E05

CORRELATION FUNCTION BEHAVIOR IN PRODUCTIVE AND NONPRODUCTIVE LAYERS

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Abstract. A mathematical technique is suggested to calculate the 3-D spatial correlation functions for elasticity tensor components. The technique is based on the expansions of elasticity modulus fluctuations into the Fourier series. This method is applied for field data obtained at the seismic frequency. The results of calculations show the existence of maximum amplitudes and minimum of correlation radius of the correlation functions associated with enhanced inhomogeneities at some depth. It is observed that these depths well correlate with oil and gas location indicated by other independent methods. It is shown that the experimental correlation functions cannot be fitted with accepted accuracy by spherical Gaussian functions.

Introduction. Previously, based on the method of pair correlation functions, a technique has been developed that makes it possible to predict elastic wave behavior along the well at given frequency from logging data obtained at higher frequency (Roy *et al.*, 2001). The frequency dependence in this case is caused by attenuation by inhomogeneities. Comparison of the difference between calculated effective and average slowness with hydrocarbon-bearing intervals obtained by independent methods indicates that the location of the enhanced differences well correlates with the productive layer location. In this work, we try to understand the reason of this effect.

The formulas for calculating the component of effective elasticity tensor and effective density have the form (Shermergor, 1977; Chesnokov *et al.*, 1996, 1998)

$$C_{ijkl}^*(\omega, \mathbf{k}) = \langle C_{ijkl}(\mathbf{r}) \rangle + \int \cos(\mathbf{k}, \mathbf{r}) \left[\frac{\partial}{\partial x_n} \frac{\partial}{\partial x_q} G_{mp}^0(\omega, \mathbf{r}) \right] B_{ijmn}^{pqkl}(\mathbf{r}) d\mathbf{r} \quad (1).$$

$$\rho^*(\omega, \mathbf{k}) = \langle \rho(\mathbf{r}) \rangle - \omega^2 \int \cos(\mathbf{k}, \mathbf{r}) G_{ii}^0(\omega, \mathbf{r}) B^{(\rho\rho)}(\mathbf{r}) d\mathbf{r}. \quad (2)$$

Here, ω is the frequency, $\mathbf{k}$ is the wave vector, G^0 is the dynamic Green's tensor depending on the medium's properties and frequency:

$$G_{ij}^0(\omega, \mathbf{r}) = \frac{1}{r} \left[h(\omega r) \delta_{ij} + g(\omega r) \frac{x_i x_j}{r^2} \right], \quad (3)$$

$$h(\omega r) \equiv \frac{1}{4\pi\rho\omega^2 r^2} \left\{ \left[\left(1 + \frac{i\omega r}{c} \right) \exp(-i\omega r/c) \right] \right|_{c_t}^{c_l} + \frac{r^2 \omega^2}{c_t^2} \exp(-i\omega r/c_t) \right\}, \quad (3a)$$

$$g(\omega r) \equiv -\frac{1}{4\pi\rho\omega^2 r^2} \left\{ \left[3 \left(1 + \frac{i\omega r}{c} \right) - \frac{r^2 \omega^2}{c^2} \right] \exp(-i\omega r/c) \right\} \right|_{c_t}^{c_l}$$

with c being the longitudinal (l) or shear (t) wave of medium, averaged over the accepted scale. The pair correlation functions of elasticity tensor components and density for the statistically homogeneous case are determined as follows:

$$\overset{(cc)}{B}_{pqmn}^{ijkl}(\mathbf{r}_1 - \mathbf{r}_2) \equiv \langle C'_{ijkl}(\mathbf{r}_1) C'_{pqmn}(\mathbf{r}_2) \rangle, \quad \overset{(\rho\rho)}{B}(\mathbf{r}_1 - \mathbf{r}_2) \equiv \langle \rho'(\mathbf{r}_1) \rho'(\mathbf{r}_2) \rangle. \quad (4)$$

The fluctuation C' and ρ' are calculated as differences between the related value at the point $\mathbf{r}$ and its value average over a given scale (the choice of the scale is discussed below).

The elasticity tensor components are determined from the field data on V_P , V_S , and ρ , depending on the X, Y (lateral coordinates), and Z (depth), as follows

$$C_{33}(\mathbf{r}) = \rho(\mathbf{r}) V_P^2(\mathbf{r}), \quad C_{44}(\mathbf{r}) = \rho(\mathbf{r}) V_S^2(\mathbf{r}). \quad (5)$$

Since at each point we have only measurements of vertical component of velocity, we assume that the medium has isotropic properties at each point of measurements. Hence, $C_{11} = C_{22} = C_{33}$ and $C_{44} = C_{55} = C_{66}$.

