METHODS FOR HISTORY MATCHING UNDER GEOLOGICAL CONSTRAINTS

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Abstract

Two geostatistical methods for history matching are presented. Both rely on the sequential simulation principle for generating geologically sound realizations. The first method relies on perturbing the sequential simulation through the perturbation of the conditional distribution models; the second method relies on the perturbation of random numbers. We show that both approaches are general in the sense that a large variety of geological scenarios can be generated while history matching. However, the conditional probability method is more efficient due to the ability to change the random path during the history matching procedure. We demonstrate these methods on two synthetic examples: a first example demonstrates how history matching can be performed under a training image based geological model constraint using multiple-point geostatistics; a second example shows how a combinations with an existing streamline-based history matching algorithm can provide efficient history matching yet maintaining geological consistency.

Introduction

In the practice of reservoir modeling, history matching refers to the process of obtaining a reservoir model based on a wealth of flow data such as well pressures, fractional flow, flow meter or RTF data. However the name “history matching” is poorly chosen since the objective is not to merely fit such data, but to calibrate a permeability model that is not only consistent to flow data but also to other types of information. While historic dynamic data is an important constraint to the reservoir model, it is often not fully constraining, a large degree of freedom still remains.

This requires the assumption of a prior model, or in reservoir terms, an assumption of geological heterogeneity or choice of geological scenario. Many types of geological heterogeneities may lead to equally adequate history matched models. However, neglecting to state a prior model is equivalent to assuming a maximally smooth prior hence often leads to overly smooth permeability models that have no prediction power.

Many authors have established this problem but no single method exists that can history match under a wide variety of geological scenarios. Most algorithms are limited by the assumption of variogram-based spatial variability, or worse rely strongly on the assumption of multi-Gaussianity, which is rarely adequate in heterogeneous reservoirs exhibiting strong connectivity or curvi-linear correlations. In this paper we present two, similar, approaches that meet the objective of reproducing a wide variety of geological constraints while history matching. Our methodology is based on the concept of building reservoir models using the sequential simulation principle, which is reviewed first.
Sequential Simulation

Sequential conditional simulation (Deutsch and Journel, 1998) is one of the most used geostatistical algorithms. The reason for this is CPU speed, robustness and the ability to generate a wide variety of geological scenarios conditioned to static information. The principle is simple: any hard (direct) information is assigned to a regular or irregular grid of choice, then each node along a predefined random path is simulated sequentially by random drawing from a conditional distribution model. Three elements are required:

1. A random path visiting each node \( u \)
2. A set of sequentially estimated conditional distributions (ccdf), termed \( P(A|B) \) from which values are drawn. Each \( P(A|B) \) models the conditional probability of the value to be simulated, namely \( A \), conditional to both hard data and previously simulated values, termed as \( B \).
3. A series of random numbers \( v \) used to draw from the ccdf models.

The type of the distribution determines the type of geological heterogeneity reproduced. For example in sequential Gaussian simulation (sgsim, Deutsch and Journel, 1998) the ccdf \( P(A|B) \) is Gaussian with mean and variance determined by simple Kriging of the hard data and previously simulated nodes. Sgsim only require the marginal distribution and covariance function to be known, hence is limited in terms of the geological heterogeneity it reproduces (maximum entropy). A recent new sequential simulation algorithm, termed snesim (single normal equation simulation, Strebelle, 2001), relies on a training image for the quantification of geological patterns. The method is straightforward: the conditional probability \( P(A|B) \) is modeled by scanning the training image for replicates of the joint data event \( (A,B) \). The ccdf \( P(A|B) \) is estimated by calculating the frequency of the event \( A \) occurring given the event \( B \) occurs. An efficient procedure for performing such scanning and estimation is outlined in Strebelle (2002). Figure 1e shows an example training image of ellipsoidal shapes, while Figures 1(a,b,c) are three realizations constrained to 5 well data. This method allows generating a wide variety of geological scenarios, including non-object shapes (Caers and Zhang, 2002) as well as conditioning to a wide variety of seismic data (Caers et al., 2001; Journel, 2002).

