E12 ANALYSIS OF OIL INFILTRATION INTO WATER-UNSATURATED HORIZONTALLY LAYERED POROUS MEDIA

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Abstract

Constant and continuous oil infiltration was considered from a point source into a low permeable horizontal layer of relatively small grain sizes and small pores with an underlying high permeable layer of relatively large grain sizes and large pores. Both porous media have a similar structure, i.e. similar pore structure but different mean pore sizes. The interface between both layers acts as a capillary barrier, and consequently, the oil plume spreads above the interface. An ordinary differential equation was derived for the oil pressure at the interface, which led to an approximation for the flow regime above the interface. The oil flow could be characterized by 5 dimensionless numbers based on fluid and porous medium properties and the position of the interface. Three-phase relative oil permeability was approximated assuming low oil saturation. A simple numerical procedure was used to obtain the oil pressure at the interface. Comparison with numerical simulation showed that the semi-analytical approximation provides a good estimate of the oil pressures and spreading at the interface.

Introduction

Many sedimentary deposits are composed of laterally continuous layers, of different grain sizes, and consequently, different porous medium properties like permeability (k) and porosity (ϕ). In this paper the effect is studied of these heterogeneous porous medium properties on the gravity-driven infiltration and redistribution of oil. We focus on oil infiltration from a point source into a low permeable horizontal sandy layer of relatively small grain sizes and small pores with an underlying high permeable layer of relatively large grain sizes and large pores. Oil is the intermediate wetting fluid in the presence of the non-wetting fluid gas and the wetting fluid water. Water saturation is low, and therefore, oil infiltration can be considered as an imbibition process (oil displaces gas). With respect to the imbibing oil, the interface between the layers forms a so-called capillary barrier (e.g. Ross, 1990). The coarser sand has larger pores, and therefore, it has for a given oil-gas capillary pressure a lower oil saturation than the finer grained sand. Since relative permeability is written in terms of fluid saturation, the relative oil permeability is discontinuous at the interface, i.e. the relative permeability is much smaller in the coarser grained porous medium than in the fine grained porous medium. The fluid migration will be held up and the oil will spread above the interface.

The objective of this paper is to develop an analytical approximation that provides an estimate for oil pressures and saturation in the area above the interface where the oil plume spreads, given a constant continuous source at a steady state situation. The analytical approximation is a follow up of the approximation for air sparging in layered porous media of Van Dijke and Van der Zee

(1998). Both the analytical model presented in this paper, and the model of Van Dijke and Van der Zee account for the effect of a barrier due to discontinuity of porous medium properties, however, whereas their model considers non-wetting fluid (air) spreading above a fine grained layer in a 2 fluid-phase system (water and air), the model presented in this paper considers spreading of the intermediate wetting fluid (oil) above a coarse grained layer in a 3 fluid-phase system.

This paper is organized as follows. We first introduce the transient model equations and the geometry of the domain. Next, we reformulate the problem in dimensionless form and identify governing dimensionless numbers. Subsequently, we derive an ordinary differential equation to describe oil pressures near the interfaces and the oil spreading above the interface. Finally, we compare the approximated semi-analytical pressure profiles and numerically obtained pressure profiles at the interface and discuss differences.

Figure (1) Schematic of the domain

Model equations

The flow problem is defined in an axial symmetric semi-infinite domain as illustrated in Fig.(1). A fine grained, low permeable horizontal layer of thickness $z = z^*$ lays above a coarse grained, high permeable layer. Below the layer interface, at $z = h$, the water table is situated. Oil is introduced at $z = 0$ and $0 < r < \frac{1}{2}d$ with an infiltration rate of $u = u_{in}$. d is the source diameter.

We assumed that each porous medium layer is isotropic, that water and oil are incompressible and immiscible and that gas is infinitely mobile at constant pressure ($P_a = 0$) and saturation S_a . The governing equations for the flow of water (w) or oil (o) are the mass balance equation

$$\phi \frac{\partial \bar{S}_i}{\partial t} + \nabla u_i = 0 \quad i = w, o \quad (1)$$

and the flux equation

$$u_i = \frac{k k_{ri}}{\mu_w} \nabla (P_i + \rho_i g z) \quad i = w, o, \quad (2)$$

where u_i is the Darcy flux of phase i and ϕ is the effective porosity, k is the intrinsic permeability tensor, $\bar{S}_i$, k_{ri} , μ_i , p_i and ρ_i are respectively the effective saturation, the relative permeability, the viscosity, the pressure and the density of phase i . g is gravity, z is the depth below soil surface

