Direct Calculation of Large-Scale Properties for One-Phase Flow in Random Fractured Systems

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ABSTRACT

Double-porosity, double permeability models are often used to describe flows in fractured systems, or systems with bi-modal permeability density probability functions. The relationship between local-scale properties, and large-scale properties has been revisited from the point of view of the large-scale volume averaging technique. The resultant large-scale two-equation model represents an extension of double-porosity, double permeability models. This approach is particularly attractive since local-scale closure problems are available that can be solved to calculate the large-scale properties such as the large-scale permeabilities, and the inter-porosity flow factor.

Examples of such calculations are provided for simple 2D unit cells representative of fractured systems. Numerical experiments are presented in order to test the theory. Finally, special unit cells representing systems with randomly distributed fractures are studied. The behaviour of the resulting large-scale parameters is studied as a function of the statistical properties of these systems.

1. INTRODUCTION

Flows in heterogeneous porous media have an important role in petroleum industry, hydrology, pollution and decontamination of aquifers, chemical engineering, civil engineering, etc. ... Many natural heterogeneous media are fractured and some of the world's most productive oil and gas reservoirs are in naturally fractured formations. Fractured porous media can be considered as a particular and rather complex case of heterogeneous porous media, in which the geometry of the discontinuities representing the fractures has an important impact on the connectivity of the fracture network. In this work, we will focus our attention on single-phase flow of a slightly compressible fluid in a well-connected fractured system (we are above the percolation threshold).

At the reservoir or aquifer scale, flows in heterogeneous porous media are often treated in terms of

one-equation models, thus assuming local mechanical equilibrium between regions possessing highly contrasted properties. When the assumption of local mechanical equilibrium is not verified, one must call upon n -region models. As fractured formations are composed of two regions (matrix and fracture) with rather contrasted properties, we can expect the behaviour of such a medium to be very different from a conventional porous medium and seek a suitable model for studying them.

Two-region models have been introduced empirically by Barenblatt and Zheltov (1960) a few decades ago and different variations have been developed since (see review in Chen, 1989). In these models, flow is described by two transport equations for the regional averaged pressures that are coupled through an exchange term. As an extension to this model, the large-scale averaging method has been used to derive a two-region model starting from transport and continuity equations for each homogeneous region at Darcy scale. By this method additional coupling terms appear in the two-region model that did not exist in the primary developments and the permeability tensors and exchange coefficients can be determined theoretically without using any adjustable parameters.

In this work, we consider 2D and 3D unit cells representative of fractured systems. The corresponding large-scale permeability tensors and exchange coefficients are determined using the large-scale averaging method for a fractured porous medium. Large-scale properties obtained are compared to the ones obtained using an industrial simulator by performing well tests. In order to get closer to physical reality, randomly fractured porous media have been generated. By a statistical study over a large number of realisations, the behaviour of large-scale two-equation model properties for random fractured media has been investigated.

2. TWO EQUATION MODEL

Assuming pseudosteady-state interporosity flow, Barenblatt and Zheltov (1960) described the flow of a homogeneous and slightly compressible fluid in a double-

porosity double-permeability medium by a system of two coupled equations for the matrix and fracture pressures:

$$\phi_m c_m \frac{\partial P_m^*}{\partial t} = \nabla \cdot \left(\frac{K_{mm}}{\mu} \cdot \nabla P_m^* \right) - \frac{\alpha}{\mu} (P_m^* - P_f^*) \dots (1)$$

$$\phi_f c_f \frac{\partial P_f^*}{\partial t} = \nabla \cdot \left(\frac{K_{ff}}{\mu} \cdot \nabla P_f^* \right) - \frac{\alpha}{\mu} (P_f^* - P_m^*) \dots (2)$$

where c_m and c_f are total compressibilities, and ϕ_m and ϕ_f are the matrix and fracture volume fractions respectively. In addition to diffusive and accumulation terms, these equations include coupling terms modelling the exchanges by pressure diffusion between the two regions. The assumption of pseudosteady-state regime means that these coupling terms do not depend explicitly on time and the corresponding exchange coefficient (α) is a constant. This condition limits the validity of the model to a quasi-stationary evolution of the pressure.

