E07 FACIES SIMULATION USING MEMBERSHIP FUNCTIONS AND MULTI P-FIELD

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Abstract

Stochastic simulation techniques are increasingly used for modelling spatial distribution of categorical attributes (such as rock types) or continuous attributes (such as porosity or permeability). This procedure consists in generating alternative, equally probable 3D realizations, that mimic the heterogeneity expected in the real media and inferred from the available data.

For a couple of years, several works have been carried out to develop new methods for building realistic facies models, such as Indicator Kriging or multipoint geostatistic methods. Others methods based on a P-Field have been also developed to account for spatial variability of facies. Most of them do not take into account trends or transition probabilities between one facies and the others.

Using Discrete Smooth Interpolation (DSI), the method proposed in this paper interpolates a membership function and is inspired from P-Field ones. P-Field based method consists in first estimating in 3D a local facies distribution and second to generate a random probability field accounting spatial structures. In the presented work, each local facies distribution will be modelled using one membership function, which corresponds to the probability of a given node to belong to a given facies. The estimation of the membership function can also take into account several constraints, such as vertical or horizontal proportion maps, trends or transition probabilities. It is possible to combine the so defined probabilities with classical stochastic simulators in order to generate equiprobable realizations of the facies distribution in the reservoir. Multi P-Fields methods can be used to account for complex, spatial variability of facies (anisotropy which depends of facies type).

Introduction

It is now become a fairly common practice to simulate stochastically 3D spatial distributions of reservoir properties such as rock-types, porosity or permeability. The main objectives of stochastic simulation procedures consist in creating the uncertainty fields due to the lack of subsurface data. To achieve this, a large set of alternative, equiprobable models of subsurface property repartition is generated and used for statistical analysis. The resulting 3D models are generally denoted to realizations and have to satisfy the available dataset generally composed of

geostatistical parameters inferred from the available data, facies observations along wells, vertical trends / proportions, or seismic related soft data.

Two broad categories of stochastic simulation methods are commonly used to model facies distributions: the boolean object-based approach and the pixel-based approach. The method presented in this paper belongs to the second one. It can be considered as an alternative method to classical approaches such as Sequential Indicator Simulation [1,2], Truncated Gaussian [3] or Pluri-Gaussian [4] methods. This approach was inspired by the Multi Binary Sequential Indicator Simulation introduced by Tom Tran. As in the multi binary SIS Implementation, it consists in simulating one facies at a time, using a given Probability Field [5] with a given covariance specifying the spatial correlations of the facies.

After shortly presenting the basic theoretical required components of the proposed method, we will detail the “multi P-Field simulation method”, and we will illustrate the obtained results on a case study.

Membership function

Definition

Let us consider Ω the set of all the nodes of a grid G , $\{F^1, F^2, \dots, F^n\}$ a finite partition of Ω for which F^i corresponds to the set of nodes of Ω belonging to the facies i , and (Ω, P) the associated probability space. Let $\alpha \in \Omega$ be a node of G . At each location $\alpha \in \Omega$ and for each facies F^μ , the membership function, $\varphi^\mu(\alpha)$, corresponds to the probability for the location α to belong to the facies F^μ

$$\varphi^\nu(\alpha) = IP(\alpha \in F^\nu)$$

The *membership function* φ [6,7] is the n -components probability vectorial function, defined for the location α as follows :

$$\forall \alpha \in \Omega \quad \xrightarrow{\varphi} \quad \varphi(\alpha) = \begin{bmatrix} \varphi^1(\alpha) \\ \vdots \\ \varphi^\nu(\alpha) \\ \vdots \\ \varphi^n(\alpha) \end{bmatrix} = \begin{bmatrix} IP(\alpha \in F^1) \\ \vdots \\ IP(\alpha \in F^\nu) \\ \vdots \\ IP(\alpha \in F^n) \end{bmatrix}$$

In other words, the “membership function” φ characterizes the distribution of facies over the whole studied domain.

