Two-Phase Flow Upscaling for 3D Sedimentary Structures

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ABSTRACT

The "geopseudo" method for flow upscaling aims to capture the effects of geological structure at different scales (e.g. lamina-scale; bed-scale) so that they may be included in reservoir-scale simulations. This process may be time consuming, since the scale-up must be carried out in several stages. At each stage detailed grids of the permeability variations must be created, and a two-phase simulation must be performed to calculate the pseudo flow parameters.

The method can, however, be speeded up while still honouring the fine-scale permeability structure. We have developed a new procedure for calculating the effective flow parameters at the bed-scale, by combining numerical methods with analytical techniques. At the finest scale, surfaces are used to model lamina structures. As an example we use a combination of sine functions to model 3D trough crossbeds. The effective permeability of sets of parallel laminae is estimated analytically, and then the effective permeability of the whole bed is calculated using a numerical simulation.

We have extended this technique to calculate two-phase flow parameters when capillary equilibrium extends over a whole bed. In our crossbed example, the presence of low permeability bottomsets is important. These may cause oil trapping, and must be modelled carefully. The original method has been modified to take account of these bottomsets, and examples show how oil trapping depends on the end points of the capillary pressure curves.

INTRODUCTION

Sedimentary rocks exhibit a variety of complicated structures. At the mm-cm scale, most clastic rocks are laminated, and the laminae are grouped together into metre-sized beds. The shape of the laminae within the beds depends on the conditions (fluid velocity and viscosity, and grain size) when the sediments were deposited. Frequently, they may form spoon-shaped structures, as shown in the model of a trough crossbedded sandstone, in Figure 1. Such bedding obviously requires 3D modelling, in order to assess the correct flow path of fluids through the medium.

In many reservoir models features at the lamina and bed scales are ignored, either because the effect of the lamination is not appreciated, or because of the work involved in scale-up. However, there may be large permeability contrasts between laminae, which will affect flow. The permeability contrasts between laminae generally lie in the range from 1:4 to 1:10 (Hurst and Rosvoll, 1991 and Corbett and Jensen, 1993).

Several studies have highlighted the importance of including lamina-scale structure, both using simulations (Jones et al, 1993) and using laboratory measurements (Huang et al 1995). At the cm scale, capillary forces tend to dominate over viscous and gravity forces, and the non-wetting phase may become trapped in high permeability zones which are surrounded by low permeability regions. This means that oil recovery is reduced if a rock is water-wet. The amount of capillary trapping depends on the morphology of the laminae: whether low permeability regions are continuous or otherwise. It is therefore important to model the precise structure of the lamination in order to estimate the recovery.

In this paper, we generate 3D trough crossbedding using a method similar to that of Rubin (1987). A new technique is applied to simulate single-phase flow, using a combination of analytical and numerical methods. We then demonstrate how this method can be extended to two-phase flow, when capillary equilibrium is assumed.

THE GENERATION OF 3D SEDIMENTARY BEDS

Rubin(1987) has shown that a variety of realistic looking sedimentary structures may be modelled using sine
functions. For example, a straight-crested ripple (2D structure) may be created using an equation of the form:

$$z = z_0 + \alpha_{x1} \sin \left( \frac{2\pi x}{\lambda_{x1}} + \phi_{x1} \right) + \alpha_{x2} \sin \left( \frac{2\pi x}{\lambda_{x2}} + \phi_{x2} \right), \quad (1)$$

where $x$ is distance, measured in the paleo-current direction, $z$ is height, $\alpha_{x1}$ and $\alpha_{x2}$ are amplitudes, $\lambda_{x1}$ and $\lambda_{x2}$ are wavelengths, $\phi_{x1}$ and $\phi_{x2}$ are phases, and $z_0$ is a zero-point. In order to generate 3D beds, plan-form sinusoidal variations are added, of the form:

$$x' = x_0 + \alpha_{y1} \sin \left( \frac{2\pi y}{\lambda_{y1}} + \phi_{y1} \right) + \alpha_{y2} \sin \left( \frac{2\pi y}{\lambda_{y2}} + \phi_{y2} \right), \quad (2)$$

where $y$ is distance measured in the horizontal direction perpendicular to $x$. The height of the ripple is then calculated using Equation 1, substituting $x'$ in place of $x$.