Since the mathematical model proposed by Barenblatt and Zheltov (1960), many authors have worked on the analytical solution of this model and its derivations or extensions (Barenblatt et al., 1960; Warren and Root, 1963; Kazemi, 1969; Kazemi et al., 1969; de Swaan, 1976; Arbogast et al., 1990). For a review of these works see Chen 1989.

As an extension to the Barenblatt and Zheltov model (1960), large-scale averaging method has been used to derive a two-region model starting from the equations for each homogeneous region at Darcy scale (Quintard and Whitaker, 1996a and b)

$$c_m \frac{\partial P_m}{\partial t} = \nabla \cdot \left(\frac{K_m}{\mu} \cdot \nabla P_m \right) \text{ in the matrix} \dots (3)$$

$$c_f \frac{\partial P_f}{\partial t} = \nabla \cdot \left(\frac{K_f}{\mu} \cdot \nabla P_f \right) \text{ in the fracture} \dots (4)$$

with the following boundary conditions at the matrix-fracture interface (A_{mf})

$$C.1 \quad P_m = P_f \dots (5)$$

$$C.2 \quad \mathbf{n}_{mf} \cdot (\mathbf{K}_m \cdot \nabla P_m) = \mathbf{n}_{mf} \cdot (\mathbf{K}_f \cdot \nabla P_f) \dots (6)$$

Large-scale quantities are defined using the following spatial averages for $\alpha = m$ or f :

$$\{P_\alpha\} = \frac{1}{V_\alpha} \int_{V_\alpha} P_\alpha dv = \phi_\alpha \{P_\alpha\}^\alpha \dots (7)$$

where $\phi_\alpha = V_\alpha/V$ with V large-scale averaging volume and V_α the α -region volume contained in V .

The following decompositions (Gray, 1975) are then introduced:

$$P_m = \{P_m\}^m + \hat{P}_m \dots (8)$$

$$P_f = \{P_f\}^f + \hat{P}_f \dots (9)$$

Large-scale averaging of local equations lead to a problem containing three source terms in which the deviations remain. Three closure problems are obtained by seeking expressions of the deviations as a function of the sources terms as follows:

$$\hat{P}_m = \mathbf{b}_{mm} \cdot \nabla \{P_m\}^m + \mathbf{b}_{mf} \cdot \nabla \{P_f\}^f - S_m (\{P_m\}^m - \{P_f\}^f) \dots (10)$$

$$\hat{P}_f = \mathbf{b}_{ff} \cdot \nabla \{P_f\}^f + \mathbf{b}_{fm} \cdot \nabla \{P_m\}^m - S_f (\{P_f\}^f - \{P_m\}^m) \dots (11)$$

This procedure leads to the following large-scale equations for single-phase flow of a slightly compressible fluid in a medium composed of symmetric unit cells:

$$\phi_m c_m \frac{\partial \{P_m\}^m}{\partial t} = \nabla \cdot \left(\frac{K_{mm}}{\mu} \cdot \nabla \{P_m\}^m + \frac{K_{mf}}{\mu} \cdot \nabla \{P_f\}^f \right) - \frac{\alpha}{\mu} (\{P_m\}^m - \{P_f\}^f) \dots (12)$$

$$\phi_f c_f \frac{\partial \{P_f\}^f}{\partial t} = \nabla \cdot \left(\frac{K_{fm}}{\mu} \cdot \nabla \{P_m\}^m + \frac{K_{ff}}{\mu} \cdot \nabla \{P_f\}^f \right) - \frac{\alpha}{\mu} (\{P_f\}^f - \{P_m\}^m) \dots (13)$$