Figure 1 : Defining the membership function φ over the Ω space

Estimation and properties of the membership function

In the proposed approach, membership functions are used as support for modelling facies distribution at each location $\alpha \in \Omega$

In practice, the membership function φ is only known at the sampling location where the facies are observed (usually along wells). At other locations, the value of φ has to be estimated. This step is crucial for the simulation, because during this step must be integrated all the available data. Two main groups of constraints must be taken into account:

- Intrinsic constraints: the membership function is similar to a probability distribution. Consequently, some intrinsic constraints detailed here must be honoured

- $\varphi^\nu(\alpha) \in [0, 1] \quad \forall \alpha, \forall \nu$

- $\sum_{\nu=1}^n \varphi^\nu(\alpha) = 1 \quad \forall \alpha$

- Hard and soft data: various types of constraints can be used to estimate the value of the membership function. For example, it may happen that the sedimentologist can provide an average map of proportions corresponding to the mean value of the MF along an axis of the stratigraphic grid. Also, constraints induced by seismic attributes, corresponding to conditional probabilities of associations between “geophysical” facies and “geological” facies, or transition probabilities, can be taken into account.

The DSI method [8] is used to interpolate the membership function φ over Ω . One important advantage of using DSI is the possibility to condition the membership function interpolation by a

large set of constraints. Moreover, each constraint introduced into the interpolator can be weighted, according to the data related degree of confidence.

Principle of the Multi P-Field Simulation Method

The purpose of the multi P-Field facies simulation is to simulate sequentially lithofacies at each location α such that proportion maps, trends, transitions and spatial structures are as well as possible respected.

Let $\{f_1; \dots; f_n\}$ be a set of facies encountered in the study domain. It is assumed that an initial model for the membership function has been provided over the whole domain. In addition, the n variograms γ_i defining for each facies indicator are supposed to be previously estimated.

The goal is now to use the initial membership function to generate an equiprobable realization of the distribution of the facies at each node of the grid. For this purpose, a five step sequential algorithm, that will be detailed thereafter, can be proposed:

- Provide an initial solution for the membership function ϕ for each node α of the grid. The set of facies $\{F^1, F^2, \dots, F^n\}$ is assumed to be sorted according to a given order defined by the user.
- For each facies i ,
 - Generate a P-Field $P^i(\alpha)$ using a Non Conditional Sequential Gaussian Simulation (NCSGS) honouring a prior variogram γ_i describing the spatial structure of the facies F^i . (figure 2-A)
 - All over the grid, a “binary” membership function ϕ^* is defined. This membership function considers only two types of facies: the current simulated facies f_i and a facies $\bar{f}_i$, composed of all the other facies. (figure 2-B)

For each node α (see figure 2-C) :

- If $P^i(\alpha)$ is lower than $\phi^{*i}(\alpha)$, then assign the facies f_i at the node α and update the membership function such as $\alpha \in F^i$. the node α becomes a “pseudo data”.

$$P^i(\alpha) < \phi^i(\alpha) \implies \begin{cases} \phi^i(\alpha) = 1 \\ \phi^j(\alpha) = 0 \quad \forall j \neq i \end{cases}$$

Following this modification of the membership function, “dynamic” additional constraints must be added such as, for example, a “property control node” specifying that the new value of $\phi(\alpha)$ defined above must remain unchanged by subsequent applications of the interpolation algorithm to the membership function, or a transition probability constraint specifying that the values of the membership function in the neighbourhood of the node α must depend on the value of $\phi(\alpha)$.

- If $P^i(\alpha)$ is higher than $\varphi^{*i}(\alpha)$, then update the membership function as described below for imposing that the facies F^i will no longer be observed at node α .

$$P^i(\alpha) > \varphi^i(\alpha) \implies \begin{cases} \varphi^i(\alpha) = 0 \\ \varphi^j(\alpha) = \varphi^j(\alpha) / \sum_{k \neq i} \varphi^k(\alpha) \quad \forall j \neq i \end{cases}$$

A property control node specifying that the new value of $\varphi^i(\alpha)$ defined above must remain unchanged by subsequent applications of the interpolation needs to be installed on this node.