Equations 1 and 2 generate a surface representing one lamina. To create a whole bed, a series of laminae are created, and these are displaced in the $z$-direction to allow for deposition, and in the $x$-direction to represent migration. In reality, the layer of sand which is deposited drawn when the $z$-value (i.e. height) is lower than the layer and moving back in time. A new layer is only created, and these are displaced in the $z$-direction to allow for deposition, and in the $x$-direction to represent migration. Rubi (1987) solves this problem by starting the simulation with the top-most layer and moving back in time. A new layer is only drawn when the $z$-value (i.e. height) is lower than the previous value, at that location.

Figure 1 shows an example of a trough crossbed model. In this case, two wave-forms which were 180 out of phase were generated each lamina. The two sets of $z$-values, calculated using Equation 1, were compared at each point and the highest value was taken. The bed was then assembled from a series of such laminae, going back in time, as described above.

**SINGLE PHASE FLOW**

Beds such as those in the model trough crossbed in Figure 1, may be gridded, so that they can be used in conventional fluid flow simulations. However, to obtain a grid which is fine enough to resolve the laminae, requires at least $10^5$ grid blocks. This is too many to carry out capillary dominated, two-phase flow using current workstation capabilities, within a reasonable time scale.

Instead of using a fine grid to model a crossbed, with several grid blocks for one lamina, we superimpose a coarser grid on the structure, which contains several laminae within one block (referred to here as a coarse block). See Figure 2. We then calculate the effective permeability of the lamina sets analytically. We assume that the laminae within each coarse block can be approximated by a series of parallel planes.

The effective permeability of a model of infinite parallel layers may be calculated analytically, using the arithmetic average, $k_a$, for flow along the layers, and the harmonic average, $k_h$, for flow across the layers:

$$k_a = \frac{\sum_{i=1}^{n} t_i k_i}{\sum_{i=1}^{n} t_i} \quad (3a)$$

$$k_h = \frac{\sum_{i=1}^{n} t_i}{\sum_{i=1}^{n} t_i k_i} \quad (3b)$$

where $i=1, 2, \ldots n$ is the number of laminae types, $k_i$ is the permeability of the $i$th lamina type, and $t_i$ is the relative thickness of the $i$th lamina. Usually crossbed laminae consist of two types: a high permeability, sandy lamina and a low permeability micaceous one. In the examples considered here, we have kept $k_a$ and $k_h$ constant throughout the bed, although it may be more realistic to introduce a trend with height, or to vary them stochastically.

In the model considered here, the coarse blocks are not infinite in extent, so Equations 3a and b do not hold exactly. If there are enough grid blocks in the model, the change in slope of the laminae from one block to the next will be slight, and errors should be slight. (However, see the comments on the effects of boundary conditions in the section describing two-phase flow.)

The analytical values of the effective permeability refer to a coordinate system in which the $x$- and $y$-axes are aligned with the plane of the lamina set, and the $z$-axis is perpendicular. The effective permeability is a diagonal tensor, of the form:

$$\underline{k} = \begin{bmatrix} k_a & 0 & 0 \\ 0 & k_a & 0 \\ 0 & 0 & k_h \end{bmatrix}$$

We need to perform coordinate rotations to calculate the effective permeability in the reference frame of the bed, which will, in general, be a full tensor. The slopes of the laminae, which are calculated analytically, by differentiating Equations 1 and 2, are used to calculate the rotation angles, as described in the Appendix.