and t is time. The set of equations is completed by the constitutive relations $\bar{S}_w + \bar{S}_o = \bar{S}_t$ and $\bar{S}_t + \bar{S}_a = 1$. $\bar{S}_t$ is the effective total liquid saturation. Furthermore, the pressure difference at the interface between the different fluid phases i and j is $p_c^{ij} = p_i - p_j$, which is the capillary pressure. We assumed that fluid wettability follows the sequence water > oil > gas. We used p_c - S relations with a distinct entry pressure that are described by scaled variants of the empirical function of Brooks and Corey (1966). The 2-phase gas-water relation is defined by:

$$\bar{S}_w = \left(\frac{p_e}{p_c^{aw}} \right)^\lambda, \quad (3a)$$

where λ and p_e are Brooks and Corey porous medium properties of which p_e is the entry pressure. If $p_c^{aw} \leq p_e$ we have $\bar{S}_w = 1$. The 3-phase gas-oil-water relations are defined by

$$\bar{S}_w = \left(\frac{p_e}{\beta_{ow} p_c^{ow}} \right)^\lambda \text{ and } \bar{S}_t = \left(\frac{p_e}{\beta_{ao} p_c^{ao}} \right)^\lambda, \quad (3b)$$

if $\beta_{ow} p_c^{ow} \leq p_e$ we have $\bar{S}_w = 1$ and if $\beta_{ao} p_c^{ao} \leq p_e$ we have $\bar{S}_t = 1$. β_{ow} and β_{ao} are scaling coefficients that relate respectively the oil-water curve and the air-oil curve to the air-water curve, $\beta_{ow} = \frac{\sigma_{aw}}{\sigma_{ow}}$ and $\beta_{ao} = \frac{\sigma_{ao}}{\sigma_{ow}}$. σ_{ij} is the interfacial tension between the fluids i and j . For the relative permeability, k_r , we apply the relationships proposed by Mualem (1976):

$$k_{rw} = \bar{S}_w^{2.5+2\lambda}, \quad (4)$$

$$k_{ro} = \bar{S}_o^{0.5} \left(\bar{S}_t^{1+\frac{1}{\lambda}} - \bar{S}_w^{1+\frac{1}{\lambda}} \right)^2. \quad (5)$$

Hysteresis is not included (no entrapment).

Analysis of steady state oil infiltration

The intrinsic permeability in the high permeable layer is referred to as k^+ and in the low permeable layer it is referred to as k^- . $k^+ = \gamma^2 k^-$. γ is the heterogeneity factor which is larger than 1. If we assume similar media, then the relation between capillary pressure (p_c) and saturation (S) can be written in term of fluid and porous medium properties according to: $p_c = \sigma \sqrt{\phi/k} J(S)$, where $J(S)$ is the dimensionless Leverett J function (Leverett, 1941). This implies that $p_e^+ = p_e^- / \gamma$.

Figure (2) shows a numerically obtained example saturation profile at steady state flow using a finite domain. The horizontal dimension was chosen such that the infiltrating oil did not reach the boundary in the area of interest. The used parameters are equal to the parameters used in Figure (4). Spreading is observed above the interface.

Figure (2) Example saturation profile at steady state

With respect to the analytical approximation, we only considered oil flow. Water is assumed at hydrostatic pressure, hence $p_w(z) = \rho_w g z$.

Dimensionless formulation

We defined the dimensionless variables:

$$T = \frac{t U_c}{\phi d} \quad R = \frac{r}{d} \quad Z = \frac{z}{d} \quad U = \frac{u}{U_c}, \quad (6)$$

where $U_c = \frac{\mathbf{k}^- \rho_o g}{\mu_o}$ is the intrinsic oil flux of the fine grained, low permeable layer. Hence, the fine grained layer is the reference layer. The source diameter, d , is chosen as the characteristic length. Dimensionless oil pressure is defined as:

$$P_o = \frac{p_o}{p_e}, \quad (7)$$

furthermore, we defined the dimensionless constants:

$$N_g = \frac{\mathbf{k}^- \rho_o g}{\mu_o u_{in}} \text{ and } N_c = \frac{\mathbf{k}^- p_e}{\mu_o u_{in} d}. \quad (8)$$

N_g is the gravity number and N_c is the capillary number.