A comparison of these equations with the ones proposed by Barenblatt and Zheltov (1960) reveals additional coupling terms containing the cross effect permeability tensors K_{mf} and K_{fm} (with $K_{mf} = K_{fm}$). The permeability tensors and exchange coefficients can be determined theoretically by solving the associated closure problems (Quintard and Whitaker, 1996a) over a representative unit cell by only using local scale properties and the geometry of the heterogeneities. The expressions of these large-scale properties as a function of the closure variables (S_m , S_f , $\mathbf{b}_{mm}$, $\mathbf{b}_{mf}$, $\mathbf{b}_{ff}$) are given below:

$$\alpha = \frac{1}{V} \int_{A_{mf}} \mathbf{n}_{mf} \cdot \mathbf{K}_m \cdot \nabla S_m dA = - \frac{1}{V} \int_{A_{mf}} \mathbf{n}_{mf} \cdot \mathbf{K}_f \cdot \nabla S_f dA \dots (14)$$

$$\mathbf{K}_{mm} = \mathbf{K}_m \cdot \left(\phi_m \mathbf{I} + \frac{1}{V} \int_{A_{mf}} \mathbf{n}_{mf} \cdot \mathbf{b}_{mm} dA \right) \dots (15)$$

$$\begin{aligned} K_{mf} = K_{fm} = K_m \left(\frac{1}{V} \int_{A_{mf}} \mathbf{n}_{mf} \cdot \mathbf{b}_{mf} dA \right) \\ = K_f \left(\frac{1}{V} \int_{A_{mf}} \mathbf{n}_{fm} \cdot \mathbf{b}_{fm} dA \right) \end{aligned} \quad (16)$$

$$K_{ff} = K_f \left(\phi_f I + \frac{1}{V} \int_{A_{mf}} \mathbf{n}_{mf} \cdot \mathbf{b}_{ff} dA \right) \quad (17)$$

We must point out that the periodic assumption which leads to the calculation of these parameters on a single unit cell is widely used in science. The domain on which we perform our calculations is therefore considered to be a unit cell in a spatially periodic model of a heterogeneous porous medium. The periodicity condition can only be justified if the variations of K_m and K_f and some large-scale averaged quantities are negligible within the unit cell as it is emphasised in Quintard and Whitaker (1990).

We must also note that another difference between this procedure and the Barenblatt and Zheltov (1960) model is that this technique is not restricted to isotropic systems either at Darcy scale or at the large-scale. In addition, the form of the heterogeneities in a unit cell is no obstacle for the resolution of the closure problems. It must be noted that this development has been possible under the assumption of scales separation and some additional time scale constraints. In particular, mapping quantities in Eqs. 10 and 11 are not time dependent. The local-scale pressure fields are time dependent through their relationship (the closure problems) with the large-scale source terms expressed by these equations. This of course imposes some limitations to this two equation model, especially in the case of large transient effects. As in the Barenblatt and Zheltov model (1960), this closure-level quasi-steady state assumption leads to a constant exchange coefficient α .

The closure problems obtained can be solved analytically for a regularly shaped fractured porous medium (an array of low permeability blocks separated by a network of high permeability fractures, for a 2D example see figure 5) in the limiting case of infinitely permeable fractures ($K_f/K_m \rightarrow \infty$) with negligible thickness ($\phi_f \rightarrow 0$). The expressions found can be compared to several estimates commonly used in reservoir engineering that can be found in the literature (table 1). While the values proposed by Warren and Root are close to the theoretical predictions, those given by Kazemi et al. are three to four times lower. Additional tests have been performed for macroscopically 1D fractured media (1D array of 2D unit cells) using numerical resolutions of the closure problems (Quintard and Whitaker, 1996b; Ahmadi et al., 1996). Comparison between two-equation model results and 2D fine grid simulation results show that the two-equation model allows a correct representation of the pressure field. In the present work we consider macroscopically 2D flows.

Table 1 - Exchange coefficients α/K_m for fractured systems (Quintard and Whitaker, 1996b)

	Stratified System	2D-Rectangular System	3D-Parallelepiped system
Warren and Root (1963)	$12 \left(\frac{1}{l_m^2} \right)$	$16 \left(\frac{2}{l_m^2} \right)$	$20 \left(\frac{3}{l_m^2} \right)$
Kazemi et al. (1976, 1992)	$4 \left(\frac{1}{l_m^2} \right)$	$4 \left(\frac{1}{l_x^2} + \frac{1}{l_y^2} \right)$	$4 \left(\frac{1}{l_x^2} + \frac{1}{l_y^2} + \frac{1}{l_z^2} \right)$
Quintard and Whitaker (1993)	$12 \left(\frac{1}{l_m^2} \right)$	$14.22 \left(\frac{1}{l_x^2} + \frac{1}{l_y^2} \right)$	$16.54 \left(\frac{1}{l_x^2} + \frac{1}{l_y^2} + \frac{1}{l_z^2} \right)$