The effective permeability for the whole bed may be calculated numerically using the tensors. The method adopted is similar to that of White and Horne (1987) which used a 9-point finite difference scheme for a 2D model. In this case, we have used a 19-point finite difference scheme for the 3D model. (This is $3 \times 3 \times 3$ blocks, minus the 8 corners.) Periodic boundary conditions were used so that a full tensor could be calculated for the bed. (See, for example, Durlofsky, 1991, or Pickup et al., 1994.)
EXAMPLE

Laminae were generated, using Equations 1 and 2, with $z_0$ ranging from $-2\alpha x_1$ to $+2\alpha x_1$, so that there was a full coverage of laminae between heights $-\alpha x_1$ and $+\alpha x_1$, which was the extent of the gridded model. The number of laminae were set by specifying the amount of deposition per laminae, $d$. Therefore the smaller the value of $d$, the greater the number of laminae. Also specified was the amount of migration, $m$, between laminae, i.e. the change in $x_0$ from one lamina to the next. The main aim here was to generate enough laminae to be able to determine the slopes of the laminae sets with sufficient resolution, rather than to model the physical sedimentation rate. Table 1 gives the values of the parameters used in Equations 1 and 2.

The permeability of the bottomsets (Figure 1) is often lower than the permeabilities of the foreset laminae (Hartkamp-Bakker, 1993). To allow for this, the bottomset has been modelled separately from the foreset laminae. The software inserts a bottomset where there is an "erosion surface", i.e. where the heights of successive laminae have been set equal. The bottomsets are one coarse gridblock thick.

Several tests were carried out to determine the sensitivity of the effective permeability of the bed to the number of laminae and the number of coarse blocks. In finite difference models, the effective permeability tends to be under-estimated if the grid is too coarse (Pickup et al, 1994). Also the amount of crossflow is under-estimated if too few grid blocks are used. The ratio $k_{xz}/k_{xx}$, which gives an indication of the amount of crossflow was plotted against $\log(N)$, the logarithm of the number of coarse blocks, as shown in Figure 3. The symbols in the graph indicate the approximate number of laminae used in the model. Figure 3 shows that the amount of crossflow increases with both the number of laminae, and with $\log(N)$. (Note that, due to computer memory limitations, we could not run any larger models.) From these tests, a model with 81 laminae, and 75000 gridblocks was chosen. The values for the deposition and migration parameters are shown in Table 1. The single-phase effective permeability tensor for the whole bed was:

$$k = \begin{bmatrix} 50 & 0 & 7 \\ 0 & 48 & 0 \\ 7 & 0 & 19 \end{bmatrix}.$$  

There is marked anisotropy between the horizontal and vertical directions, but negligible anisotropy in the x- and y-directions. The $k_{xy}$ and $k_{yz}$ terms are zero because of symmetry in the model. There is significant crossflow in the x-z plane, as indicated by the $k_{xz}$ term.

TWO-PHASE FLOW

This method can be extended to two-phase flow if we can assume capillary equilibrium over a region that is at least as large as the gridblocks used to represent a lamina set, otherwise the phase permeabilities will vary along the laminae, and Equation 3a and b will not be valid. However, this is a reasonable assumption to make (Smith, 1991). If, in addition, we assume that capillary equilibrium holds over the whole bed, then the single-phase method, described above can be readily adapted to two-phase flow.

The capillary equilibrium method has been described by various authors (e.g. Pickup and Sorbie, 1994). The effective phase permeabilities are calculated for a range of equilibrium levels. At each level, the water saturations in each permeability region (lamina type, in this case) are calculated, from which the phase permeabilities may be obtained. The arithmetic and harmonic averages may then be calculated for each phase, and the effective permeability is obtained, for each phase, using the single-phase method described above. The effective water saturation and each level is calculated using a porosity-weighted average.