The governing steady state flow equation for the fine grained layer is now:

$$\nabla U_o = \nabla \left(k_{ro} \nabla \left(-\frac{N_c}{N_g} P_o + Z \right) \right) = 0 \quad Z < Z^* . \quad (9)$$

For the coarse grained layer, steady state oil flow is given by:

$$\nabla \gamma^2 U_o = \nabla \left(\gamma^2 k_{ro} \nabla \left(-\frac{N_c}{N_g} P_o + Z \right) \right) = 0 \quad Z > Z^* . \quad (10)$$

The water pressure is defined by:

$$P_w(Z) = \frac{N_g}{N_c} \frac{\rho_w}{\rho_o} Z, \quad (11)$$

where $P_w = p_w / p_e$. The 2-phase gas-water relation is defined by:

$$\bar{S}_w = \left(\frac{1}{-P_w} \right)^\lambda, \quad (12a)$$

if $-P_w \leq 1$ we have $\bar{S}_w = 1$. The 3-phase gas-oil-water relations are defined by

$$\bar{S}_w = \left(\frac{1}{\beta_{ow}(P_o - P_w)} \right)^\lambda \text{ and } \bar{S}_t = \left(\frac{1}{-\beta_{ao}P_o} \right)^\lambda, \quad (12b)$$

if $\beta_{ow}(P_o - P_w) \leq 1$ we have $\bar{S}_w = 1$ and if $-\beta_{ao}P_o \leq 1$ we have $\bar{S}_t = 1$. Oil is introduced with an infiltration rate of $U_{in} = u_{in} / U_c = 1 / N_g$ between $R = 0$ and $R = 0.5$ at $Z = 0$. Hence the total infiltrating flux is $\pi / 4 N_g$. At the interface between the two layers continuity of pressure, $P_o(R, Z^*)$, exists and there is continuity of vertical flux across the interface, $U_z(R, Z^*)$.

Figure (3) Schematic of the analytical approximation

Analytical model

The model flow of the analytical approximation is illustrated in Fig (3). In contrary to the transient model we assumed the oil to enter the domain from the left side through a tube with diameter d . Following Van Dijke and Van der Zee (1998) we considered the vertical velocity component of the lower layer to be dominated by buoyancy forces, and therefore, we considered $\partial P / \partial Z$ to approximate zero due to the low relative permeability, hence, from Eq.(10) follows: $U_z = \gamma^2 k_{ro}$. Subsequently, we assumed that above the interface the radial spreading of the oil is much larger and, because vertical flow velocities are very small, we assumed that the oil is approximately at vertical hydrostatic pressure. This implies that above the interface:

$$P_o(R, Z) = P_o(R, Z^*) - \frac{N_g}{N_c} (Z^* - Z). \quad (13)$$

$P_o(R, Z^*)$ is the oil pressure at the interface. Consequently, the oil pressure at the interface defines the pressure in the entire area.

The thickness of the plume above the interface is limited by Z_b , belonging to a certain R where $P_o = P_o^c$:

$$Z_b(R) = Z^* + \frac{N_c}{N_g} (P_o^c(Z^*) - P_o(R, Z^*)) \quad (14)$$

P_o^c is the oil pressure at Z_b for $S_l = S_w$, where $S_o = 0$, and it is equal to $(\beta_{ow} P_w)/(\beta_{ow} + \beta_{ao})$. We assumed that the height of the hydrostatic pressure area is small, and that P_o^c can be considered as constant over the layer and equal to the P_o^c at Z^* .

The pressures at Z^* can be derived by vertically integrating Eq.(9) from Z_b to Z^* , which yields:

$$\int_{Z_b}^{Z^*} \left(\frac{\partial R U_R}{\partial R} + R \frac{\partial U_Z}{\partial Z} \right) dZ = 0 \quad \text{or} \quad (15)$$

$$\frac{\partial}{\partial R} \left(R \int_{Z_b}^{Z^*} U_R(R, Z) dZ \right) + R U_Z(R, Z^*) = 0. \quad (16)$$

U_R is U_o in the R -direction and U_Z is U_o in the Z -direction. To evaluate the first term of Eq.(16), i.e. the effective horizontal flux, we derive from Eq.(13):

$$\frac{\partial P_o(R, Z)}{\partial R} = \frac{dP_o(R, Z^*)}{dR} \quad \text{and} \quad dZ = \frac{N_c}{N_g} dP_o(R, Z^*), \quad (17)$$

using Eq.(9) this leads to:

$$\int_{Z_b}^{Z^*} U_R dZ = -\frac{N_c}{N_g} \int_{Z_b}^{Z^*} k_{ro} \frac{\partial P_o(R, Z)}{\partial R} dZ = -\frac{N_c^2}{N_g^2} D(P_o(R, Z^*)) \frac{dP_o(R, Z^*)}{dR} \quad (18)$$

$D(P_o)$ is defined as $D(P_o) = \int_{P_o^c}^{P_o} k_{ro}(\xi) d\xi$, where ξ is a dummy variable for integration over the range of P_o values. The second term in Eq.(16) is the effective vertical flux. Eq (16) can be rewritten as:

$$-\frac{N_c^2}{N_g^2} \frac{d}{dR} \left(R D(P_o) \frac{dP_o}{dR} \right) + R \gamma^2 k_{ro}(\gamma P_o) = 0 \quad (19)$$

where P_o is the pressure at Z^* .