3. NUMERICAL RESOLUTION OF CLOSURE PROBLEMS

The three above-mentioned closure problems have been solved in the 2D and 3D case, using a finite volume formulation on regular grid-blocks.

A number of preliminary tests have been performed in order to validate the application of the numerical model to fractures oriented in any direction. The matrix and fracture regions are considered homogeneous and isotropic with $K_m = IK_m$ and $K_f = IK_f$. The differences between large-scale exchange coefficients obtained by the two models illustrated in Fig. 1 are studied as a function of grid refinement for different values of $R = K_f/K_m$ (10, 100 and 1000), and ϕ_f (volume fraction of the fracture region). The simulations are more rapid and more precise for important values of R and ϕ_f . In all cases studied, for 180 x 180 grids in each unit cell, the difference between the exchange coefficient obtained by the two models is less than 1%. The model is therefore suitable for simulations with fractures oriented in any direction as long as a sufficient number of grid blocks are used.

4. COMPARISON WITH WELL TESTS

A number of 2D Numerical well-test simulations are performed using an industrial simulator (SIMTEST). The pressure response given by SIMTEST is then inverted to determine the most suitable set of two-equation model parameters. The matching is based on analytical solution of a model of the type Warren and Root and is adjusted by non-linear regression. The objective is to compare these results with the ones obtained by the resolution of the closure problem. We have therefore chosen to perform numerical simulations on a surface containing a "sufficient" number of rectangular unit cells. It must be noted that the number of representative unit cells used in the numerical simulations is limited by computation costs (time, memory). For each test case, identical number of grid blocks are used for modelling a unit cell in the SIMTEST simulation and in the two-equation model.

The fluid is injected at constant flow rate in the first grid block (in the fracture) located at the bottom left of the medium. In this manner, by imposing zero flux at the

lower and left boundaries, the simulation corresponds to a medium four times the modelled region in which the fluid is injected at the center (Fig. 2).

4.1. RESULTS

The numerical simulations provide us with pressure fields over the medium for any time and the pressure profile at the well as a function of time. A study of the pressure fields reveals a 1D diffusion phenomenon at small times, then flows involving several unit cells. Matrix-fracture exchanges are only observed at greater times. It is therefore important to choose suitable simulation parameters, in order for the matrix-fracture exchanges to occur before reaching the limits' effects.

Indeed, some physical criteria must be taken into account in order to be able to observe the matrix-fracture exchanges. Consider a fractured medium of dimension $L \times L$, in which the matrix blocks are squares of dimension $l_m \times l_m$. The unit cell is a square of dimension $l_c \times l_c$ with $l_c = l_m + l_f$ (see figure 5). With l_f small compared to l_m we have $l_m \approx l_c$. We can define two characteristic times:

- characteristic diffusion time corresponding to the minimum time necessary for the diffusion to reach the limits of the medium:

$$t_D = O\left(\frac{L^2}{k_f}\right) \dots\dots\dots (18)$$

with $k_f = K_f/(\epsilon_f \mu c_f)$.

- characteristic exchange time corresponding to the minimum time necessary for the matrix-fracture exchanges to occur:

$$t_E = O\left(\frac{l_m^2}{k_m}\right) \dots\dots\dots (19)$$

with $k_m = K_m/(\epsilon_m \mu c_m)$.

