Gradients Method Constrained by Geological Bodies for History Matching

RAHON, Daniel, Institut Français du Pétrole
BLANC, Georges, Institut Français du Pétrole
GUERILLOT, Dominique, Institut Français du Pétrole
and HÉLIOS Reservoir Group


ABSTRACT

Conventional gradient methods have already been applied to reservoir engineering for matching the history of former field performances. The key point of these methods is to select the best areal reservoir zoning for reduction of the amount of reservoir parameters to be identified. In this paper we propose a zoning based on reservoir lithofacies, thus making a more natural than geographical choice.

The gradients of the bottom hole well pressure relating to the reservoir characteristics to be identified are required by all history matching processes. These gradients are calculated using either a perturbation method or a so-called analytical method. Without any reservoir zoning both methods require as many problem solving jobs as grid blocks.

In this article we propose computing gradients relating to the lithofacies. Starting with a geological model in lithofacies that is generally from a seismic interpretation and pixel or object-based geostatistical simulation we try to quantify the effect of a lithofacies on well pressure response or production history. The influence of a lithofacies is measured through its petrophysical parameters. This method drastically reduces the number of parameters to be identified and allows the use of the natural partition of the reservoir in geological bodies.

The gradient calculation relating to the lithofacies has been successfully implemented in an implicit single-phase fluid flow model. This model can be used for well-tests simulations.

By introducing gradients to minimize an objective function that measures the difference between observed and simulated well pressure responses, we can effectively achieve the inversion of petrophysical lithofacies parameters.

By fixing the value of petrophysical parameters, the influence of the geometry can be studied by varying the geologic body's dimensions.

Several examples of inversion are given at the end of the article to illustrate the effectiveness of this gradient method.

INTRODUCTION

The characterization of a reservoir is a delicate problem that is confronted by geophysicists, geologists and reservoir engineers. Interpreting production results, interference tests and well-tests plays a very important role in attempting to specify the description of the geologic model and to reduce the attenuating uncertainties.

Some of these interpretation techniques are based on inversion procedures in which, by using a meshed geologic model, an attempt is made to calibrate numerical simulations of field measurements by varying the parameters of this model. The variable parameters are both geometric (position and size of geologic bodies) and petrophysical (value of porosity, absolute permeabilities, and points of relative-permeability curves, etc.).

To vary the petrophysical characteristics automatically, one solution is to use a method of gradients relating to these parameters [1, 2, 3, 4]. In a meshed model used by a flow simulator in porous media, the number of parameters is often a multiple of the number of meshes. To reduce the number of these parameters, the reservoir is cut up into influence zones in which they vary uniformly.

This article describes an original gradient method using the description of the reservoir model in lithofacies. This method is particularly well suited for an approach that is used more and more often. It consists first of all in building a geologic model by using the structure in lithofacies before quantifying it in petrophysical data.
This approach does away with cutting up the reservoir into zones of the meshed model, by using a natural zoning in geologic bodies and by measuring directly the influence of a lithofacies, through its petrophysical characteristics, on the pressure response of a well-test or on production curves.

After describing the gradient method in relation to the lithofacies, we will see how it has been programmed into a numerical well-test simulator, then validated and tested by simple geologic models.

THE GRADIENT METHOD APPLIED TO FLOW SIMULATIONS

The Model Problem

To model flow in an oil reservoir, we take into account:

- Darcy's generalized equations,
- the law of mass conservation.

to which we associate:

- borehole conditions,
- boundary conditions,
- initial conditions.

We thus obtain a system of nonlinear equations with partial derivatives, in which the unknowns are pressures \( P \) and saturations \( S \) for each phase and, possibly, the concentrations of each constituent in the phases.

If we name \( U \) the set of unknowns and \( d \) the set of parameters influencing the solution, the system of discretized equations can symbolically be written in the following way:

\[
f(U^{n+1}, U^n, d) = 0
\]

in which \( U^{n+1} \) solution at \( t^{n+1} \) time depends on the solution at \( t^n \) time and on \( d \).