The input relative permeabilities and capillary pressure curves were derived from the following formulae, which represent a water-wet rock:

$$S_{wc} = 0.6 - 0.165 \log_{10}(k_{abs})$$

$$S_{wor} = 0.7$$

$$S_c = (S_w - S_{wc})/(S_{wor} - S_{wc})$$

$$k_{rw} = 0.3 S_e^3$$

$$k_{ro} = 0.85 (1 - S_e)^3$$

$$\phi = 0.076 + 0.05 \log_{10}(k_{abs})$$

$$P_c = 3.0 \sqrt{\phi/k} S_e^{-2/3}.$$  

Figure 4a shows the average input relative permeabilities, and Figure 4b shows capillary pressure curves. The pseudo phase permeabilities are illustrated in Figure 5. Again, there is anisotropy between the horizontal and vertical directions, but not within the horizontal plane. The $k_{yo}$ term decreases to zero at $S_w = 0.65$, indicating slight oil trapping. The cause of the oil trapping is the oil permeability decreasing to zero in the low permeability region, so no further oil can flow. However, we would expect oil trapping in the x- and y-directions as well, because the high permeability zones are completely surrounded by low permeability regions (Figure 6a). The problem lies in the fact that boundary conditions have been ignored while calculating the effective permeabilities for the coarse blocks. The method assumes that the laminae extend to infinity, as shown in Figure 6b. To correct for this, we have modified $k_a$, the arithmetic average of the laminae permeabilities, using a method shown in Figure 6c. The $k_a$ was harmonically averaged with the bottomset permeability, using Equation 3b. The thickness of the bottomset was taken as one coarse block. The pseudo phase permeabilities for this model are shown.

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in Figure 7. The oil phase permeability now decreases rapidly to zero at $S_w = 0.5$, indicating significant oil trapping.

Capillary trapping will only take place if there is a pressure difference between the end points of the two $P_c$ curves. As an example where no capillary trapping occurs, the capillary pressure curve for the low permeability regions was altered, so that the end point was the same as that for the high permeability laminae. Figure 8 shows the resulting phase permeabilities. In this case there is no oil trapping, as expected.

**DISCUSSION**

This new method for estimating the effective permeabilities of 3D sedimentary structures is relatively fast because analytical calculations are used at the smallest scale, and the size of the grid for numerical calculation is greatly reduced. However, there are some problems with the method, in that the boundary conditions (in the form of the bed bottomsets) are ignored while calculating the effective permeabilities of the sections of the foreset laminae. (Similar problems may also occur for any numerical simulations carried out on lamina sets, without proper attention being paid to boundary conditions.) We have developed a quick solution to this problem, by harmonically averaging the permeability along the foresets with the permeability of the bottomset. Further work is required to check the validity of this correction.

The difference in recovery between the model in Figure 7 and that in Figure 8 is very significant. In Figure 7, less than half of the moveable oil is recovered. It is obviously important to be able to estimate the amount of trapping correctly. This depends on the geometry of the laminae - whether the high permeability regions are completely enclosed with low permeability regions, and also on the wettability, which may be heterogeneous (Huang et al., 1995). In addition, the amount of trapping depends on the viscous/capillary ratio. Here we have assumed steady-state, and ignored the viscous pressure gradient. In practice, the oil will have some viscous pressure, which will help it to over come the capillary pressure to a certain extent.

We have assumed that capillary equilibrium holds over a whole bed, i.e. a scale of several metres. In practice, $P_c$ equilibrium may hold over a range of a few 10s of cm. This is sufficient to use the analytical approach for estimating the effective phase permeabilities of the coarse blocks. The method can be modified by carrying out an unsteady-state two-phase flow simulation to calculate the water saturation in each coarse block.

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**REFERENCES**


APPENDIX : COORDINATE ROTATIONS

Suppose that a sedimentary rock has a bed is described by a coordinate system with x and y in the horizontal plane and z vertically upward (Figure A1a).