A necessary condition for having a double-region behaviour, that is for the matrix-fracture exchanges to have the time to occur is:

$$t_E \ll t_D \dots\dots\dots (20)$$

leading to the following condition:

$$\left(\frac{l_m}{L}\right)^2 \ll \frac{k_m}{k_f} \dots\dots\dots (21)$$

We must also point out that at small times, that is for $t < l_m^2/k_f$, the simulation corresponds to 1D flow in the fractures. For very small times, on the order of l_m^2/k_f , the flow takes place in a small number of unit cells and the assumption of separation of scales does not hold. In this latter case, the bulk averaged equations (whatever the one or two-equation models used) are no longer valid. This call for a specific treatment of the boundary region near the well which is beyond the scope of this paper. Taking into account the above-mentioned conditions, the

following properties have been considered for the medium:

The medium is composed of 20×20 representative unit cells. Each unit cell is composed of 15×15 numerical grid-blocks ($l_g/L=0.05$ and $l_g/l_c = 1/15$). The matrix and the fracture are considered homogeneous and isotropic with $K_m = IK_m$ and $K_f = IK_f$ with $K_f = 100$ mD and $K_m = 1$ mD ($K_m/K_f = 0.01$). The matrix and fracture have the same porosity. By positioning the well at an angle between two zero flux boundaries we gain a factor 4 on the grid-block sizes.

The resolution of the closure problems corresponding to the two-equation model leads to the large-scale permeability tensors and the exchange coefficient.

In this particular case, the cross effect permeability tensors are found to be negligible (see Quintard and Whitaker, 1996) and the large-scale permeability tensors K_{ff}^* and K_{mm}^* are isotropic.

The two-equation model gives the following results:

$$K_{ff} = 6.8 \text{ mD}; K_{mm} = 0.82 \text{ mD}; \alpha = 0.011$$

The properties obtained by direct numerical simulation and inversion of the well pressure results are:

$$K_{ff} = 7.53 \text{ mD}; K_{mm} = 0.17 \text{ mD}; \alpha = 0.014$$

First of all, the precision of our comparison is limited by calculation costs and computer capacities. In particular, while the closure problems can be solved over a *single unit cell*, field-scale numerical tests involve many unit cells and achieving a good accuracy is limited due to computational costs. By a comparison of the initial pressure data and the matched one, we notice that the matching is not completely satisfactory. The main differences observed in the pressure data is at small times. It is encouraging to notice, however, that the results obtained by the well test are of the same order of magnitude as the ones obtained by the two-equation model. This emphasises the fact that classical well test interpretative techniques can have a poor accuracy. It could be interesting to double check the results with direct predictions over representative unit cells.

5. RANDOM FRACTURED MEDIA

Natural geological structures are most often found to be fractured in a random manner. In order to get closer to this physical reality, stochastically fractured media have been generated.

The study of random fractured media calls upon the concept of percolation. The percolation threshold corresponds to the volume fraction of fractures necessary for the fracture network to be statistically connected.

In a square reservoir $L \times L$ containing fractures of length l and thickness ξ , if we denote the density of fractures by ρ , the number of fractures is given by:

$$N = \rho L^2 \dots\dots\dots (22)$$

We can therefore define "fracture disks" (in the 2D case) as being circles centered at the center of the fracture and of diameter equal to the fracture length. The fracture network is connected if the ensemble of "fracture disks"

fill the reservoir completely. This leads to the following definition of N_c the number of fractures corresponding to the percolation threshold.

$$N_c \pi \left(\frac{l}{2} \right)^2 \approx L^2 \quad (23)$$

The critical fracture density will therefore be:

$$\rho_c \approx \frac{4}{\pi l^2} \quad (24)$$

Thus giving the following expression for the critical fracture volume fraction corresponding to the percolation threshold:

$$\phi_c = \rho_c l \xi \approx \frac{4\xi}{\pi l} \quad (25)$$

In our case with $L = 40$, $l = 8$ and $\xi = 2$, ϕ_c will approximately be 25%.

5.1 RESULTS

For given fracture density and fracture-matrix permeability ratio, the large-scale two-equation model properties are calculated over a number of realisations of the same medium (Fig. 3).

For given fracture density and permeability ratio, a statistical study (over 51 realisations of the same reservoir) has been performed. Large-scale two-equation model properties are calculated for each realisation. The following discussion is based upon the mathematical expectations of the large-scale two-equation model properties.