The conventional method used to solve this type of system is Newton's method which has the following algorithm

\[
U^{(0)} = U^n \\
\left( \frac{\partial f}{\partial U^n} \right)^{(k)} (U^{(k+1)} - U^{(k)}) = -f(U^{(k)}, U^n, d) \quad (2)
\]

\[U^{n+1} = U^{(k)} \text{ when stop criterion is verified}
\]

The Gradient Method

The idea is to determine the influence of a parameter \( d \) on the values of \( U \) at each time step.

By deriving Equation (1) in relation to parameter \( d \), we obtain an equation in \( \frac{\partial U^{n+1}}{\partial d} \):

\[
\frac{\partial f}{\partial U^{n+1}} \frac{\partial U^{n+1}}{\partial d} + \frac{\partial f}{\partial U^n} \frac{\partial U^n}{\partial d} + \frac{\partial f}{\partial d} = 0
\]

(3)

The value of the gradients of the set of unknowns \( U \) in relation to parameter \( d \) at \( t^{n+1} \) time, namely

\[Y^{n+1} = \frac{\partial U^{n+1}}{\partial d}, \text{ is then the solution of the linear system of matrix } \left( \frac{\partial f}{\partial U^{n+1}} \right):
\]

\[
\frac{\partial f}{\partial U^{n+1}} Y^{n+1} = -\left( \frac{\partial f}{\partial f} - \frac{\partial f}{\partial U^n} Y^n \right)
\]

(4)

No matter what parameter \( d \) may be, the matrix of the system is the same as that of Newton's last iteration to solve the equation in \( U \). This matrix is computed only once.

However, the second member changes for each \( d \). We thus have to solve at each time step, in addition to Newton's iterations, as many linear systems as parameters.

To reduce the number of these parameters, the "conventional" gradient method, such as is programmed in SCOREGRAD, consists of cutting up the reservoir into zones to which constant petrophysical values are assigned. We then compute parameter contribution for each zone by solving a system of type (4).

The gradient method described below proposes to decrease the number of parameters by using a modeling of the reservoir in lithofacies.

GRADIENT METHOD BY LITHOFACIES

Principle of the Method

During the construction of a meshed model of an oil field, at one stage we have a geologic model and a description of this reservoir in lithofacies. This description may be the construction of a geostatistical model or a deterministic construction of an geologic object model or the interpretation of well-tests.

The principle of the gradient method by lithofacies is to use the natural zoning of the field, as defined in this geologic description.

The influence zones are therefore defined a priori, and we compute the influence of lithofacies on the pressure and saturation ranges in the field via the petrophysical parameters that are associated with it.
This gradient method by lithofacies has been initially developed to constrain geological modeling by well-tests. It therefore applies to single-phase flows in which the unknown is the pressure range.

If we call \( \{ f_i, i = 1, N \} \) the lithofacies of the geological model and \( \{ \phi_{f_i}, K_{f_i}, i = 1, N \} \) the associated porosities and permeabilities, we can compute \( \frac{\partial P}{\partial \phi_{f_i}} \) and \( \frac{\partial P}{\partial K_{f_i}} \) for each lithofacies \( f_i \).

### Derivative of Discrete Equations in Single-Phase Flow

For the single-phase flow of a relatively incompressible fluid, the equation system is reduced to:

\[
C \phi \frac{\partial P}{\partial t} - \text{div} \left( \frac{K}{\mu} \text{grad}(P) + \rho g z \right) = q_w
\]

with zero flow conditions or pressure imposed at the edges. \( C \) represents the full compressibility (pores and fluid), assumed to be constant in the model, \( \mu \) the fluid viscosity, \( g \) the gravity, \( \rho \) the fluid density and \( q_w \) is a sink term.

