E25 A BLACK-OIL RESERVOIR SIMULATOR FOR HEXAHEDRAL MULTIBLOCK GRIDS

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Abstract

A black oil reservoir simulator was developed for hexahedral multi-block grids. An object-oriented approach for the design and implementation of the simulator was adopted. We discuss the general features and object-oriented (O/O) design of the simulator.

Complexity in geological or well features in a reservoir model (e.g., faults, inclined and multi-lateral wells) entails non-orthogonal and/or unstructured grids in place of conventional Cartesian grids. To obtain flexibility in gridding as well as numerical efficiency, hexahedral multi-block grids are employed. These multiblock are locally structured and globally unstructured.

In general, one can always generate multiblock grids with matched grids at block boundaries, and that leads to simple data structures and communication patterns between blocks. Nevertheless, when geometrical boundaries (e.g. fault surfaces) become complex, multiblock grid with matched grids tends to create many blocks with small cells. We generalize the multiblock grid to the case where the gridlines do not match at block boundaries. The general quality and characteristics of multiblock grids, with matched or unmatched grids at the block boundaries, are discussed.

Multi-point discretization schemes are necessary to accommodate non-orthogonal grid and tensor permeability. A flux-continuous finite difference scheme, which was previously developed for hexahedral grids with tensor permeability, is extended to multiblock systems with unmatched grids. We also present efficient iterative solution schemes that take advantage of the locally structured character of the computational grid.

We demonstrate the capabilities of this generalized multiblock approach using examples of practical interest: (1) An SPE comparative study and (2) a real reservoir model with multiple complex faults and internal surfaces.

1. Introduction

Real reservoir models often involve highly detailed description of reservoir properties, complex fault surfaces, non-conventional and inclined or multi-lateral wells. It has become increasingly more difficult to accommodate complex internal and external boundary surfaces with conventional Cartesian grids, which are used in traditional reservoir simulators. The most general, flexible gridding method involves fully unstructured triangular grids in two dimensions and tetrahedron grids in three dimensions. Their application in reservoir simulation is still limited because a number of important prerequisites are necessary in order to benefit from the flexibility of general unstructured grids. These include (1) rigorous discretization schemes, (2) improved solution algorithms, (3) improved numerical accuracy and convergence for large aspect-ratio grids, and (4) more powerful computational and visualization resources.

Multiblock grids, as an alternative to fully unstructured grids, can be general and flexible in describing complex geometry by allowing different gridding (structured or unstructured) in each

block with non-matching gridlines at block boundaries [15]. Hexahedral multiblock methods have been applied to reservoir flow problems. In our previous paper [9] we considered hexahedral multiblock grids where the gridlines at block interfaces are matching. They are somewhat less flexible in describing complex geometry than fully unstructured grids or more general multiblock grids with mismatched grids at block boundaries; however, they possess many desirable properties from the perspective of reservoir flow modeling. We found, however, that this matching gridline constraint entails many blocks with small cells in gridding a reservoir model with fault surfaces showing bedding juxtaposition. We thus relax the matching gridline constraint and develop simple discretization scheme for unmatched hexahedral grids. We also implemented a back-oil formulation in our multiblock simulator to identify any weak links of this new discretization and gridding technology in handling complex reservoir flow simulation problems.

This paper discusses major technical issues in developing a black-oil multiblock simulator. We first present the formulation and the solution algorithm of a multi-block black oil simulator. Then the applicability of hexahedral multiblock grids for general reservoir problems is investigated by generating grids of geometrically complex models such as pinchouts and a real reservoir model with complex fault surfaces. We generated hexahedral multiblock grids, using the GridPro commercial program [13], for various reservoir model problems. These examples demonstrate the wide flexibility of the gridding technology.

We developed a black-oil multiblock simulator that was written in C++ with Object/Oriented design. We present a brief description of the design architecture of the multiblock black-oil simulator. Multi-point discretization scheme is used for the flux formulation in order to treat grid non-orthogonalities and anisotropic permeability fields correctly. A general finite-volume discretization scheme for nonorthogonal hexahedron grids with tensor permeability leads to wide stencils (9-pt in 2D [8,10] and 27-pt in 3D [1,11]). We employ the wide stencil derived by the finite-volume method of Lee et al. [11] in the numerical calculations of this paper. We note that since the transmissibilities are computed only once in the initialization step of reservoir simulation, the relatively complex transmissibility computation of the flux-continuous method should have a minimal impact on overall numerical performance.

