, to maintain the conditioning to the well permeability. Using a general purpose optimisation scheme, we determine the optimal correction terms at the master points for minimising the mismatch criterion. For gradient optimisers, this requires the calculation of gradients which can either be done by finite differences, using a gradient approximation technique (Broyden, 1965, Madsen et al., 1991) or in the simulator using sensitivity coefficients (Bissell, 1994) or the adjoint state method (Chavent, 1994). Next, the coarse grid correction terms are projected onto the fine GRF grid using kriging. Because kriging is an exact interpolator, the correction field is zero at the wells, thereby preserving the conditioning to static well data. Since kriging is a smooth interpolator, the correction field is smooth, thereby maintaining the local spatial statistics of the initial GRF throughout the process. Rather, regionally the level of the permeability field is modified by the correction fields.

When convergence of the algorithm is attained, we have a permeability field that satisfies all three type of conditioning, i.e. conditioning to spatial statistics, static and dynamic well data.

RESULTS

To demonstrate the method, we applied it to a 1D test problem of injection of water into an oil reservoir. The production data was synthetically generated from a truth case permeability field (see Figure 1). A fine grid of 100 grid blocks was used. The base case master point grid contained the two well at the outer ends and 5 master points equally spaced between the wells. Using the sequential Gaussian simulation algorithm from GSLIB, an initial realisation was generated. Assuming piston-like displacement, the pressure drop across the reservoir for a given flow rate could be calculated explicitly. The gradients were calculated using finite differences. Finally, kriging of the permeability correction terms was done using the kriging algorithm from GSLIB. Results for data given in table 1, showed that for 50-75% of the initial permeability fields converged to a successful history match. Figure 2 shows in summary the results of a pass through the conditioning method. The initial permeability field is marked with crosses. The corresponding pressure drop as a function of time is marked with pluses. The solid/dotted lines shows the decrease of the mismatch criterion throughout the iterations. Initially there is a rapid decrease of the mismatch criterion, which then more slowly decreases. The final permeability field is marked with squares. The pressure drop versus time for the final

permeability field is denoted with diamonds and matches well with the truth case pressure drop (solid line). Note that the initial and final permeability fields have the same local spatial variability, but the level has been lifted in the left half and the right quarter of the reservoir. Note also that at the two ends, where wells are located, the permeability has not changed, i.e. conditioned to static well data.

Permeability fields which did not converge, seemed to have a totally different global structure with respect to the truth case permeability field, e.g. high and low permeability region were swapped.

Figure 3A shows the prediction of pressure drop when the permeability fields are only conditioned to static well data and spatial statistics. Figure 3B shows the prediction of pressure drop when the permeability fields have also been conditioned to the history of observed pressure drops (markers). Clearly, uncertainty in prediction has significantly reduced because of the conditioning.

Varying the number of master points from 2 (only the wells) to 12 shows only little change in the final converged permeability field. Thus the algorithm does not seem to be sensitive to the number of master points selected.

Finally, we also applied the method to a 3D black oil problem in which water is injected to produce oil in a quarter of a five-spot pattern. For this case, we used the QFIVE test case delivered with ECLIPSE™. Figure 4 shows the pressure drop for the converged permeability fields and the truth case pressure drop (diamond markers). Also for this case 50-75 % of the initial permeability fields converged.

CONCLUSIONS

A method for conditioning Gaussian Random Fields to dynamic production data has been presented. The method starts with a GRF conditioned only to static well data and spatial statistics, and iteratively modifies the permeability field to match also to observed production data. It has been shown that for 50-75 % of the initial permeability fields the iteration converges. Through the conditioning, a significant reduction in the uncertainty of predictions of production performance has resulted.

Further work can be done on recognising earlier, when an initial permeability field will not result in convergence. Also, the general principle of iteratively modifying geostatistical realisations to match production data without destroying spatial statistics and static well data, can be applied to other types of geostatistical simulation methods.

ACKNOWLEDGMENTS

This research has been funded in part by the European Commission under the Joule II, Geoscience II programme. We acknowledge GeoQuest for the use of the ECLIPSE™ code for our research.

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Table 1. Data used for the first test problem

Q	$= 432 \text{ m}^3/\text{d}$	μ_o	$= 5 \cdot 10^{-3} \text{ Pa.s}$	n_x	$= 100$
L	$= 250 \text{ m}$	μ_w	$= 0.5 \cdot 10^{-3} \text{ Pa.s}$	m_x	$= 7$
W	$= 250 \text{ m}$	ϕ	$= 0.2$	n_{obs}	$= 14$
H	$= 50 \text{ m}$	Δt	$= 10 \text{ d}$		

Figure 1. Permeability field and pressure drop for the truth case.

Figure 2. Summary of a typical run of the direct conditioning method. Markers denote initial (squares) and final (crosses) permeability, initial (pluses) and final (diamonds) pressure drop versus time, truth case pressure drop (solid line) and decrease of mismatch criterion (solid/dotted line).

Figure 3. Decrease in uncertainty in pressure drop prediction from (A) no conditioning with pressure history, to (B) conditioning with pressure history.

Figure 4. Pressure drop for converged permeability fields compared to truth case (diamond markers) using a black oil quarter five-spot flow model.