After discretizing in time and space using a finite-volume method, the equation becomes:

\[
CV_i \phi_i \left( \frac{P_i^{n+1} - P_i^n}{t^n} - \frac{1}{\mu} \sum_{v(i)} T_{iv(i)} \right) \left[ \left( P_{v(i)}^{n+1} - P_i^{n+1} \right) + \rho g (z_{v(i)} - z_i) \right] = q_w \delta_{iw}
\]

with:
- \( i \) grid-block index
- \( V_i \) volume of grid-block \( i \)
- \( v(i) \) set of neighbors of grid-block \( i \)
- \( \delta_{iw} = 1 \) if well \( w \) is in grid-block \( i \)
- \( T_{n(i)} \) the transmissivity between grid-blocks \( i \) and \( v(i) \)

If the system is written in matrix form, calling \( A \) the matrix and \( B \) the second member, we obtain:

\[
A^n P^{n+1} = B^n
\]

In addition, if we want to compute \( \Delta P \), the variation of the pressure range at each time step, is as follow:

\[
A^n \Delta P^{n+1} = B^n - A^n P^n
\]

To determine the influence of the permeability of lithofacies \( f_j \) on the values of the pressure range, we attempt to compute \( \frac{\partial P}{\partial K_{f_j}} \).

From Equation (6) in relation to \( K_{f_j} \), let us derive:

\[
\frac{\partial P}{\partial K_{f_j}} = \frac{C \phi_i V_i}{(r^{n+1} - r^n)} \left[ \frac{\partial P_{v(i)}^{n+1}}{\partial K_{f_j}} - \frac{\partial P_i^{n+1}}{\partial K_{f_j}} \right] + \frac{1}{\mu} \sum_{v(i)} \left( \frac{1}{\partial P_{v(i)}^{n+1}} - \frac{1}{\partial P_i^{n+1}} \right)
\]

By stating

\[
Y_{K_{f_j}}^n = \frac{\partial P^n}{\partial K_{f_j}} \quad \text{and} \quad \Delta Y_{K_{f_j}}^{n+1} = \frac{\partial P^{n+1}}{\partial K_{f_j}} - \frac{\partial P^n}{\partial K_{f_j}}
\]

the system of derivative equations can be written in a matrix form:

\[
A^n \Delta Y_{K_{f_j}}^{n+1} = D_{K_{f_j}}^{n+1}
\]

of which

\[
d_{K_{f_j}}^{n+1} = \frac{1}{\mu} \sum_{v(i)} \left( \frac{1}{\partial P_{v(i)}^{n+1}} - \frac{1}{\partial P_i^{n+1}} \right) + \frac{1}{\mu} \sum_{v(i)} \left( \frac{1}{\partial P_{v(i)}^{n+1}} - \frac{1}{\partial P_i^{n+1}} \right)
\]

is the second member of the gradient.

If we presume that the transmissivities have the form:

\[
T_{iv(i)} = \frac{\alpha K_i K_{v(i)}}{\beta K_i + \lambda K_{v(i)}}
\]

in which \( \alpha, \beta, \lambda \) are the functions of the geometry of grid-blocks \( i \) and \( v(i) \), we can explain the derivatives in relation to the permeabilities. Four cases are possible depending on what lithofacies are present in the grid-blocks \( i \) and \( v(i) \).

### IMPLEMENTATION OF THE METHOD IN A NUMERICAL MODEL FOR WELL-TESTS

This gradient method by lithofacies has been programmed in a simulation model of well-tests with the following characteristics:
Gradients Method Constrained by Geological Bodies for History Matching

ECMOR V, 1996

- the diagram is implicit,
- the wells are coupled,
- the relation between grid-block pressure and the bottomhole flow pressure is defined by a numerical productivity index (NPI) function taking into account the skin, the permeability of the grid-block and the equivalent radius of the borehole computed by the Peaceman rule [9,10],
- the discretized system is expressed in pressure difference, \( \Delta P \), between \( t \) and \( t + \Delta t \) times,
- the boundaries may be either with imposed flow or with imposed potential.

This numerical simulation model called SIMTESTW has been presented by G. Blanc et al [5].

Derivative of Borehole Equations

Additional equations, one per borehole, due to borehole coupling, are derived in relation to the permeability of the lithofacies of the well grid-block.

They are written in the following form:

\[
\sum_i NPI(i)(P_i^{n+1} - P_w^{n+1}) = q_w \tag{7}
\]

with \( i \) = grid-block index in which the borehole is open; \( NPI(i) \) borehole function and \( q_w \) flow rate of borehole \( w \).