Perturbing sequential simulations

Any history matching method relies on a perturbation mechanism that moves some initial geostatistical realization to history match. However, such perturbation should not destroy the geological concept intended by the applied sequential simulation algorithm, be it variogram or training image-based. Our aim is to perturb sequential simulation realizations by perturbing either:

1. The probability models \( P(A|B) \) used to draw simulated values.
2. The series of random numbers used to draw from \( P(A|B) \)

The former is a novel approach, the latter has been proposed by Hu et al. (2001), but a new gradient-based version is presented.
Perturbing the probabilities $P(A|B)$

**Methodology**

Perturbations of the conditional probabilities should be such that the resulting sequential simulation honors the assumed prior geological concept expressed in the training image. Sequential simulation can honor a wide variety of spatial constraints, such as the correlation structure between the variable simulated and some soft data (e.g., seismic) through co-kriging models or block average constraints through block-kriging. However, sequential simulation cannot integrate directly with production data, which is observed at a point (well) and moreover varies in time. The approach taken in this paper is to turn production data into a spatial constraint and use sequential simulation to integrate that constraint into a geologically sound realization.

Consider for simplicity and clarity of presentation a binary indicator random function $I(u)$ which is defined as follows:

$$I(u) = \begin{cases} 
0 & \text{if facies 0 occurs} \\
1 & \text{if facies 1 occurs}
\end{cases}$$

Define a non-stationary Markov chain on the entire set of $I(u)$ $\forall u$, starting from an initial realization $i^{(0)}(u)$, generating iterations $i^{(t)}(u)$ till convergence (history match). The production data (pressures, flow) is denoted by $\mathbf{D}$. We define a 2x2 transition matrix as follows:

$$
P(I^{(t+1)}(u) = 1 | \mathbf{D}, I^{(t)}(u) = 0) = r_D P(I^{(t)}(u) = 1) = r_D P(A)$$

$$
P(I^{(t+1)}(u) = 0 | \mathbf{D}, I^{(t)}(u) = 1) = 1 - r_D P(I^{(t)}(u) = 0) = 1 - r_D (1 - P(A))$$

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$$
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The transition matrix quantifies the probability of switching state (=facies) given only the production data (not the geology). It is parameterized by a single parameter $r_D$ that be freely chosen. Note that in case $r_D = 1$ the transition from facies 1 to facies 0 is simply based on the prior $P(A)$, i.e. the proportion of facies 1, which is equivalent to stating that a maximum change should be made at $u$, while $r_D = 0$ entails that no change should be made.

From the transition probabilities we can easily derive the following conditional probability

$$P(A | \mathbf{D}) = (1 - r_D) i^{(t)}(u) + r_D P(A) \quad (1)$$

which is the conditional probability of drawing “facies 1” at any location $u$, given the production data and is expressed as a mixture of the current realization and the marginal proportion $P(A)$ of facies 1. $r_D$ depends on the production data $\mathbf{D}$ and is found through a simple optimization process as follows. First, we generate realization of the random function $I(u)$ using a sequential simulation method, e.g., snesim. However, we use the probability model $P(A|\mathbf{D})$ as a “soft” data set by recombining the single conditional probability models $P(A|B)$ and $P(A|\mathbf{D})$ into a joint conditional probability $P(A|B, \mathbf{D})$ using a Journel’s ratio independence hypothesis (Journel, 2002):
\[
\frac{x-b}{b} = \frac{d-a}{a} \quad \text{with} \quad x = \frac{1 - P(A|B,D)}{P(A|B,D)}, b = \frac{1 - P(A|B)}{P(A|B)}, d = \frac{1 - P(A|D)}{P(A|D)}, a = \frac{1 - P(A)}{P(A)}
\]

\[
\Rightarrow P(A|B,D) = \frac{1}{1+x} = \frac{a}{a+bd}
\]

