DIRECT AND INVERSE PROBLEMS OF THE FLUID DISPERSION IN THE REAL RESERVOIR

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ABSTRACT

Movement of the fluids, saturating porous media of the real reservoirs, can be, in many cases, especially by miscible displacement, considered as a dispersion process. This process may be caused not only by molecular and convective diffusion, but as well by strata heterogeneity. As examples, solutions of the dispersion equation for layered and fractured-porous reservoirs are given. On the base of this solutions inverse method is developed, which permits to evaluate heterogeneous strata properties.

INTRODUCTION

Dispersion of the substances, moving in the homogeneous porous media, takes place due to the velocity difference of molecule and globula in porous space.

By viscosity difference of filtrating substances usual convective diffusion will be intensified by "viscosity difference diffusion" in porous media. Nevertheless, given above imaginations reflect observed in the real oil and gas reservoirs dispersion not completely. Very important influence on the reservoir fluid dispersion has rock heterogeneity factor. Caused by this factor dispersion can be called "heterogeneous quasidiffusion".

DIFFERENTIAL EQUATIONS

Dispersion processes are described mathematically by diffusion differential equation. It includes as well members, characterizing transport of additives, dissolved in the main substance, moving in the porous media. Relationship between effective diffusion coefficient $D_E$ and true average velocity of the movement of the substances in porous media $w = w/m$ ($v$ - filtration velocity, $m$ - porosity) is linear one.

Thus

$$D_E = k_v w$$

$k_v$ - experimental coefficient.

If we consider transport in porous media of substance, dissolved, for example, in water, displacing oil, for description of this process is needed not only convective diffusion equation, but as well equation of displacement of oil by water. In this case we have the following system of equations

$$k_v \left( \frac{\partial}{\partial x} w x \frac{\partial C}{\partial x} + \frac{\partial}{\partial y} w y \frac{\partial C}{\partial y} + \frac{\partial}{\partial z} w z \frac{\partial C}{\partial z} \right) = w \cdot \text{grad}(C) = \frac{\partial(C_s)}{\partial t}$$

$$\text{div} v_w + m \frac{\partial s}{\partial t} = 0$$

$$\text{div} v_o - m \frac{\partial s}{\partial t} = 0$$
Direct and Inverse Problems of the Fluid dispersion in the real Reservoirs  

\[ v_w = \frac{k(x, y, z) k_w(s)}{\mu_w} \text{grad} p \]

\[ v_0 = \frac{k(x, y, z) k_o(s)}{\mu_o} \text{grad} p \]  

In equations (1)-(5) \( S \) and \( C \) are water saturation of the porous media and dissolved in water substance concentration correspondingly, \( \mu_o \) and \( \mu_w \) - viscosities of oil and water, \( k_w(s) \) and \( k_o(s) \) - relative permeability functions, \( p \) - reservoir pressure, \( x, y, z \) - coordinates, \( t \) - time.

In the case of displacement of oil by miscible liquid, having another viscosity in comparison with oil, it is convenient to use half empirical relationships, based on results of corresponding experiments. Good agreement of calculated and experimental results was obtained [1], when in the convective diffusion equation (2), instead of diffusion coefficient \( D_E \), one complex coefficient \( D \), was introduced determined as

\[ D = D_E \left( 1 + k_{\mu} \text{grad} \frac{\mu_C}{\mu} \right) , \quad \mu_C = \mu_1 \mu_2 ^{1-C} \]  

where \( \mu_1 \) and \( \mu_2 \) - viscosities of displacing and being displaced liquids, \( k_{\mu} \) - viscosity dispersion coefficient. According to experimental results was found, that \( k_{\mu} = 2.45 \times 10^4 \text{m/Pa s} \), \( k_V = 0.09 \text{m/s} \) (equation (1)).

OVERCOMING OF FALSE DIGITAL DISPERSION

Of most interest are the solutions of direct and inverse dispersion problems in the cases of complicated geometry of real reservoirs. Obtaining of this solutions is connected with digital calculations, using present day computers.

In this case calculated saturation and concentration profiles become often similar to those processes, where strong diffusion occurs. Really in these processes could be only small diffusion. So false diffusion or digital dissipation arises in solutions. To get over this difficulty some known methods were applied and tested: methods of Lax P.D. and Wendroff B.[2], MacCormack R.W.[3], Zalesak S.T.[4], Warming R.F. and Beam R.M. [5] and others.

