E18 POROUS MEDIA DEPOSITION DAMAGE FROM INJECTION OF WATER WITH PARTICLES

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Abstract

Severe fall of injectivity in porous rock occurs from the practice in offshore fields of injecting sea water containing organic and mineral inclusions. In general, injection of a poor quality water in a well curtails its injectivity. The injectivity loss is assumed to be due to particle retention in the porous rock.

A model for porous rock damage due to retention in deep filtration during injection of water containing solid particles is formulated. The model contains two empirical functions that affect loss of injectivity - filtration coefficient and damage coefficient versus deposited particle concentration.

We show how to solve the inverse problem for determining the first function based on effluent particle concentration measurements in coreflood tests.

The second inverse problem is the determination of the formation damage coefficient from the pressure drop history on a core. These two methods allow determining from laboratory tests the information necessary for prediction of well impairment.

1. Introduction

Injectivity reduction due to the injection of water containing solid particles takes place in most waterflood projects to some degree.

In designing a waterflood project, the level of water treatment necessary to minimize formation damage must be assessed, so it is important to know the performance of an injector as a function of injection water quality [1]. Therefore, substantial efforts have been devoted to model the injectivity decline due to injection of water with solid particles.

The basic mathematical model for deep filtration with particle retention consists of a particle mass balance equation and a kinetic equation for clogging [2-4]. An analytical model for diffusion-free flow was developed in [2] under the assumptions that accumulation of suspended particles can be ignored and that the suspended concentration distribution is in steady state.

In order to predict injectivity decline in wells, the mathematical model for radial flow requires two empirical functions – filtration coefficient and permeability versus deposited concentration. The problem of determining these functions from laboratory deep bed filtration requires solving two inverse problems.

Methods for determining constant filtration and formation damage coefficients from outlet particle concentration were presented in the literature [4,5]. However, the general inverse problems for determining filtration coefficient and permeability damage versus deposited concentration from deep bed filtration laboratory tests have not been investigated.

In the current paper we derive an exact analytical solution for 1-D linear problem for diffusive-free particle flow accounting for particle capture, without other limitations. The explicit solution of the direct problem is the basis for finding unique, stable solution of the inverse problems.

A well-posed and stable sequence of two procedures for determining the filtration and formation damage functions from outlet particle concentration and pressure drop measurements is formulated. Closed equations are derived. The solution of the two inverse problems allows complete tuning of the model from laboratory test data enabling prediction of well injectivity behaviour.

The assumptions in the model are the following. The water and particles are incompressible. The volume of the entrapped particles is negligible compared to the effective porosity ($\sigma \ll \Phi'$). The kinetics of particle capture is linear, and diffusion is negligible.

2. Mathematical model for flow of water with suspended particles

The mass conservation equation for retained and deposited particles in linear flow of water is (Fig.1):

$$\frac{\partial}{\partial t}(\Phi' c' + \sigma') + U \frac{\partial c'}{\partial x} = 0, \quad (2.1)$$

where $\Phi' = \Phi(1 - s_{or})$ is the effective porosity. Here $c'(x, t)$ and $\sigma'(x, t)$ are the volumetric concentrations of suspended and retained particles (Fig. 2).

The rate of retention is proportional to concentration of particles in suspension and to flow velocity:

$$\frac{\partial \sigma}{\partial t} = \lambda'(\sigma') U c. \quad (2.2)$$

The constant of proportionality $\lambda'(\sigma')$ is the “filtration coefficient” function. Darcy’s law includes the “permeability damage function” $k(\sigma)$:

$$U = - \frac{k_0 k(\sigma)}{\mu} \frac{\partial p'}{\partial x}. \quad (2.3)$$

We introduce dimensionless time, space, suspended and retained concentrations and pressure:

$$T = \frac{U t}{\Phi' L}, \quad X = \frac{x}{L}, \quad c = \Phi' \frac{c'}{c_0'}, \quad \sigma = \frac{\sigma'}{c_0'}, \quad p = \frac{k_0}{\mu L U} p'. \quad (2.4)$$

The concentrations have been non-dimensionalised by division by injected concentration c_0' (constant). Using Eq. (2.4), Equations (2.1), (2.2) become:

$$\frac{\partial c}{\partial T} + \frac{\partial c}{\partial X} = \frac{\partial \sigma}{\partial T}, \quad \frac{\partial \sigma}{\partial T} = \lambda(\sigma) c. \quad (2.5)$$

The system of equations (2.5) determines the unknowns $c(X, T)$ and $\sigma(X, T)$. Initial and boundary conditions for (2.5) simulating laboratory coreflood tests are (Fig. 1):

$$T = 0: c = 0, \sigma = 0 \text{ for all } X; \quad X = 0: c = 1, \text{ for all } T. \quad (2.6)$$

It is easy to show that the typical penetration length of both c and σ in the core is $X \cong 1/\lambda(0)$.