- Re-interpolate the membership function $\varphi(\alpha)$ using DSI conditioned to all the specified constraints.
- Repeat the above steps on the next facies following the facies simulation order.

Note that at each step the simulation of the facies F_i depends on the realization of the simulation for facies $j, j < i$. In other words, the latest the facies is simulated, the stronger its dependency on previous simulated facies is. Note that the last simulated facies is totally defined at each location α by the realizations of the $n-1$ others facies.

In application of the modelling of hierarchical sedimentary architectures (such as fluvial deposits), this procedure becomes particularly interesting as soon as it makes the different facies be simulated in a hierarchical order.

Figure 2 : Different steps of the Multi P-Field simulation method

Results

To illustrate the flexibility of the proposed approach, two case studies are presented. The first one is a stationary anisotropic case, and the second one illustrates the notion of transitions between facies.

Case study #1

In this example, the sedimentary formation is composed of 4 equiprobable facies with different spatial structures. The sedimentary environment is assumed to be stationary in the whole domain. The variograms used in this study have been estimated for each facies (spherical ones, without nugget). Results obtained using the Multi P-Field simulation method are presented in figure 3.

Figure 3 : A Realization of facies simulation

We can note that *a priori* proportions are reproduced in the simulation during the simulation process (facies are equally represented in the realization), and that directions of anisotropy are well honoured.

Case study #2

The second case study illustrates the results obtained when transition probability constraints are taken into account in the simulation.

Let us consider a node α belonging to a facies F^i . The transition probability T_{FiFj} may be defined as the probability for a node β neighbouring α to be located into facies Fj . The transition probabilities may be computed from wells, or *a priori* given; for exemple, facies may often be ordered into a stratigraphic sequence, making some occurrences such as facies F_i close to facies F_j impossible. Such a case can be handled using transition constraints ($T_{FiFj} = 0$).

In the proposed example, only vertical transition probabilities have been defined, such as $T_{F1F2} = 0.9$ and $T_{F1F3} = 0.1$. Results are presented in figure 4. Since both realizations have been performed using the same seed number, the same configuration for facies F^1 may be observed in both realizations. When transition probability constraints are used, facies F^3 is more represented on the nodes neighbouring facies F^1 .

Figure 4 : Facies Realization : A – without the transition constraint B – with a transition constraint.

Discussion

Unlike classical geostatistical methods, the proposed approach interpolates a membership vectorial function describing the probabilities of occurrence of each facies at a given node of a grid, while reproducing a specific variogram per facies is honoured.

To summarize, the advantages of the membership function method are the following ones :

- The method can be directly used to simulate lithological facies;
- The obtained realizations account for the spatial anisotropy of each facies.
- The method is very flexible and can be adapted to take into account external constraints such as proportion map, transitional probabilities, vertical proportions, seismic attributes.
- It can follow a predefined stratigraphic sequence of facies (conditioning properties).

It may be noted that the Probability Fields used during the simulation take only into account the spatial correlations and does not include the hard data. In this context, using P-Fields does not introduce any limitations or artefact.

However, the limitations are the following:

- The simulations do not take into account cross spatial structures of facies.
- The result depends on the stratigraphic sequence defined on the facies. Different sequence of facies simulation may lead to different results.

Conclusions

The presented method seems to be particularly well adapted to model accurately the spatial architecture of fluvial deposits (deep sea fans, flood plain facies). Unlike sequential indicator simulation, the membership function based simulation here proposed allows reproducing the spatial structures and transition probabilities between facies. This method combines the flexibility of DSI to account for external constraints defined at a local (well data) or global scale (proportion maps) with the ability of Probability Field methods to account for prior information coded at local probabilities of occurrences of the different facies. The algorithm requires to select a predefined sequence order according to the successive facies are simulated. In this way, it is close to the Truncated Gaussian Simulation. Finally, it is important to note that the proposed algorithm is relatively fast and easy to implement.

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