Better results were obtained using method of Warming R.F. and Beam R.M., but its disadvantage is connected with introduction too small space and time steps. Finally, was used implicit scheme of second order approximation, which gave the best accuracy.

HETEROGENOUS QUASIDIFFUSION EXAMPLES

Usual convective diffusion is caused by velocity differences at the pore level. Heterogeneous quasidiffusion can be considered as a high order diffusion in comparison with porous diffusion. Its quantitative description can be made on the base of equation (2).

LAYERED RESERVOIR

Let us imagine one linear form reservoir, which contains some layers, having different absolute permeability. The reservoir can be saturated by gas or water (Fig.1). In the moment of time \( t = 0 \), the reservoir of such a type, purely water saturated, we started to inject water, containing nonsorbing completely dissolved in water additive with initial concentration \( C \). The permeable layers are separated one from another by nonporous layers. Distribution of the relative thickness of layers \( h_i/h \) as function of absolute permeability \( k \) is shown on the fig.2 (\( h \)-thickness of all layers, \( h_i \)- thickness of \( i \)-layer). For quantitative consideration of example we use the following data: size of reservoir in the direction \( l=200 \text{m} \), width \( b=200 \text{m} \), thickness \( h=20 \text{m} \), porosity \( m=0.2 \), injection rate \( q \) is equal to \( 692,1 \text{m}^3/ \text{day} \) and thus average filtration velocity \( v=q/(bh)=692,1/200*20=0.173 \text{m/day} \). True water velocity \( v=m=0.2 \times 10^{-5} \text{m/c} \) substances diffusion in each separate layer is described by equation of usual convective diffusion (2). For convenience we can use instead of mathematically exact solution of diffusion equation, an approximative solution, obtained by means of Barenblatt G.I. method [6]. On the base of this solution we obtained following formulae for determination of substance concentration in each layer

\[ C_i(t) = 0.25 \left[ 2-3 \frac{\xi_1^3(t)}{\lambda_1(t)} + \frac{\xi_3^3(t)}{\lambda_3(t)} \right] \]

\[ \xi_i = x-w_i t, \quad \lambda_1 = (8 k_V w_i t)^{0.5}, \quad w_i = \frac{q_i}{h_i/h} \]  

Let us consider conventionally, that each layer continue by \( x > 1 \) (fig.1, dashed line) and determine average concentration of additive on the exit from reservoir \( C_1 \) as

\[ C_1 = \frac{\sum C_i q_i}{q} \]  

where \( \nu \)-number of layers, to which came miscible zone.
Then we can determine time \( t_k \), which passed from appearance of additive on exit from most permeable layer to the moment, when additive concentration on the exit from reservoir became equal to \( C_0 \). Substance concentration distribution on exit from reservoir one can imagine as those, which takes place by heterogeneous quasidiffusion with coefficient

\[
D_H = \left( \frac{2\lambda}{8kt} \right)^2
\]

In the considered example \( D_H = 2.22 \times 10^{-2} \text{ m}^2/\text{c} \) by \( D_E = k \_V \ w = 10^6 \text{ m}^2/\text{c} \)

Then \( D_H / D_E = 2.22 \times 10^4 \).

**FRACTURED-POORUS RESERVOIR.**

Let us consider fractured-porous reservoir (Fig.3), initially saturated by gas-condensate mixture, which is in single-phase state by reservoir conditions. At the time \( t>0 \) from the edge of reservoir \( x=0 \) we started to inject dry gas (for example, methan \( \text{CH}_4 \)) with volume rate \( q \). From the exit of reservoir gas-condensate mixture is produced, which contains "fat" components (\( \text{C}_3\text{H}_8 \), \( \text{C}_4\text{H}_8 \) and others). Concentration of the last components in the whole production drops and methan concentration increases. In an large enough time interval all fat components will be "washed out" from reservoir and on its exit \( x=1 \) we will produce only methan.

Let us assume, that in the whole period of movement of the gas-condensate mixture condensate precipitation in porous media does not occur.