Resulting effective medium (at lower frequency) has transversely isotropic type of symmetry as consisting of a set of isotropic layers. For vertical direction, we have

$$\begin{aligned} V_P^*(\omega) &= \sqrt{\frac{C_{33}^*(\omega)}{\rho^*(\omega)}}, & V_S^*(\omega) &= \sqrt{\frac{C_{44}^*(\omega)}{\rho^*(\omega)}} & \text{for effective velocities and} \\ T_P^*(\omega) &= 1/V_P^*(\omega) & \text{and} & & T_S^*(\omega) = 1/V_S^*(\omega) & \text{for effective slowness.} \end{aligned}$$

The difference between effective and average slowness is determined by the second terms in relations (1) and (2). Numerical analysis of the behavior of Green functions (determined by formula (3)) along the traces (or along a well, in the 1-D case) showed that the function produces no anomalies for the hydrocarbon-bearing intervals. Its behavior is rather smooth governing by the averaged velocities. However, it was not the case for the correlation functions. They exhibited significant increases in amplitude for the productive layer location. Below, we analyze this effect in detail and find a reason for the observed anomalies in the correlation function behavior.

Calculation of the pair correlation functions in the 3-D case. We have a set of seismic traces for a region (P. Anno, 19...). The distance between the traces was fixed and equal to a . The total number of trances for X direction was N . The same number of traces was given for Y direction. Along each trace, we have measurements for compression and shear wave slowness and density versus depth.

In order to obtain effective characteristics depending on depth, we apply the averaging with moving window. The choice of the window size is not a simple problem. However, numerical tests performed for a set of wellog data gave evidence that the results on the effective characteristic determination become stable for window size equal to the wavelength of compression wave for given frequency. Thus, for calculating the averaged values, we use the size of moving window equal to the wavelength calculated for given frequency at the current point from measured velocity.

In order to calculate the 3-D correlation function, we consider a “box” whose height is equal to the wavelength for frequency at which the effective parameters are calculated. The box sides are equal to the distances between the first and last measurements of seismic traces. For our case, the half-length of box side is $L_x = L_y = a(N-1)/2$. The half-height is $L_z = L_w/2$, where L_w is the compression wavelength for frequency at which the effective values are found. We assume that all averaged and effective values are related to the central point of this box. As a result, using the moving window, we obtain the effective characteristics for central trace of the considered set.

The formula for correlation function $B_{ijkl}^{pqmn}(\mathbf{r}_1 - \mathbf{r}_2) = \langle C'_{ijkl}(\mathbf{r}_1) C'_{pqmn}(\mathbf{r}_2) \rangle$ for the considered box takes the form ($\mathbf{r}_1 - \mathbf{r}_2 = \mathbf{r} = \{x, y, z\}$):

$$B_{ijkl}^{pqmn}(\mathbf{r}) = \int_{-L_x}^{L_x} \frac{dx'}{2L_x} \int_{-L_y}^{L_y} \frac{dy'}{2L_y} \int_{-L_z}^{L_z} \frac{dz'}{2L_z} C_{ijkl} \left(x' + \frac{x}{2}, y' + \frac{y}{2}, z' + \frac{z}{2} \right) C'_{pqmn} \left(x' - \frac{x}{2}, y' - \frac{y}{2}, z' - \frac{z}{2} \right) \quad (6)$$

According to commonly accepted rules, the correlation functions can be found using the expansion of fluctuations into the Fourier series. In this case, we have

$$B(\mathbf{r}) = \sum_{\alpha} \sum_{\beta} \sum_{\gamma} f_{\alpha\beta\gamma} \phi_{\alpha\beta\gamma}^* \exp(i\alpha x + i\beta y + i\gamma z), \quad (7)$$

$$\alpha = \pi n_x / L_x, \quad \beta = \pi n_y / L_y, \quad \gamma = \pi n_z / L_z;$$

$$n_x = -N_x, \dots, N_x, \quad n_y = -N_y, \dots, N_y, \quad n_z = -N_z, \dots, N_z$$

where f and ϕ are, respectively, the coefficients of expansion into the Fourier series of fluctuations C'_{ijkl} and C'_{pqmn} ; the star indicates the complex conjugation; N_x, N_y , and N_z are the chosen number of harmonics for each coordinate. Hereinafter, we assume that the medium is statistically homogeneous, and the correlation functions depend only on the difference $\mathbf{r}$ between the points.

Correlation functions for productive and non-productive layers. When calculating the correlation functions along the trace (or well), we notice that a difference exists in correlation functions calculated for productive and non-productive layers (indicated with the help of independent methods). Figures 1 and 2 show the function $\langle C'_{11} C'_{11} \rangle$ calculated for these layers at the 70-Hz frequency. As seen, the hydrocarbon-bearing places exhibit enhanced values of the correlation functions. The difference in the correlation function $\langle C'_{11} C'_{11} \rangle$ calculated for productive and non-productive layers can attain an order of magnitude. Note that this function has the greatest amplitude as compared to the functions $\langle C'_{44} C'_{44} \rangle$, $\langle C'_{12} C'_{12} \rangle$, and mixed correlation functions (for example, $\langle C'_{12} C'_{44} \rangle$).