Relative permeability approximation

Under the condition that P_o approximates P_o^c , the relative permeability, k_{ro} , can be expanded and written in terms of $P_o - P_o^c$:

$$k_{ro} = \begin{cases} C(P_o - P_o^c)^{5/2} & \text{for the fine grained layer} \\ \gamma^{-2.5\lambda-4.5} C(P_o - P_o^c)^{5/2} & \text{for the coarse grained layer} \end{cases} \quad (20)$$

where $C = (1 - 1/\lambda)^2 \lambda^{5/2} (-P_w)^{-2.5\lambda-4.5} (\beta_{ow} + \beta_{ao})^{5/2}$. Given Eq.(18), the integral $D(P_o)$ is: $\frac{2}{7} C (P_o - P_o^c)^{7/2}$. Because oil saturation is observed to be low, Eq.(18) can be used to obtain the solution of the flow problem.

Boundary Conditions

Integration of the horizontal effective flux over R gives the boundary condition at $R = 1/2$:

$$-2\pi \frac{N_c^2}{N_g^2} RD(P_o) \frac{dP_o}{dR} \Big|_{R=1/2} = \frac{\pi}{4N_g}. \quad (21)$$

At $R = f$, $Z_b = Z^*$. Z_b is tangential to the flow direction. Following Van Dijke and Van der Zee(1998), we obtain the boundary condition at $R = f$:

$$\frac{dP_o(f)}{dR} = -\gamma^{-1.25\lambda-1.25} \frac{N_g}{N_c}. \quad (22)$$

Furthermore, the oil pressure at the plume boundary, $P_o(f)$ is P_o^c .

The boundary value problem (Eq(20) to (22)) can be solved using a fourth order Runge-Kutta routine.

Results and Discussion

In Fig (4), the pressure difference, $P_o - P_o^c$, at the interface is given for the analytical approximation presented in Eq (20) to (22) as well as the numerical solution presented in dimensional form in the beginning of this paper using finite boundaries. For the numerical solution we made transient computations of which the flow reached a steady state situation. We used the following dimensionless parameters: $N_g = 500$, $N_c = 1263$, $P_w = 3.23$, $P_o^c = 1.8$, $\lambda = 1.8$, $\beta_{ow} = 2.25$, $\beta_{ao} = 1.8$, $C = 6.45 \cdot 10^{-4}$ and $\gamma = 2.0$.

The figure shows that the analytical approximation performed well and the extension of the plume at the interface is accurately estimated by the approximation. Differences between both solutions may be attributed to the boundary condition at the center of the plume. With respect to the analytical approximation the oil is assumed to flow into the domain through a vertical tube, and instead, with respect to the numerical solution the oil flows from the upper boundary into the domain. In addition, in the analytical approximation we assumed hydrostatic pressure above the interface and buoyancy dominated flow below the interface. Despite that the numerical solution approximates both pressure regimes, it may be that the assumption should be relaxed.

Observe that the boundary value problem represented by Eq(20) to Eq(22) shows that the steady state infiltration of flow can be characterized by two dimensionless numbers; N_g , N_c that are composed of porous medium, fluid and infiltration parameters in cooperation with γ the degree of heterogeneity between the layers and P_o^c that corresponds to the position of the interface with respect to the water table. With respect to the heterogeneity factor, γ , the boundary value problem shows that γ affects in two ways the effective vertical flux. On one hand, the flux is increased γ^2 accounting for the higher permeability of the coarse grained layer, and on the other hand, the flux is decreased $\gamma^{2.5\lambda-4.5}$ accounting for the relative permeability that is much lower in the coarse grained layer. Similarly, the derivative at $R = f$ represented by Eq(21) is on one hand increased by the higher permeability of the coarse layer and on the other hand decreased by the lower relative permeability in the coarse grained layer.

Future work will be done on the sensitivity of the solution to these dimensionless parameters and on the agreement to the numerical solution.

Figure (4) Comparison numerical and approximated pressure difference, $P_o - P_o^c$, at the interface $Z = Z^*$.

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