Fig. 1. Scheme of laboratory test.

Fig. 2. Suspended and deposited particles.

3. Analytical solution for 1-D linear flow of water with particles in porous media

Eq. (2.5b) is transformed to the following form:

$$\frac{\partial \Psi(\sigma)}{\partial T} = c, \quad \text{where (see Fig. 3):} \quad \Psi(\sigma) = \int_0^\sigma \frac{dy}{\lambda(y)}. \quad (3.1)$$

Substituting $c(X, T)$ from (3.1a) into (2.5a) and integrating from 0 to T , we obtain

$$\frac{\partial \Psi(\sigma)}{\partial T} + \sigma + \frac{\partial \Psi(\sigma)}{\partial X} = \left(\frac{\partial \Psi(\sigma)}{\partial T} + \sigma + \frac{\partial \Psi(\sigma)}{\partial X} \right)_{T=0}. \quad (3.2)$$

From the initial conditions (2.6), the right hand side of Eq. (3.2) is zero, so

$$\frac{\partial \Psi(\sigma)}{\partial T} + \frac{\partial \Psi(\sigma)}{\partial X} = -\sigma \quad \text{with} \quad \Psi(\sigma(X, 0)) = 0; \quad \Psi(\sigma(0, T)) = T. \quad (3.3)$$

The initial and boundary conditions for the equation (3.3) are obtained from the fact that at $T = 0$, $\sigma = 0$ and that at $X = 0$, the concentration c equals 1, and then we use (2.5) and (3.1).

Introducing the integral Π below, Eq. (3.3a) along characteristic lines $X = T + \text{const}$ becomes (see Fig. 4):

$$\frac{d}{dX} \Pi(\sigma) = -1, \quad \text{where (see Fig. 3):} \quad \Pi(\sigma) = \int_0^\sigma \left(\frac{1}{\lambda(\sigma)} - \frac{1}{\lambda(0)} \right) \frac{d\sigma}{\sigma} + \frac{\ln \sigma}{\lambda(0)}. \quad (3.4)$$

Integrating equation (3.4) from 0 to X , we obtain $c(X, T) = \sigma(X, T) = 0$ for $T < X$ and

$$\Pi(\sigma(X, T)) = \Pi(\sigma(0, T-X)) - X, \quad \text{for } T > X, \quad (3.5)$$

where the values of $\sigma(0, T)$ are determined by inverting Eq. (3.3c). Eq. (3.5) determines the retained particle concentration $\sigma(X, T)$.

In order to obtain the concentration of particles suspended in water $c(X, T)$, we differentiate both sides of Eq. (3.5) with respect to T . Accounting for the boundary condition (3.3) and the expression (3.4), we obtain

$$\frac{c(X,T)}{\sigma(X,T)} = \frac{1}{\sigma(0,T-X)}, \quad (3.6)$$

which determines the expression for $c(X,T)$. Finally, the formula for pressure distribution along the core during flow can be obtained by integrating the scaled version of the pressure gradient equation (2.3).

Fig. 3. Shape of functions in analytical model

Fig. 4. Concentration profiles in (X,T) plane.

4. Behaviour of concentration waves

The solution $\sigma(X,T)$ and $c(X,T)$ is shown in Fig. 4 in the plane (X,T) . The concentration front $X=T$ moves along the reservoir with unit speed. Ahead of this front $c(X,T)=0$, $\sigma(X,T)=0$. The concentration of deposited particles equals zero along the front, $\sigma(T,T)=0$. Thus, from Eq. (2.5), the concentration of suspended particles along the front $c(T,T)$ is always:

$$c(T,T) = \exp(-\lambda(0)T). \quad (4.1)$$

Fig. 4 shows the distributions of c and σ ahead and behind the front $X=T$.

5. Determination of filtration coefficient from coreflood tests – first inverse problem

The first inverse problem is the determination of the filtration coefficient $\lambda(\sigma)$ from the history of effluent suspended particle concentration at the core sample outlet $c(1,T)$ (Figs. 1-4).