By deriving (7) in relation to \( K_{f_j} \), we obtain:

\[
\sum_i NPI(i) \left( \frac{\partial H_i^{n+1}}{\partial K_{f_j}} - \frac{\partial H_p^{n+1}}{\partial K_{f_j}} \right) = \\
- \sum_i \frac{\partial NPI(i)}{\partial K_{f_j}} (H_i^{n+1} - H_p^{n+1})
\]

with \( \frac{\partial NPI(i)}{\partial K_{f_j}} = \frac{NPI(i)}{K_{f_j}} \) if grid-block \( i \) contains the lithofacies \( K_{f_j} \) (if not, 0).

Algorithm

As seen above, the matrix of the gradient system is the same as the one for the pressure system. However, the second member is different.

To compute the gradient range at \( t^{n+1} \) time, we have to know the pressure range at \( t^{n+1} \) time and the gradient range at \( t^n \) time because they intervene in the second member of the system.

The algorithm of the model, modified for computing the gradients, is as follows:

- Loop for \( t^{n+1} \) times.
  - Computing matrix \( A \) of the system (if the time steps are constant, the matrix remains constant for single-phase flow),
  - Computing the right hand side of system in \( \Delta P \),
  - Solving the system in \( \Delta P \),
  - Loop for \( d(i) \) parameters in relation to which \( P \) is derived.

\[
< \text{Computing the right hand side of the gradient system making } P^{n+1} \text{ and } \frac{\partial P^n}{\partial d(i)} \text{ intervene,}
\]

\[
< \text{Solving the system in } \frac{\partial \Delta P}{\partial d(i)},
\]

- End of the loop on \( d(i) \)

- End of the time loop.

The programming of the computing part of the gradients simply requires the addition of subroutines for constructing the right hand sides and the transmissivity derivation functions. The matrix of the pressure system is not modified, and so the same solver can be used for the different gradient systems. Likewise the use of a good preconditioning algorithm, which will be run once, is very interesting in order to save computation time.

In the present version, we are interested in the pressure gradients relating to the horizontal and vertical permeabilities and porosity associated with each lithofacies. We have also developed methods to compute gradients relating to the well skin and to the well bore storage since these variables appear explicitly in the equations.

Pressure gradients relating to mean value and standard deviation of a normal-log permeability and porosity distribution associated to a lithofacies have also been developed.

The general case of non-uniform petrophysical distribution assigned to a lithofacies can also be carried out. We then compute gradients relating to a coefficient of translation and a porosity or permeability distribution multiplier.

All these developments will be presented at the 1996 SPE Fall meeting in Denver.

VALIDATION

The simplest validation test consists of comparing the results from the solving of the system of derivative equations to the values obtained by "numerical" gradient computing.

286
The principle of the numerical gradient is to compute two different potential ranges for two neighboring values $d_1$, $d_2$ for a given parameter.

The gradient is then approached by:

$$\frac{\partial P}{\partial d} \approx \frac{P(d_2) - P(d_1)}{d_2 - d_1}$$

The results are presented in graph form of showing the evolution during well-test simulation of the pressure-gradient values at the borehole relating to the horizontal or vertical permeability of lithofacies.

Case 1 - Homogeneous Reservoir

This first model is a homogeneous monolayer reservoir horizontally 1350 m by 1350 m and 10 m thick. The porosity is set to 20%.

The well, of a 7.85 cm radius, is located at the center of the model and fully penetrates the reservoir. The well bore storage and the skin are set to 0.

The meshed model consists of 33x33x1 grid-blocs all containing the same lithofacies.

The well-test for the validation is a simulated draw-down of about 1 day with a constant flow-rate of 50 m$^3$/day.

The numerical pressure gradient is computed by taking two successive horizontal permeability values of 100 and 100.1 mD. The analytical pressure gradient is computed by solving the system of derivative equations corresponding to $K_h = 100$ mD.

The comparison of the results gives a maximum difference of less than 1% (Figure 1).

Case 2 - Centered Channel Model

The same test is performed for a model with 2 lithofacies as a channel parallel to the X axis and centered in the reservoir.