First of all the large-scale tensors obtained are basically diagonal, the extra-diagonal terms obtained by the numerical procedure are several orders of magnitude smaller than the diagonal terms.

Effective permeability of fractures

The macroscopic permeability of the fractures increases as a function of the average fracture volume fraction and tends to the local-scale fracture permeability (Fig. 4). One should note that the fracture permeability is not equal to zero below the percolation threshold for the fracture network (i.e., the system when $K_m = 0$). This is due to the fact that the matrix contributes to the permeability of the fractures, something that should not be forgotten when performing such analyses. Of course, this phenomenon is not important when R tends to infinity, and results with $R=1000$ do have the percolation threshold limit.

We have compared our results to those reported by Quintard and Whitaker (1996b) for a regularly shaped porous medium of the type Warren and Root (Fig. 5). They have calculated the large-scale properties for $R = 10$ and $R = 1000$ (Fig. 6). The ratio $K_{mm}/\phi_m K_m$ tends to 0.5 for the regularly fractured medium case; obviously this limit is not respected in the case of stochastic porous media. This emphasises the importance of the geometry of the unit cell. As a consequence, one must be careful

when using estimates of the effective properties based only on the permeabilities, the fracture volume ratio, and some characteristic length.

Effective permeability of the matrix

The macroscopic matrix permeability is very close to the product of the matrix volume fraction by the initial matrix permeability (Fig. 7). In particular, for a natural reservoir having important fracture permeability and very low fracture volume fraction, this identification is perfect. In addition, we notice that the behaviour of large-scale matrix permeability of the random fractured media is similar to a regularly fractured medium (Warren and Root Model) (Fig. 8). In particular for $R = 1000$, the results for random and regular fractured media are identical. The percolation threshold has therefore no influence on this property.

Exchange coefficient

The closure problems have been written in dimensionless form. Thus the exchange coefficient calculated by the resolution of the closure problems (α_{num}) is related to α in the following manner:

$$\alpha = \alpha_{num} \frac{K_m}{l_c^2} \quad (26)$$

where l_c is the characteristic length of a representative unit cell. The value found for α_{num} is rather small for values of the fracture volume fraction near zero (Fig. 9). This is not surprising because we approach a homogeneous medium with a permeability equal to the matrix permeability. The increase of the fracture volume fraction corresponds to a decrease of the size of the matrix blocs and this enhances the matrix-fracture exchanges. These exchanges are maximal for a fracture volume fraction of about 60%, and decrease for larger values of fracture volume fraction. For fracture volume fractions near unity, the medium behaves as a homogeneous medium with the fracture permeability.

For regularly shaped fractured media (Warren and Root model), l_c is naturally equal to $l_m + l_f$ (Fig. 5). However, for random fractured media the unit cell is chosen arbitrarily and is then filled with a certain number of fractures. It would therefore be interesting to find an equivalent regularly shaped fractured medium for which the exchange coefficient would be the same as our random fractured medium, that is

$$\alpha_{stoch} = \alpha_{reg} \quad (27)$$

So we have

$$\left(\alpha_{num} \frac{K_m}{l_c^2} \right)_{stoch} = \left(\alpha_{num} \frac{K_m}{l_c^2} \right)_{reg} \quad (28)$$

We can therefore estimate the ratio $(l_c)_{stoch}/(l_c)_{reg}$ in the following manner:

$$\frac{(l_c)_{\text{stoch}}}{(l_c)_{\text{reg}}} = \left[\frac{(\alpha_{\text{num}} K_m)_{\text{stoch}}}{(\alpha_{\text{num}} K_m)_{\text{reg}}} \right]^{1/2} \dots\dots\dots (29)$$

For a given fracture volume fraction, we can therefore calculate the size of the representative unit cell of an equivalent regularly shaped fractured porous medium. In figure 10, we have illustrated the ratio $(l_c)_{\text{stoch}}/(l_c)_{\text{reg}}$ as a function of fracture volume fraction.

These results suggest that order of magnitude can be estimated by following this approach, it seems however that they cannot be used as reliable values in simulations. This confirm that geometrical effects may be important, and that several parameters describing the system may be necessary. However, this problem needs further investigation since the realisations should be obtained independently of the unit cell size, which may not be the case with the procedure we used for generating the random fracture system.