Introducing the inverse function $\sigma = g(\Psi)$ of the function $\Psi = \Psi(\sigma)$ found in Eq. (3.3c), we obtain

$$\sigma(0,T) = g(T). \quad (5.1)$$

Let us integrate (3.1a) from 1 to T for $X=1$:

$$\Psi(\sigma(1,T-1)) = C(T-1) \text{ or } \sigma(1,T-1) = g(C(T-1)), \text{ where } C(T-1) = \int_0^{T-1} c(1,\ell+1) d\ell. \quad (5.2)$$

Substituting expressions (5.1) and (5.2) into equation (3.6) with $X=1$ and making $\tau = T-1$, we obtain the following functional equation for the function $\sigma = g(\Psi)$:

$$g(C(\tau)) = C'(\tau)g(\tau-1), \text{ where } C'(\tau) = c(1, T), \quad 0 < \tau < \infty. \quad (5.3)$$

The “initial data” for equation (5.3) are obtained from (4.1).

$$\lambda(0) = -\ln(c(1, 1)). \quad (5.4)$$

Remark 5.1. Notice that $\lambda(\sigma) > 0$ in Eq. (2.5) implies that $c < 1$, so that $C'(\tau) < 1$, so that $0 < C(\tau) = \int_0^\tau C'(\ell) d\ell < \tau$ for $\tau > 0$.

Lemma 5.1. The solution of the filtration inverse problem (5.3), (5.4) is given by Eqs. (5.10), (5.11).

Proof. To compute $g(\tau_0)$, for any $\tau_0 > 0$, we define the two infinite sequences

$$\begin{aligned} \tau_1 = C(\tau_0), \quad \tau_2 = C(\tau_1), \dots, \quad \tau_n = C(\tau_{n-1}), \dots, \quad \text{or} \quad \tau_n = C^{(n)}(\tau_0); \\ q_1 = C'(\tau_0), \quad q_2 = C'(\tau_1)q_1, \dots, \quad q_n = C'(\tau_{n-1})q_{n-1}, \dots, \quad \text{or} \quad q_n = \prod_{k=0}^{n-1} C'(\tau_k). \end{aligned} \quad (5.5)$$

We will see that $\lim_{n \rightarrow \infty} \tau_n = 0$ monotonically. The monotonicity follows from Remark 5.1:

$$\tau_n = C(\tau_{n-1}) < \tau_{n-1}, \quad \text{so} \quad \tau_n < \tau_{n-1}. \quad (5.6)$$

Now, we want to show that $\tau_n \rightarrow 0$. Let us assume that $\tau_n \rightarrow \bar{\tau} > 0$. Now, $\tau_n = C(\tau_{n-1})$; let $n \rightarrow \infty$; since C is continuous, $\bar{\tau} = C(\bar{\tau})$; because of Remark 5.1, $\bar{\tau} = 0$.

Notice that $\tau_1 = C(\tau_0)$ is continuous in τ_0 , and so is $\tau_n = C^{(n)}(\tau_0)$ and $q_n(\tau_0)$.

From the functional equation (5.3), we have that

$$g(\tau_k) = g(C(\tau_{k-1})) = C'(\tau_{k-1})g(\tau_{k-1}) \quad \text{or} \quad g(\tau_k) = \frac{q_k}{q_{k-1}} g(\tau_{k-1}). \quad (5.7)$$

So, by repeated use of any of the two formulas above for $k = n, n-1, \dots, 1$ we obtain:

$$g(\tau_n) = g(\tau_0) \prod_{k=0}^{n-1} C'(\tau_k) \quad \text{or} \quad g(\tau_n) = q_n g(\tau_0). \quad (5.8)$$

On the other hand, using Eq. (5.8), the definition of derivative of g , and $g(0) = 0$ (arising from Eq. (5.3)),

$$g'(0) = \lim_{n \rightarrow \infty} \frac{g(\tau_n) - g(0)}{\tau_n - 0} = \lim_{n \rightarrow \infty} \frac{g(\tau_n) - 0}{\tau_n - 0} = \lim_{n \rightarrow \infty} \frac{g(\tau_n)}{\tau_n}. \quad (5.9)$$

Substituting Eq. (5.8) in Eq. (5.9), we see that

$$g'(0) = \lim_{n \rightarrow \infty} g(\tau_0) \frac{\prod_{k=0}^{n-1} C'(\tau_k)}{\tau_n} \quad \text{or} \quad g(\tau_0) = g'(0) \lim_{n \rightarrow \infty} \frac{\tau_n}{\prod_{k=0}^{n-1} C'(\tau_k)} \quad \text{or} \quad g(\tau_0) = g'(0) \lim_{n \rightarrow \infty} \frac{\tau_n}{q_n}. \quad (5.10)$$

The derivation of formula (5.10) is complete. This is the solution of the functional equation, where $g'(0)$ is given in Eq. (5.11).