2. Governing Equations and Formulation

The governing equations for black-oil without capillary pressure are given by

$$\frac{\partial}{\partial t}(\phi b_o S_o) = \nabla \cdot (\lambda_o \cdot (\nabla p - g \rho_o \nabla z)) - q_o$$

$$\frac{\partial}{\partial t}(\phi b_w S_w) = \nabla \cdot (\lambda_w \cdot (\nabla p - g \rho_w \nabla z)) - q_w$$

$$\frac{\partial}{\partial t}(\phi (b_g S_g + R_s b_o S_o)) = \nabla \cdot (\lambda_g \cdot (\nabla p - g \rho_g \nabla z) + R_s \lambda_o \cdot (\nabla p - g \rho_o \nabla z)) - q_g - R_s q_o$$

Here, ϕ is the porosity, b_i is the inverse of volume formation factor ($=1/B_i$), $\lambda_i = k_{r,i} k b_i / \mu_i$ with the relative permeability, $k_{r,i}$, the permeability tensor k and the viscosity μ_i . In addition, g is the gravitational acceleration, ρ_i the density, z the depth of the reservoir, p the reservoir pressure, S_i the saturation and q_i the source term. The subscripts o, w and g stand for oil, water or gas, respectively. The pressure equation is obtained by taking the sum of the equations multiplied with the factors α_i :

$$\alpha_o = 1 - \frac{b_o}{b_g} R_s, \quad \alpha_w = \frac{b_o}{b_g} \quad \text{and} \quad \alpha_g = \frac{b_o}{b_w}$$

The resulting pressure equation contains no saturation at the new time level $n+1$ and therefore can be used as a pressure equation for fully implicit and IMPES as well. The pressure equation and saturation equation are discretized in conservative form, and the source terms are treated implicitly to yield a stable scheme. The discretized equations are solved numerically.

Figure 1. Multiblock grids: (a) a pinchout (b) a complex reservoir model with three fault planes.

3. Hexahedral Multiblock Grid

Complex geometrical features in reservoir models include boundaries of wide variation in length scale, complexity, and level of uncertainty. As a result, deciding on an optimal grid system for a given reservoir model is a challenging task. Ultimate flexibility in describing every detail is possible with fully unstructured grids (triangular and tetrahedron grids in two and three dimensions). However, accurate description of the geometry should not be the only measure of grid quality. One must also understand the relationship among the grid, discretization operators, and quality of the numerical solutions obtained for the reservoir model and flow process of interest. Issues of computational efficiency are also important. Here, an alternative to fully unstructured gridding is considered. We investigate whether hexahedral multiblock grids, which are less flexible than tetrahedral grids yet more computationally efficient, are adequate for most flow simulation needs that involve complex geometrical features.

The focus is on a particular multiblock gridding approach where the reservoir model is divided into blocks; the blocks form a globally unstructured grid. Each block is discretized using a structured hexahedron grid. In general, the resulting grid is nearly orthogonal and the gridlines at block interfaces are generally matching. However, this constraint of matching grid lines at block boundaries tends to generate many small blocks for a reservoir model with complex fault planes. We thus relax this constraint in Section 7 and develop multiblock models with unmatched grid lines. The attraction of hexahedral multiblock grids is that one can easily adapt proven algorithms and software that have been developed for conventional (globally structured) reservoir simulation.

The multiblock grids in this paper were all obtained using the GridPro [13] commercial package. Two gridding examples, (1) a pinchout and (2) a highly faulted real reservoir model, are shown in figure 1. More examples of multiblock gridding can be found in our previous papers [9,12].

Figure 2. Corners and edges of two neighboring blocks (blue) glued together with Glue objects

4. Object-Oriented Design of the Simulator

The black-oil simulator was written in C++ and designed in a modular way using OO-patterns, while keeping efficiency in mind.

There is a Simulator class which has pointers to instances of Block, Surface and Well. The Block class represents the locally structured grids with all of its data; the base class contains all nonspecific implementation while the reservoir simulation specific methods are implemented in the concrete class Block_Res (an instance of BlockFactory tells Simulator how to build an instance of Block). The single blocks are linked by face pointers and figure 2 shows how the nodes and edges of the blocks are glued together by Glue objects (this is necessary for multi-point discretization schemes).