The lateral extent of the reservoir are 2700 m on X direction and 900 m on Y direction and its thickness is of 10 m. The channel is 290 m wide. The porosity of each lithofacies is set to 20%.

The well, of 7.85 cm radius, is located at the center of the model and fully penetrates the reservoir. The well bore storage is set to 10$^{-2}$ m$^3$/bar and the skin effect to -1.

The mathematical model is made up of 8649 irregular grid-blocks:
- 33x33 horizontal grid-blocks,
- 2 vertical grid-blocks per layer.

The well-test for the validation is a simulated draw-down of about 1 day with a constant flow-rate of 50 m$^3$/day.

The vertical permeability values used to compute the numerical gradient of well pressure with respect to the lithofacies #2 (middle layer lithofacies) are 2 mD and 2.01 mD. The analytical pressure gradient is computed by solving the system of derivative equations corresponding to $K_v = 2$ mD.

A comparison of the results shows a difference of less than 1% (Figures 2) between the numerically estimated gradient and the one obtained by solving the system of derivative equations in relation to the permeability.

Case 3 - Multilayer Model

To validate the computing of the gradient relating to a vertical permeability, we test a "homogeneous multilayer" case.

The reservoir is made up of 3 layers each corresponding to a lithofacies and allowing vertical flow involving vertical permeability values.

The horizontal extent is 1350 m in each direction and the thickness of the 3 layers is respectively 7, 12 and 10 m from the bottom to the top. The porosity of each layer is set to 20%.

The horizontal permeability of the 3 lithofacies is respectively 10, 200, 50 mD and the anisotropy ratio between vertical and horizontal permeability is set to 0.01 for each lithofacies.

The well, whose radius is always 7.85 m, partially penetrates the reservoir and is only open in the middle layer. It's located in the center of the model.

The well bore storage is set to 10$^{-2}$ m$^3$/bar and the skin effect to -1.

The mathematical model is made up of 6534 irregular grid-blocks:
- 33x33 horizontal grid-blocks,
- 2 vertical grid-blocks per layer.

The well-test for the validation is a simulated draw-down of about 1 day with a constant flow-rate of 50 m$^3$/day.

The vertical permeability values used for computing the numerical gradient of well pressure with respect to the lithofacies #2 (middle layer lithofacies) are 2 mD and 2.01 mD. The analytical pressure gradient is computed by solving the system of derivative equations corresponding to $K_v = 2$ mD.

The comparative results are shown in Figure 3. As we can see the results are similar and the analytical solution is much more regular than the numerical one.
FIRST APPLICATION CASES

Several results obtained with our gradients method are given here. The examples presented here are based on the synthetic models detailed above in which we try to reverse the petrophysical parameters by simulating well-tests. More realistic application cases were presented at the SPE/NPF European Conference held in Stavanger in April 1996 [6].

The following method is used to generate synthetic well-tests:

• a meshed geologic model is constructed with lithofacies,
• constant permeabilities are associated with each lithofacies,
• an initial pressure reference simulation is computed,
• the initial petrophysical values are changed, and we attempt to recover them again by using an objective function measuring the difference between the well pressure computed by simulation and the one referenced.

A least-squares objective function is calculated as follow:

\[ J(K_f) = \frac{1}{2} \sum_{i=1}^{n} w_k (P_w^c - P_w^o)^2 \]

in which:

• \( K_f \) represents the vector of permeabilities associated with lithofacies,
• \( n \) is the number of data points measured,
• \( w_k \) is positive weight applied to each data point,
• \( o \) and \( c \) refer respectively to observed and computed data.

Minimizing this objective function means cancelling out its gradients relating to \( K_f \). By deriving the \( J \) function relating to \( K_f \), the \( \frac{\partial P}{\partial K_f} \) appear.

Different efficient optimization algorithms can be used when analytical gradients are supplied. We performed these tests with the Gauss-Newton, Powell and Levenberg-Marquardt algorithms [7,8].

The iterative optimization process stops when the objective function value is smaller than a given threshold.