The iterative procedure for solving the functional equation (5.2) is shown graphically in Fig. 5. It is very easy to implement numerically. Curves 1, 2 and 3 in this figure correspond to the functions:

$$C(T) = \int_1^T c(1, \ell) d\ell, \quad c(1, T), \quad T-1, \text{ respectively.}$$

The value of $g'(0)$ follows from differentiating Eq. (5.1) and (3.3c), using (3.6) near $T = X = 0$:

$$g'(0) = \lambda(0). \quad (5.11)$$

The filtration coefficient $\lambda(\sigma)$ can be found from the function $\sigma = g(\Psi)$ using equation (3.1).

Fig. 5. Iterative procedure for solving the first inverse problem.

Fig. 6. Determination of filtration and formation damage coefficients from laboratory data.

6. Formation damage function from coreflood tests— second inverse problem

The second inverse problem is to calculate the formation damage function from measurements of flow velocity U and pressure drop on the core (Fig. 6). The problem and the solution method are analogous to these developed for precipitation of paraffins, asphaltenes and mineral salts [8].

Let us calculate pressure drop on the core from equation (2.3) after the scaling (2.4):

$$\int_0^1 \frac{dX}{k(\sigma(X, T))} = \Delta p(T). \quad (6.1)$$

Once the first inverse problem is solved, $\lambda(\sigma)$ has been determined, and therefore $\sigma(X, T)$ can be found by solving (2.5), (2.6). If we measure $\Delta p(T)$, then (6.1) is an integral equation that determines the positive function $k(\sigma)$. If a good guess of $k(\sigma)$ is available, Tichonov's regularization can be used as iterative procedure to solve (6.1). Optimization methods can be used to find the initial guess. The details of this procedure are the subject of a future paper.

7. Example of recovery of filtration coefficient function from effluent concentration data

Coreflood tests measuring two particle concentrations at the core inlet and outlet have been performed in work [6]. In Figs. 7a, 8a, the effluent concentration history for two corefloods are shown. In Figs. 7b, 8b, the corresponding filtration coefficient obtained by using formulas (5.10)-(5.11) is displayed. Using this filtration coefficient, the direct problem was solved numerically for the dispersed concentration; the resulting effluent concentration history at the core outlet is displayed by dots and it is very close to the original history in the plots shown in Figs. 7a, 8a.

The outlet concentration in Fig. 7.a increases with time, so the filtration coefficient decreases during retention (Fig. 7b). This fact can be explained by gradual reduction of “clean” pore surface available for retention of particles.

On the other hand, the outlet concentration decreases in Fig. 8a. The filtration coefficient increases with retention up to some limit and decreases afterwards (Fig. 8b). This can be explained by assuming that particles are attracted to the pore surface and to each other. Thus, retained particles stack up to a point when increased interstitial water velocity prevents further retention.

Fig. 7a. Outlet concentration history.

Fig. 7b. Recovered $\lambda(\sigma)$.

Fig. 8a. Outlet concentration history.

Fig. 8b. Recovered $\lambda(\sigma)$.

Figs. 7, 8. Determining the filtration coefficient vs. deposited concentration $\lambda(\sigma)$ from the effluent concentration history at the outlet.

8. Conclusions

1. An analytical model for flow of water with suspended particles is developed for any shape of the filtration coefficient function
2. The inverse problem of determination of the filtration function from data on concentration of particles on the core outlet is well-posed
3. The formation damage function can be determined from the pressure drop data. The corresponding integral equation is derived and can be solved numerically
4. The analytical model developed suggests the following procedure for prediction of well injectivity decline:
 - plan and design coreflood test (choice of flow velocity, frequency of sampling, number of samples, etc.) using the analytical model for 1-D linear flow;

- determine filtration coefficient from coreflood pressure data by solving the first inverse problem;
- determination of formation damage function from coreflood test data by solving the second inverse problem;
- once all this information is available, is it possible to calculate the evolution of the retained concentration distribution and of injectivity using the analytical model for radial flow.

Acknowledgement

D. M., G. H. and A. A. were partially supported by Grants FINEP/CTPETRO/SINJE 65.00.0424.00, PETROBRAS/CTPETRO/SINJE 650.4.021.01, FINEP/CTPETRO/SAINJA 21.01.0276.00, PETROBRAS/CTPETRO/SAINJA 650.4.040.01-1 and CNPq 300204/83-3. P. B. was partially supported by PETROBRAS/CTPETRO/PTPI 65990468.00 and 65000327.00 and FINEP/CTPETRO/SAINJA 21.01.0386.00. They also thank B. Gundelach for assistance in preparing the manuscript, and Petrobras for allowing its publication.

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