Since it often is desirable to change the implementation of a functionality or to switch between various versions the strategy pattern is extensively used in the design of Res++. Block has strategy pointers to interfaces to read the geometry, properties and initial conditions (InputGeom, InputProperty and InputFluid). There are further strategy pointers from Block to Solver (to allow for different implementations of the linear solver) and to BoundCond (to allow for various types of boundary conditions). Some of the strategies are reservoir simulation specific and are owned by Block_Res; these are the strategies for property calculation, i.e. density, viscosity, formation volume, solution gas, relative permeability and porosity. Simulator has an output strategy (Output), Well has a well condition strategy (WellCondition) and Surface has a surface condition strategy (SurfaceCondition). Surface condition objects represent a collection of adjacent surfaces and treat it as a single

5. Multi-Point Flux Continuous Finite-Difference Scheme

For an orthogonal grid with diagonal tensor permeability the discretization scheme for the pressure equation usually leads to a 5-point stencil in two dimensions and a 7-point stencil in three-dimensions. The flow between two neighboring cells can be approximated by the pressure difference between these two cells. The proportional coefficient (transmissibility) becomes a harmonic mean of two cell permeabilities. This formulation is not valid, however, if the grid is non-orthogonal or if permeability is characterized by a full tensor. In this case, other connections arise, leading to multi-point fluxes (e.g., a nine-point stencil in two dimensions and a 27-point stencil in three dimensions).

Flux-continuous methods, designed to handle full tensor permeability and/or non-orthogonal grids, have been presented previously for two dimensional [8,10] and three dimensional systems [1,11]. Though there are differences between these various schemes, they all provide continuity of flux across cell boundaries and reduce to the usual weighted harmonic mean transmissibilities in the appropriate limit (orthogonal grid, diagonal permeability tensors). The development of our extended stencil was presented previously [11].

It is important to note that determination of the transmissibilities is a pre-processing step that needs to be done only once. The computational cost of this calculation is minor; the real cost of the 9- and 27-point schemes occurs during the course of the actual simulation and is mainly due to the linear solver, as will be discussed below. The matrix resulting from the above flux-continuous finite volume scheme is denser than that from the normal 7-pt scheme, and is not an M-matrix [1]. However, we found that the matrix is well conditioned and can be easily inverted using many iterative methods. A discussion of linear equation solvers follows in the next section.

6. Linear Solvers for Multi-Point Stencil

Multiblock models are globally unstructured, locally structured with matching gridlines across the blocks. The characteristics of the pressure equation are quite different from the usual globally structured grids that are handled efficiently using existing general purpose reservoir simulators. As a result, simulation of multiphase flow in multiblock reservoir models requires special numerical tools and solution techniques.

Multiblock gridding yields a data structure that is naturally suited for domain decomposition solution methods. We assume that the number of blocks is small compared to the aggregate number of cells in the entire grid. That is, we rely on the premise that a block contains a large enough number of cells to warrant use of block-based solution methods. This assumption is easily satisfied in practice. Of course, different blocks may contain different numbers of cells. Each block is wrapped with an additional layer of cells. Thus, the wrapped block holds information one-cell deep into the adjacent blocks. If a side of a block is adjacent to a real boundary, or inactive cells, a ghost layer is introduced.

In our previous paper [12], we described our two-level data structures and preconditioning scheme, which were designed to make full use of features intrinsic to this class of multiblock grids. A Block-Jacobi was employed as the first level (local) preconditioner and a BEPS-like method [5] for the second level (global) preconditioner. We noted that the second-level correction improves convergence drastically.

In order to lower the computational effort, Aavatsmark et al. [2] suggested using the extended stencil only locally based on the importance of the local contribution from the non-7-pt gridblocks. Edwards [7] proposed operator splitting schemes for dealing with these wider stencils. He retains the two-point flux, which yields a 7-pt stencil, and a remainder. The remainder is updated from the previous time step, or iteration, so that he can employ the regular 7-pt stencil with a modified right hand side. Edwards describes a general framework for his splitting scheme, and he applies it to structured and unstructured grids. [7,8] also showed that his splitting operator possesses many desirable properties. Using a variety of two-dimensional multiphase flow problems, he showed that his scheme converged to the full matrix solution.

The pressure equation in matrix form can be written as

$$\mathbf{A}\mathbf{p} = \mathbf{f}$$

where $\mathbf{A}$ is the full 27-pt coefficient matrix and $\mathbf{f}$ is the right-hand side. We define an iteration scheme based on a regular splitting as follows.

$$\mathbf{M}\mathbf{p}^{n+1} = (\mathbf{M} - \mathbf{A})\mathbf{p}^n + \mathbf{f}$$

where $\mathbf{M}$ is an approximate matrix with a standard 7-pt structure and $(\mathbf{M} - \mathbf{A})$ defines the remainder. An equivalent, but more intuitive form is given by

$$\mathbf{p}^{n+1} = \mathbf{p}^n + \mathbf{M}^{-1}\mathbf{r}^n$$

where $\mathbf{r} = \mathbf{f} - \mathbf{AP}$ is the residual. Lee et al. [12] discussed various splitting schemes that aim to invert a 7-pt stencil in the process of converging to the full 27-pt stencil solution.