1. Consider a rotation of $\theta$ anti-clockwise round the y-axis. The new axes are $x'$, $y'$ and $z'$ (Figure A1b). The equation for this rotation of axes is (Wong, 1991):

\[
\begin{bmatrix}
x' \\
y' \\
z'
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\]

(1)

2. Next, consider a rotation of $\phi$ round the $x'$ axis, to give the $x''$, $y''$, $z''$ axes (Figure A1c). The equation for this rotation is:

\[
\begin{bmatrix}
x'' \\
y'' \\
z''
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & \sin \phi \\
0 & -\sin \phi & \cos \phi
\end{bmatrix}
\begin{bmatrix}
x' \\
y' \\
z'
\end{bmatrix}
\]

(2)

Multiplying the two matrices, gives us the equation for the combined rotation:

\[
\begin{bmatrix}
x'' \\
y'' \\
z''
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
-\sin \theta \sin \phi & \cos \phi & \cos \theta \sin \phi \\
-\sin \theta \cos \phi & \sin \phi \cos \phi & \cos \theta \cos \phi
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\]

(3)

Suppose that in this coordinate system the laminae are in the $x''$-$y''$ plane, so the equation of this plane is $z'' = 0$. From the equation (3) this means that

\[-x \sin \theta \cos \phi - y \sin \phi + z \cos \theta \cos \phi = 0.\]

\[z = x \tan \theta + y \tan \phi / \cos \theta.\]

\[\frac{dx}{dx} = \tan \theta, \quad \text{and} \quad \frac{dx}{dy} = \tan \phi / \cos \theta. \quad \text{(4a,b)}\]

The slopes, $\frac{dx}{dx}$ and $\frac{dz}{dy}$, are already known by differentiating Equations 1 and 2, and evaluating at particular x and y locations. Hence we can calculate the required rotation angles from Equations A4a and b. In the $x''$, $y''$, $z''$ system the permeability tensor is diagonal with $k_{xx} = k_{yy} = k_{zz}$, and $k_{zz} = k_h$. In order to calculate the tensor in the bed axes (x, y, z) we need to rotate back into the x, y, z axes using a rotation of $-\theta$ about the x'' axis and then by $-\phi$ about the y' axis.

Table 1
Bedform parameters values for crossbed example.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
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<tr>
<td>$\lambda_{x1}$</td>
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</tr>
<tr>
<td>$\phi_{x1}$</td>
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</tr>
<tr>
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</tr>
<tr>
<td>$\lambda_{x2}$</td>
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<td>$\phi_{v1}$</td>
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<td>No. of Laminae</td>
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</tr>
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<tr>
<td>Migrit./lamina</td>
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</tr>
<tr>
<td>High perm lam</td>
<td>100 mD</td>
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<tr>
<td>Low perm lam</td>
<td>10 mD</td>
</tr>
<tr>
<td>Bottomset perm</td>
<td>10 mD</td>
</tr>
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</table>
Figure 1
a) A trough crossbed showing the laminations in the x-z and y-z planes.
b) A slice through the model in the x-y plane.
Figure 2
A grid superimposed on the laminae. Only the x-z plane is shown for clarity. The slopes at the z nodes are obtained by interpolation between laminae.

Figure 3
The effect of the number of laminae and the number of coarse gridblocks on the crossflow.

Figure 4
a) Input relative permeability curves.
b) Input capillary pressure curves.
Figure 5
Pseudo relative permeabilities for the first model.

Figure 6a
Oil flow in crossbed, showing lamina trapping. In a capillary-dominated flood, the oil cannot cross the low perm laminae, after $S_w$ has reached the maximum value, and it remains trapped in the area indicated by the shading.

Figure 6b
If periodic boundaries are applied, oil can still flow through the high perm regions, because the bottomset has not been taken into account.

Figure 6c
The arithmetic average for the flow along the foreset laminae is harmonically averaged with the bottomset permeability.
Figure 7
Pseudo relative permeabilities for the second model (corrected for boundary effects), showing the effects of significant oil trapping.

Figure 8
Pseudo relative permeabilities for the third model, in which the Pc curves have been set equal, at Sw=0.7. There is no oil trapping.

Figure A1
Axis rotations