Methan, displacing gas-condensate mixture from fractured-porous reservoir, penetrates in the fracture system and, as well, in the blocks. Into each separate block gas-condensate mixture enters with concentration of methan, existing in the fracture system in a given point and time moment. Leaving the block and entering into fracture system mixture does with those concentration, which existed in the fracture system in the moment of entrance in the block of this mixture. Thus, it was supposed, that each "portion" of mixture, entering into the block system from the fracture system is "stopped" in the blocks for equal for each block time \( t \).

We obtained following equation for gas dispersion in the fractured-porous reservoir

\[
w \frac{\partial}{\partial x} \left[ \frac{\partial C}{\partial t} + mT \frac{\partial C}{\partial t} \right] + \alpha \ w \left[ C(x, t) - C(x, t - t_r) \right] = 0 \quad \text{(8)}
\]

where \( mT \) - fracture "porosity". We obtained as well following approximate expression for \( \alpha \)

\[
\alpha \approx \frac{3k_B}{k_T l_B} \quad \text{(9)}
\]

where \( k_B \) and \( k_T \) - permeabilities of block and fracture system, correspondingly, \( l_B \) - average size of the blocks.

It is more convenient to introduce, instead of \( C(x, t) \), value \( \varphi \ (x, t) \), determined as

\[
\varphi = \frac{C - \overline{C}}{1 - \overline{C}} \quad \text{(10)}
\]

where \( \overline{C} \) - initial methan concentration in the gas-condensate mixture.

The problem, considered above, can be solved analytically. But we used its digital solution, because such solutions are of a more general application. 6 - points digital approximation scheme was applied with calculation of corresponding values in the points:

\[
(i - 2, n), \ (i - 1, n), \ (i, n),
(i - 2, n + 1), \ (i - 1, n + 1), \ (i, n + 1)
\]

where \( i \) is number of cell in the space and \( n \) - number of time layer.

At fig.4 is shown profile of \( \varphi \) - function by \( l_B = 1 \text{ m} \) and \( l_B = 2 \text{ m} \) and \( \Delta x = 5 \text{ m} \) and \( \Delta x = 10 \text{ m} \). As can be seen, difference in results by increasing of step from 5 to 10 m is negligible. On fig.5 is shown \( \varphi \) - function profiles for different sizes of block \( l_B \).

We suppose, that mentioned above scheme will give good results in the high dimension cases too.

**INVERSE PROBLEMS**

Results of calculations show, that reservoir heterogeneity causes significant dispensor of filtering substances velocities. Heterogeneous quasidiffusion coefficients \( D_H \) in the real heterogeneous strata are often on many orders more, than coefficients of usual convective diffusion \( D_E \). Thereby, the more reservoir heterogeneity, the more heterogenous quasidiffusion coefficient.

On fig.6 is shown, for example, relationship between heterogeneous quasidiffusion coefficient \( D_H \) and block size of fractured-porous reservoir \( l_B \). Measuring this coefficient on the base of observed dispersion of substance concentration in one of the real processes of gas-condensate extraction and solving corresponding dispersion problems, one can obtain coefficient \( D_H \) and
then determine size of the block $l_B$, which usually is difficult for determination. Such a procedure can be attributed to the procedures of solution of inverse problems of underground hydrodynamics.

REFERENCES

Fig. 1 Scheme of the layered Reservoir
1, 2 - layers, 3 - miscible zone in a single layer,
4 - appeared miscible zone by $x > 1$

Fig. 2 Permeability distribution in the layered Reservoir
Fig. 3 Scheme of the fractured-porous reservoir
1 – porous and permeable blocks,
2 – fractures

Fig. 4 Profiles of ϕ-function
1 – block size
\( l_B = 1 \text{m} \),
2 – block size
\( l_B = 5 \text{m} \)

by \( \Delta x = 5 \text{m} \) (continual line) and \( \Delta x = 10 \text{m} \) (dashed line)
Fig. 5 Profiles of ϕ-function by $t=365$ days

1. $l_B=1\text{m}$, 2. $l_B=2\text{m}$, 3. $l_B=5\text{m}$, 4. $l_B=7\text{m}$, 5. $l_B=10\text{m}$
Fig. 6 Relationship between heterogeneous quasidiffusion coefficient $D_H$ and block size $l_B$. 