7. Multiblock Models with Non-Matched Gridlines at Block Boundaries

The original multiblock gridding method was designed to generate blocks with matched grid lines at block boundaries, which can be easily incorporated in iterative numerical scheme in block level (i.e., domain decomposition). However, this constraint tends to generate many small blocks along complex fault surfaces (e.g., fault surface showing bedding juxtaposition). If the matching grid line constraint is relaxed, we can generate much simpler block structure, although the mass transfer calculation between blocks may need interpolation which results in numerical inaccuracy. In the industry, this unmatched approach has been widely used as local grid refinements and along a fault surfaces. We thus generalize the multiblock gridding to include unmatched grid lines at the block boundaries which results in flexibility in gridding, but entails a more complex discretization scheme.

Wheeler et al. [15] applied mortar space techniques to enforce flux continuity between the blocks. Aavatsmark et al. [3] considered a general scheme for unstructured grid (triangular grid and shifted fault grid) using a triangular interpolation of pressure field. In this section we examine two simple unmatched block grids as shown in figure 3: (1) local grid refinements and (2) a shifted fault plane and derive a flux continuous discretization scheme. Here, we employ a simple approach [6,14] which is consistent with the original derivation of flux-continuous finite difference scheme. This can be more clearly illustrated with a two dimensional model in figure 4. The refined grid lines and unmatched fault lines are extended to create regular hexahedron or quadrilateral grids. Then the flux-continuous multi-point scheme in section 4 can be directly applied.

Figure 3. Simple multiblock grids with unmatched grid lines: (a) local grid refinements and (b) a fault plane

Figure 4. Extended gridlines at unmatched block boundaries: (a) local grid refinements and (b) a fault plane

8. Numerical Examples

The first example is the SPE Comparative Study [4] with a three-layered model. The model comprises $10 \times 10 \times 3$ grids, which is a very small model to today's regular simulation practice. We divided the model into four blocks and tested the Multiblock Black-Oil Simulator. The saturation distribution at 0.25 PVI is shown in figure 5(a). Lighter color indicates a mixed phase of oil and water (i.e., water saturation front).

The second example is a realistic reservoir model with three faults (one of which is sealing) and two wells (one producer and one injector). Two phase flow without capillary pressure as well as gravity was considered for simulation. The injector and the producer wells operate at constant pressure, 4062 psi and 3000 psi, respectively, and the reservoir is at an initial pressure of 3500 psi; the outer reservoir boundaries are sealed. The permeability field is heterogeneous, but isotropic. The mean of the permeability is 100 md, the coefficient of variation is 0.5 and the correlation structure is isotropic with $\ell_x = \ell_y = \ell_z = 0.3$, where ℓ is the dimensionless correlation length (normalized by the system length in the corresponding coordinate direction).

Figure 5(b) shows the saturation field obtained from the numerical calculation with the 27-pt stencil. The numerical results indicated the pressure jump across the sealing fault, whereas the pressure is continuous across the non-sealing fault planes. The saturation front is blocked by the sealing fault and moves through the communicating fault planes. The numerical results accurately represent the correct physics of fluid motion. This example also clearly demonstrates that the multiblock approach proposed in this paper can be effectively applied for complex, practical reservoir models.

Figure 5. Saturation Distribution: (a) a SPE Model (b) a Realistic Reservoir Model

9. Discussion and Conclusions

We have developed a black-oil multiblock simulator with O/O design and studied simulation of multi-phase flow in reservoir models with complex geometrical features related to the geology and wells. The focus was on developing a simulator for reservoir models gridded using a hexahedral multiblock approach. The gridding was accomplished using the commercial package GridPro [14]. The grids are unstructured globally, but retain a logical i,j,k structure locally. This allows for utilizing proven numerical methods developed for current reservoir simulation technology.

Issues related to discretization, solution quality, and computational efficiency of multiblock reservoir simulation are addressed. We employed a flux continuous 27-point discretization stencil to account for grid non-orthogonality and full tensor permeability.