The joint conditional probability \( P(A|B,D) \) is used in sequential simulation as the conditional distribution, instead of \( P(A|B) \). Denote as \( i^{(i+1)}(u) \) a single realization drawn using \( P(A|B,D) \), then we find \( r_D \) by solving the optimization problem

\[
r_D^{opt} = \min_{r_D} \left\{ \sum_{t_p} \sum_{w} \left( f_w^o(t) - f_w^s(t, i^{(i+1)}(u)) \right)^2 \right\}
\]

where \( f_w^o(t) \) is the observed flow and pressure data and \( f_w^s(t, i^{(i+1)}(u)) \) the simulated flow and pressure on a given realization \( i^{(i+1)}(u) \). Typically five flow simulations are required to find the optimal value \( r_D^{opt} \). The resulting realization \( i^{(i+1)}(u) \) will match better the production data \( D \), yet will still honor the same geological prior model. The latter is guaranteed through Eq. (2) which combines the probability \( P(A|B) \) determined from the training image with \( P(A|D) \) which depends on the production data \( D \). To provide a large space of possible realizations, the random path is changed at each iteration \( \ell \).

In case of multiple (K) facies, each facies class \( s_k \) can be modeled using a different indicator random variable

\[
I(u, s_k) = \begin{cases} 
1 & \text{if facies class } s_k \text{ occurs} \\
0 & \text{else}
\end{cases}
\]

then obtain a set of conditional probabilities

\[
P(A_k | D) = (1 - r_D) i^{(i)}(u, s_k) + r_D P(A_k) \quad \text{with} \quad A_k = \{ i^{(i)}(u, s_k) = 1 \}
\]

**Application: Global perturbations based on training images**

The snesim algorithm is used to generate a 100x100 reference model in Figure 1A based on the training image of Figure 1E. A full 5-spot configuration is used, water injecting into oil at constant rate, producers are bottom-hole pressure controlled. Mobility ratio is 5. The permeability is 1500 mD for facies 1 and 50 mD for facies 0, and is assumed known. The injector is located in the middle, producing well 1 in bottom left, well 2 in bottom right, well 3 in top left, well 4 in top right.

Figure 1(A) shows a good connection between the injector and producing wells 1 and 4, a poor connection to producing wells 2 and 3, exhibiting later breakthrough. Figure 1D shows that a good match is obtained after 8 iterations (about 40 flow simulations). The initial model and history matched model is shown in Figure 1B,C. The history matched realization in Figure 1C maintains the geological continuity of elliptical shapes common to the reference and initial model.
Perturbation of random numbers

Methodology

Hu et al. (2001) propose to perturb sequential simulations by perturbing random numbers. This method is equally general as the method above, although less efficient. Consider two random vectors $Y_1$ and $Y_2$ whose components are all standard normal and mutually independent. From these two vectors, a new vector $Y(r_D)$ is constructed as follows

$$Y(r_D) = Y_1 \cos r_D + Y_2 \sin r_D$$

Consequently, $Y(r_D)$ is also standard normal and has mutually independent components, for all possible $r_D$. Based on $Y(r_D)$, one defines a vector $V(r_D)$ as follows

$$V(r_D) = G(Y(r_D))$$

with $G$ the standard normal cumulative distribution function. All components of $V(r_D)$ are therefore uniformly distributed. Hence, for various values $r_D$ one obtains a series of vectors of uniform random numbers $v(r_D)$ that are perturbations of $v(0) = v$. Since any sequential simulation method uses a set of uniform random numbers, one can generate realizations $k_{r_D}(u)$ that are perturbations of the initial model $k_0(u)$. To obtain a history match using this method a similar objective function as Eq. (3) can be minimized. The method is often termed “gradual deformation”, although this term is only strictly applicable in the case of sequential Gaussian simulation. For discrete type variable (facies), in which case one would use the snesim and sisim algorithms, the perturbation is actually “a-gradual”, i.e. the objective function $O(r_D)$ is not a smooth function of the parameter $r_D$. This method can be applied to any type of sequential simulation, however, is less efficient due to the fact that the random path remains fixed during the entire procedure. This slows the retrieval of the full space of realizations available. A variable random path, such as used in the probability perturbation method can search more efficiently the space of all possible realizations.