Cross effect large-scale permeability

We notice, first of all, that for fractured media (small fracture volume fraction and large permeability ratio), these terms are negligible and they behave in the same manner as regular fractured formations (Figs. 11 and 12). The large-scale cross effect permeability is therefore not influenced by the percolation threshold.

We notice however that for a given fracture volume fraction, these coupling terms are more important than the ones found for the regularly shaped fractured medium. In addition, they increase as R increases and are not, in general negligible.

6. CONCLUSIONS

In the validity domain of the two-equation model presented in this paper, it has been emphasised that direct computation of the double-porosity model effective properties, over representative unit cells, is an interesting alternative to fine grid field-scale simulations in terms of accuracy and computational costs. Moreover, direct computation can be used to double-check the inverse procedure used to estimate the double-porosity model effective properties.

Results obtained for random fractured systems show that geometrical effects can be important. As a consequence, large-scale properties do not depend simply on the permeabilities, the volume fractions, and some characteristic length. However, it should be clear that this is preliminary results based on one single fracture generation method. Different random fractured systems are under investigation.

NOMENCLATURE

Roman letters

A_{mf}	area of the interface between the m and f regions contained within V , m^2
b_{ff}	vector that maps $\nabla\{P_f\}^f$ onto $\hat{P}_f$, m
b_{fm}	vector that maps $\nabla\{P_m\}^m$ onto $\hat{P}_f$, m
b_{mf}	vector that maps $\nabla\{P_f\}^f$ onto $\hat{P}_m$, m
b_{mm}	vector that maps $\nabla\{P_m\}^m$ onto $\hat{P}_m$, m
c_m, c_f	total compressibility in the m -region or f -region respectively, Pa^{-1}
k_f	$= K_f/(\varepsilon_f \mu c_f)$, m^2/s
k_m	$= K_m/(\varepsilon_m \mu c_m)$, m^2/s
K_m, K_f	matrix and fracture local-scale permeability tensors, m^2
K_m, K_f	matrix and fracture local-scale permeability when the media are isotropic, m^2
K_{mf}, K_{fm}	large scale cross effect permeability tensor, m^2
K_{mm}, K_{ff}	m -region or f -region large scale permeability in Barenblatt and Zheltov model, m^2
K_{mm}, K_{ff}	m -region or f -region large scale permeability tensor, m^2
L	dimension of the fractured medium considered, m
l	length of a fracture for a randomly fractured structure, m
l_c	characteristic length of the representative unit cell, m
l_f	characteristic length of the f -region, m
l_m	characteristic length of the m -region, m/l_x length of a regularly shaped matrix bloc in the x -direction, m
l_y	length of a regularly shaped matrix bloc in the y -direction, m
l_z	length of a regularly shaped matrix bloc in the z -direction, m
N	number of fractures

N_c	critical number of fractures at the percolation threshold
$\mathbf{n}_{mf}$	outwardly directed unit normal vector pointing from the m-region toward the f-region.
P_m, P_f	local-scale matrix and fracture pressure, Pa
P_m^*, P_f^*	average large-scale matrix and fracture pressures in Barenblatt and Zheltov Model (1960), Pa
$\hat{P}_m, \hat{P}_f$	matrix and fracture pressure deviations, Pa
$\{P_m\}^m, \{P_f\}^f$	intrinsic regional large scale average pressure for the m or f region, Pa
R	$= K_f/K_m$, ratio between local scale fracture and matrix permeabilities
S_f	scalar field that maps $\{P_m\}^m - \{P_f\}^f$ onto $\hat{P}_f$
S_m	scalar field that maps $\{P_f\}^f - \{P_m\}^m$ onto $\hat{P}_m$
t	time, s
t_D	characteristic diffusion time, s
t_E	characteristic matrix-fracture exchange time, s
V	large scale averaging volume, m^3
V_m, V_f	matrix and fracture volume contained in V , m^3