The resulting matrix from the 27-point stencil is much denser than that from 7-point stencil. We examined numerical algorithms to solve the resulting matrix efficiently. Following Edwards' work [7], we also investigated operator splitting schemes for the local problems. We compared the convergence rate of these 7-point preconditioners to find that all the iterative schemes based on the 7-point conditioners converge well. Furthermore, a two-level BEPS-like [5] method was employed to accelerate the global convergence.

Two numerical simulation results were presented to validate the simulator and the numerical methods based on multi-block approach. Application of the MBG method to a geometrically complex case illustrates the considerable flexibility and robustness of the gridding and solution procedures.

References

- [1] Aavatsmark, I., T. Barkve, and T. Mannseth: 1998a, 'Control Volume Discretization Methods for 3D Quadrilateral Grids in Inhomogeneous, Anisotropic Reservoirs', *SPEJ* **3**, 146-154.
 - [2] Aavatsmark, I., O. Boe, and E. Reiso: 1998b, 'Hybrid MPFA-Methods with Application to Field Simulation'. In *Proc. of 6th European Conf. on the Mathematics of Oil Recovery*. Peebles, Scotland.
 - [3] Aavatsmark, O. Boe, E. Reiso and R. Teigland: 2000, 'MPFA for Faults and Local Refinements with Application to Field Simulations', In: *Proc. of 7th European Conf. on the Math. Of Oil Recovery*, Lago Maggiore, Italy.
 - [4] Aziz, O. D.:1981, 'Comparison of Solution to a Three-Dimensional Black-Oil Reservoir Simulation Problem', *J. Pet. Tech.*, 13-18.
 - [5] Bramble, J.H., R.E. Ewing, J.E. Pasciak, and A.H. Schatz: 1988, 'A Preconditioning Technique for the Efficient Solution of Problems with Local Grid Refinement'. *Comp. Meth. in App. Mech. Eng.* **67**, 149-159.
 - [6] Chien, M. C.H. and E.J. Northrup: 1993, 'Vectorization and Parallel Processing of Local Grid Refinement and Adaptive Implicit Schemes in a General Purpose Reservoir Simulator'. *Proceedings of 12th SPE Symposium on Reservoir Simulation*, New Orleans, LA, pp. 279-290.
 - [7] Edwards, M.G.: 2000, 'M-Matrix Flux Splitting for General Full Tensor Discretization Operators on Structured and Unstructured Grids'. *J. Comp. Phys* **160**, 1-28.
 - [8] Edwards, M.G. and C.F. Rogers: 1998, 'Finite Volume Discretization with Imposed Flux Continuity for the General Tensor Pressure Equation'. *Comp. Geosciences* **2**, 259-290.
 - [9] Jenny, P., C. Wolfsteiner, S.H. Lee, and L.J. Durlofsky: 2001, 'Modeling Flow in Geometrically Complex Reservoirs Using Hexahedral Multi-Block Grids', *Proceedings of 16th SPE Symposium on Reservoir Simulation*, SPE Paper 66357. Houston, TX.
 - [10] Lee, S.H., L.J. Durlofsky, M.F. Lough, and W.H. Chen: 1998, 'Finite Difference Simulation of Geologically Complex Reservoirs with Tensor Permeabilities'. *SPE Res. Eval. & Eng.* **1**, 567-574.
 - [11] Lee, S.H., H. Tchelepi, and L.J. DeChant: 1999, 'Implementation of a Flux-Continuous Finite Difference Method for Stratigraphic, Hexahedron Grids', *Proceedings of 15th SPE Symposium on Reservoir Simulation*, Houston, TX, 231-241.
 - [12] Lee, S.H., P. Jenny and H. Tchelepi: 2002, 'A Finite-Volume Method With Hexahedral Multiblock Grids For Modeling Flow in Porous Media', to appear in *Comp. GeoSciences*.
 - [13] PDC: 1999, *GridPro Interactive Multi-Block Gridding* (www.gridpro.com)
 - [14] Wheeler, J.A., M.F. Wheeler and I. Yotov: 2001 'Enhanced Velocity Mixed Finite Element Methods for Flow in Multiblock Domains', *TICAM Report 01-27*, UT Austin.
 - [15] Wheeler, M.F., T. Arbogast, S. Bryant, J. Eaton, Q. Lin, and M. Peszynska: 1999, 'A Parallel Multiblock/ Multidomain Approach for Reservoir Simulation'. In *Proc. of 15th SPE Symposium on Reservoir Simulation*, Houston, TX, 51-61.
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