Homogeneous Case

The reservoir model and the mathematical model are those presented above. The well-test is a synthetic build up run with the numerical model. The permeability and porosity are set respectively to 100 mD and 20%. A constant flow rate of 50 m3/day was maintained during 105 s (about one day) to simulate a drawdown and the build up was simulated for 2 days.

The reservoir permeability and porosity are the parameters of the inversion process which was initiated with a value of \( K \) equal to 500 mD and \( \Phi \) equal to 40%.

The reference values were recovered in 5 iterations with Powell algorithm and the stop criterion set at 0.01%.

The well pressure curve and its time derivatives are given in Figure 4 with the initial guess values of the inversion process and with the final iteration values.

Figures 4 also shows the evolution of the values of \( K, \Phi \) and the objective function during the iterations.

Channel Case

The reservoir model and the mathematical model are detailed above. The well-test is a synthetic build up run with the numerical model. The permeability and porosity of lithofacies 1 (outside the channel) and 2 (channel) are set respectively to 10 mD, 500 mD and 20%. A constant flow rate of 50 m3/day was maintained during 2 days to simulate a drawdown and the build up was simulated for 5 days.

The two lithofacies reservoir permeabilities and the skin are the parameters in the inversion process which was initiated with an homogeneous value of \( K \) equal to 100 mD and a zero skin.

The reference values were recovered in 15 iterations starting with Powell algorithm and finishing with Levenberg-Marquard. The stop criterion was set to \( 10^{-8} \).

Figure 5 presents:

• the well pressure curve and its time derivatives with initial guess values of the inversion process and with final iteration values,
• the evolution of the values of \( K \), well skin and the objective function during the iterations.

Multilayer Case

The reservoir model and the mathematical model are those presented above. The well-test is a synthetic build up run with the numerical model. The permeabilities of the three lithofacies are respectively 50, 200, 10 mD from the bottom to the top layer and the porosity is set to 20%. A constant flow rate of 50 m3/day was maintained during \( 10^5 \) s (about one day) to simulate a drawdown and the build up was simulated for 2 days.

The horizontal permeability of the 3 lithofacies of the reservoir were the parameters of the inversion process which was initiated with an homogeneous value of \( K \) equal to 500 mD.

The anisotropic ratio between vertical and horizontal permeability is forced to stay at 0.01 during the inversion process.
The reference values were recovered in 8 iterations with Powell algorithm and the stop criterion set to $10^{-8}$. The results are shown on Figure 6.

**CONCLUSIONS**

A new gradient method applied to reservoir engineering, and in particular to well-tests, has been developed. This method is programmed into a numerical simulator for well-tests. It represents a type of assistance for constructing a reservoir model around the borehole.

We validated the programming by comparing the results obtained with a numerical derivation.

We checked the advantage of the method by integrating the pressure gradients obtained into a parameter-inversion process for simple cases. The computerized performances also suggest that the software should have industrial uses.

As the matrix remains constant for the linear systems in pressure and in gradients, the use of a good preconditionning algorithm largely reduce the computation time.

This technique has been tested with more complex cases, issued from actual data, to check its sturdiness and to determine the best conditions for its use [6]. All of these tests have revealed the importance of this method within the particular framework of well-tests. The inversion of production data is possible and it is undoubtably a great help in the construction of a reservoir model.

We are now focusing our researchs on the inversion of geometric parameters such as the shape of geological bodies and reservoir limits.

**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>computed data point</td>
</tr>
<tr>
<td>$d$</td>
<td>parameter</td>
</tr>
<tr>
<td>$o$</td>
<td>observed data point</td>
</tr>
<tr>
<td>$w$</td>
<td>well</td>
</tr>
</tbody>
</table>

**ACKNOWLEDGMENTS**

The authors thank Elf Aquitaine Production and the Institut Français du Pétrole for their permission to publish this article. All participants to the HELIOS reservoir project are thanked for their valuable discussions on all aspects of the work.

**REFERENCES**


Figures 1, 2, 3 - Gradients Validation
Figures 4 - Homogeneous Application Case
Figures 5 - Channel Application Case
Figure 6 - Multilayer Application Case