Application: Streamline history matching

Applying the perturbation of random numbers method requires hundreds of flow simulation if applied to the example in Figure 1. This is due to the fixing of the random path in the entire procedure as outlined above. To avoid this amount of flow calculation, we apply a streamline-based history matching method presented in Wang and Kovscek (2001).

Wang and Kovscek present a history matching algorithm based on matching time of flights (TOFs) along a set of streamlines in a black oil model. For a given producer, the fractional flow of the producer is function of the time-of-flight along all streamlines connecting to the producer. For each producer, the observed target fractional flow is converted into a set of target time-of-flights for all streamlines entering that producer. Since the time of flight of a streamline can be written as a direct function of the harmonic average of the permeability along that streamline, the problem of history matching is turned in a problem of matching harmonic averages along streamlines. Denote by $k_{SLm}(k_{r_D}(u))$ a harmonic average along a streamline $SL_m$ that depends on the permeability field $k_{r_D}(u)$. To history match, we solve the following optimization problem
\[ r_{D}^{\text{opt}} = \min \{ O(r_{D}) = \sum_{\text{all } \mathbf{Sl}_{m}} (k_{\text{Sl}_{m}} - k_{\text{Sl}_{m}}(k_{D}(u)))^{2} \} \quad (4) \]

with \( k_{\text{Sl}_{m}} \) the harmonic averages determined from the production data (see Wang and Kovscek, 2001). To solve this optimization problem, no flow simulations are required. In order to provide additional flexibility, the permeability model can be perturbed per zone. A zone could for example be defined as the set of all permeabilities that are hit by streamlines that enter a given producer, hence there are as many zones as producers. However, the perturbation should be such that the geological model continuity is maintained across zones. Instead of perturbing permeabilities directly, random numbers are perturbed differently in each zone, each perturbation parameterized with an individual \( r_{D}^{\text{zone}} \) (Hu et al., 2002). This guarantees geological continuity across zones. A gradient-based optimization method can then be used to find the optimal set \( r_{D}^{\text{zone}}, \ell = 1, \ldots, N_{\text{zone}} \) (Caers, 2002) of the problem in Eq. (4).

To demonstrate the importance of the zoning procedure we develop a 9 spot case with 4 injectors and 5 producers, as shown in Figure 2A. The reference model is a 200x200 field shown in Figure 2A and is generated using "sgsim". An initial guess is generated in Figure 2B. A streamline simulator is run on the initial model to obtain TOF and streamline geometry. Using a simple interpolation procedure, we construct a set of zones-of-influence per producer, as shown in Figure 2G. Using this zoning, we attach a parameter \( r_{D}^{\text{zone}} \) to each zone and find jointly set of parameters \( r_{D}^{\text{zone}}, \ell = 1, \ldots, N_{\text{zone}} \) that match best the target harmonic averages. The result is shown in Figure 2D. For comparison, Figure 2C shows a similar match to the target harmonic averages, but not accounting for any geology. After four iterations, a satisfactory match is found, see Figure 2F. Note that the zones vary with each iteration, Figure 2H shows the zones after the final iteration, which are different from Figure 2G. Instead of having to evaluate possibly hundreds of flow simulations, we obtain a history match using only four streamline simulations, yet a satisfactory match is obtained as shown in Figure 2I.

References


Figure 1: (A) Reference model, (B) History matched model starting from (C) Initial realization, (D) history matched results, (E) Training image used
Figure 2: (A) reference model and location of wells, (B) initial model, (C) streamline perturbations of the initial model without accounting for geology, (D) with geology, (E) last iteration, not accounting for geology, (F) history matched realization, (G) zones per producer based on the initial model, (H) zones of the final model, (I) fractional flow results.