Fig. 1a. Correlation function $\langle C'_{11}C'_{11} \rangle$ in the XY plane (Z = 0). Non-productive layer. Frequency is 70 Hz.

Fig. 1b. Correlation function $\langle C'_{11}C'_{11} \rangle$ in the XZ plane (Y = 0). Non-productive layer. Frequency is 70 Hz.

Fig. 2a. Correlation function $\langle C'_{11} C'_{11} \rangle$ in the XY plane ($Z = 0$). Productive layer. Frequency is 70 Hz.

Fig. 2b. Correlation function $\langle C'_{11} C'_{11} \rangle$ in the XZ plane ($Y = 0$). Productive layer. Frequency is 70 Hz.

Analysis of the calculated correlation function shape. Sometimes one makes an attempt to simplify the correlation functions found from experimental data and fits them by spherical Gaussian functions having the form $f^t = A \exp(-r^2/\acute{a}^2)$. We try to fit the correlation function calculated from experimental data (f^e) by unimodal spherical Gaussian functions. The parameter a was found by minimizing the functional $\psi = \sum_i \sum_j \sum_k \left((f^t / f^e)|_{i,j,k} - 1 \right)^2$, summation was

performed over 3 sets of experimental points for current window. Figure 3 and 4, respectively, show the experimental correlation function, and related difference between the experimental and fitted spherical Gaussian functions.

Fig.3. Experimental correlation function f^e

Fig.4 The relative difference between experimental and the most close spherical Gaussian correlation functions.

As seen the difference may attain 50% of the correlation function value even in the XY (lateral) plane exhibiting different character in the X and Y directions. This means that, in general, the experimental functions cannot be represented by spherical Gaussian functions. This reflects the inhomogeneity of studied medium in the X, Y, and Z directions. The oscillation character of the correlation functions seen in sections along the coordinates axes point to possible non-Gaussian character if the oscillation amplitudes are greater than the accepted level of error following from the experimental accuracy.

Reason for the increased values of correlation function in the productive layers. In order to explain the enhanced values of the correlation functions in the productive layers, we compare the value of relative inhomogeneity of the material along the trace/well and come to conclusion that the increased values of the correlation function amplitudes is caused by the increase in inhomogeneity of material.

Besides, we calculate the correlation radius behavior along the central trace. The correlation radius is specified as follows:

$$R_c = \frac{L_z^0}{L_z} \int_{-L_z}^{L_z} \frac{|B(0,0,z)|}{B(0,0,0)} dz, \quad L_z^0 = \text{const},$$

where $B(x,y,z)$ is the 3-D correlation function. The correlation radius characterizes the rate of the correlation function decrease with the distance and determines the size of so-called effective grain of inhomogeneity. The effective grain of inhomogeneity has a probabilistic meaning, namely, the values of medium's property at arbitrary points $\mathbf{x}_1, \mathbf{x}_2$ are statistically related with one another if the points belong to the grain. The absolute value of normalized correlation function $|B(\mathbf{x})|/B(0)$ "cuts" a volume of effective grain of inhomogeneity. Figure 5 compares the productive layer location, the correlation function amplitude, and correlation function radius. As seen, the productive layers are associated with the correlation radius minimum (possible reason is the enhanced inhomogeneity, since for a homogeneous medium, the correlation radius is

infinity). Numerical modeling with isolated inclusions shows that the decrease in correlation radius can be associated with the fact that the inclusions become smaller in size.

Fig. 5. Comparison of the productive layers location with the correlation function amplitude and correlation radius behavior for the central trace. The average window corresponds to the wavelength at frequency 70 Hz.

Thus, the enhanced inhomogeneity of the medium produces the increased scattering and, in turn, the increased difference in the frequency dependent term in (1). Figure 6 shows the comparison of oil location prediction produced by the standard method of correlation functions (by formula (1), analyzing the difference between effective and averaged slowness) and with the help of the correlation function amplitudes. As seen, the later method can be used for indication of the hydrocarbon location. This means that there is no sense to apply formula (1), it is sufficient to calculate only the distribution of the correlation function amplitude (and radius) along the trace/well under study.

Note that this technique is also applicable if data only for single XY location are available (for example, wellog data at acoustic frequency). In this case, the technique is simply reduced to the 1-D case, and the one-dimensional correlation function amplitudes/radius should be analyzed.

Conclusions. The pair correlation functions constructed for the elasticity tensor components derived from 3-D (or 1-D) field data can be used for the “contrast” layer indication. It was found that the maximums of the correlation functions and minimums in the correlation radius are associated with enhanced medium’s inhomogeneity typical of places of oil and gas location. The difference in the correlation function amplitude can attain an order of magnitude. In general, the

experimental functions cannot be fitted by spherical Gaussian functions, which reflect different character of the studied place inhomogeneity in different directions.

Fig. 6. Comparison of two methods for productive layer indication. (a) using the difference between effective and averaged slowness and (b) using the correlation function amplitudes.

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