Greek letters

α	exchange coefficient
ϕ_m, ϕ_f	volume fraction of the m or f region
ϕ_c	critical fracture volume fraction at the percolation threshold
μ	viscosity, Pa s
ρ	fracture density defined by N/L^2 , m^{-2}
ρ_c	critical fracture density at the percolation threshold, m^{-2}
ξ	thickness of the fractures, m
$\varepsilon_m, \varepsilon_f$	matrix and fracture porosities

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ANNEXE A: Interpretation of well tests

The well tests in the fractured case are based upon the Warren and Root model (1963). Considering $K_m/K_f \approx 0$, they consider the resolution of the following equations:

$$\alpha(P_f^* - P_m^*) = \mu \phi_m c_m \frac{\partial P_m^*}{\partial t} \quad (30)$$

$$K_{ff} \nabla \cdot (\nabla P_f^*) + \alpha(P_m^* - P_f^*) = \mu \phi_f c_f \frac{\partial P_f^*}{\partial t} \quad (31)$$

The following dimensionless parameters are used:

$$P_{Di}^* = \frac{2\pi K_{ff} h (P_0 - P_i^*)}{q\mu}, \quad i = m, f \quad (32)$$

$$r_D = \frac{r}{r_w} \quad (33)$$

$$t_D = \frac{K_{ff} t}{\mu(\phi_f c_f + \phi_m c_m) r_w^2} \quad (34)$$

$$\omega = \frac{\phi_f c_f}{\phi_f c_f + \phi_m c_m} \quad (35)$$

$$\lambda = \frac{\alpha r_w^2}{K_{ff}} \quad (36)$$

where r_w is the radius of the well and q is the flow rate. The dimensionless equation found is solved using Laplace transform method, thus giving $P_{Df} = f(t_D, \lambda, \omega)$. At a given time, the two coefficients λ and ω are sufficient to characterize the behaviour of a naturally fractured reservoir. The pressure build up data can be analysed to estimate the parameters K_{ff} , ω and λ . By successive matching we can find the best series of parameters corresponding to the pressure results.

Figures and Captions

Figure 1 - The two different models considered for the study of orientation effects.

Figure 2 - Geometry chosen for the well tests.

Figure 3 a) - Example of randomly fractured porous media generated: $\phi_f = 0.0875$

Figure 3 b) - Example of randomly fractured porous media generated: $\phi_f = 0.1606$

Figure 3 c) - Example of randomly fractured porous media generated: $\phi_f = 0.446$

Figure 6 - The mathematical expectation of $K_{H^*}/(\phi_f K_f)$ as a function of the mathematical expectation of ϕ_f for different permeability ratios ($R = K_f/K_m$): comparison with the results of Quintard and Whitaker (1996b) for regularly shaped fractured media.

Figure 4 - The mathematical expectation of $K_{H^*}/(\phi_f K_f)$ as a function of the mathematical expectation of ϕ_f for different permeability ratios ($R = K_f/K_m$).

Figure 5 - Representative unit cell considered for the regularly shaped fractured porous medium.

Figure 7 - The mathematical expectation of $K_{mn}/(\phi_m K_m)$ as a function of the mathematical expectation of ϕ_f for different permeability ratios ($R = K_f/K_m$).

Figure 8 - The mathematical expectation of $K_{mmf}/(\phi_m K_m)$ as a function of the mathematical expectation of ϕ_f for different permeability ratios ($R = K_f/K_m$): comparison with the results of Quintard and Whitaker (1996b) for regularly shaped fractured media.

Figure 11 - The mathematical expectation of K_{fm}/K_m as a function of the mathematical expectation of ϕ_f for different permeability ratios ($R = K_f/K_m$).

Figure 9 - The mathematical expectation of α_c^2/K_m as a function of the mathematical expectation of ϕ_f for different permeability ratios ($R = K_f/K_m$).

Figure 12 - The mathematical expectation of K_{mf}/K_f as a function of the mathematical expectation of ϕ_f for different permeability ratios ($R = K_f/K_m$): comparison with the results of Quintard and Whitaker (1996b) for regularly shaped fractured media.

Figure 10 - Ratio $(I_c)_{stoch}/(I_c)_{reg}$ as a function of fracture volume fraction.