E29**TWO-PHASE FLOW THROUGH FRACTURED POROUS MEDIA**P.M. ADLER ¹, I.I. BOGDANOV ^{1,2}, V.V. MOURZENKO ², J.-F. THOVERT ²⁽¹⁾IPGP, tour 24, 4 Place Jussieu, 75252 Paris Cedex 05,⁽²⁾LCD-PTM, SP2MI, BP 179, 86960 Futuroscope Cedex, France**Abstract**

Two-phase flow in fractured porous media is investigated by means of a direct and complete numerical solution of the flow equations in a three-dimensional porous medium straddled by a fracture network. The numerical model applies to arbitrary fracture network geometry, and to arbitrary distributions of permeabilities in the porous matrix and in the fractures. Calculations relative to two sets of networks are presented. An approximate model can be proposed to derive the macroscopic relative permeabilities. The macroscopic capillary pressure is not significantly different from the one of the porous medium only.

Introduction

Consider a set of fractures which are embodied in a porous solid matrix. Examples of such media are displayed in Figure 1; both the matrix and the fractures are permeable, with permeabilities which may vary with space. Hence, when a two-phase flow takes place through such a medium, the fractures and the porous matrix interact.

Historically, this complex situation was first addressed by [1,2]; though these papers motivated many further works [3-5], progress in this area has been slow and so far no complete solution of this problem for arbitrary fracture network and for arbitrary permeability distribution in the porous matrix has been described to the best of our knowledge in the literature. A more thorough coverage of the literature is made in [6-9].

The present work is based on a three-dimensional discrete description of the fracture network. Hence, arbitrary fracture network geometry, various types of boundary conditions and of distribution of the fracture and matrix properties can be addressed, without simplistic approximations. The main drawback of this direct approach used to be its computational requirements, but thanks to the progress of the computers it is not a real issue anymore. The numerical tools described here are able to handle several hundreds of fractures. They can be applied to case studies, in an arbitrary setting, or as done in the present case, to determine the macroscopic properties of a fractured medium to be used in an upscaled description.

The purpose of this paper is to briefly present the methodology and the first results obtained in the determination of the two-phase flow properties of fractured porous media.

Figure 1. The network of $N_{fr}=16$ fractures in the sample used for the simulations in Figures 3 and 4 (a). The saturation maps in Figure 3 correspond to the horizontal marked plane Π . The three-dimensional meshes of the same fractured medium (b) and of another sample with $N_{fr}=32$ (c). Distances are normalized by the fracture circumscribed radius R . Both samples are spatially periodic, with cell size $L=4R$. The tetrahedral volume elements in the cubic unit cell $-2 < x, y, z < +2$ are displayed. The protruding fractures in (b,c) sit astride the boundaries with the neighboring cells. Several periodic replicas are shown for some of them. For the sake of clarity, the edges and the intersection lines of the fractures have been thickened.

Equations for two-phase flow

Transport equations

Let the porous rock matrix have a porosity ϵ_m and a bulk permeability $K_m [L^2]$ that can vary with space. The flow in the matrix is described by a generalized Darcy law for each phase, with relative permeabilities $K_{r,i}$ ($i=w,n$). Subscripts w and n refer to the wetting and non-wetting fluids, respectively.

The local seepage velocities $\bar{v}_i$ are given by

$$(1) \quad \bar{v}_i = -\frac{K_m K_{r,i}}{\mu_i} \nabla (P_i - \rho_i g z) \quad (i=1,2)$$

where μ_i is the viscosity, ρ_i is the density, and P_i is the pressure for fluid i . For concision, denote Φ_i the potential $P_i - \rho_i g z$ and $\Lambda_i = K_{r,i}/\mu_i$ the phase mobilities. Two continuity equations and a global condition on the saturations S_i can be written

$$(2a) \quad S_1 + S_2 = 1$$

$$(2b) \quad \epsilon \frac{\partial \rho_i S_i}{\partial t} + \nabla \cdot (\rho_i \bar{v}_i) = 0 \quad (i=1,2)$$

Equations similar to (1,2) are applied for the flow in the fractures. We assume that the hydraulic properties of a fracture can be described by an effective conductivity $\sigma [L^3]$. The in-plane flow rates $j_{s,i}$ per unit width are related to the surface pressure gradients $\nabla_s P_i$ by two-dimensional generalized Darcy laws

$$(3) \quad j_{s,i} = -\frac{\sigma \sigma_{r,i}}{\mu_i} \nabla_s \Phi_i \quad (i=w,n)$$

where $\sigma_{r,i}$ are the relative permeabilities of the fractures. λ_i denotes the fluid mobility in the fracture $\sigma_{r,i}/\mu_i$. The conductivity σ can be position dependent, and it can differ for different fractures. For a fracture

which can be viewed locally as a plane channel of aperture b , filled with a porous material with permeability K_f , σ is given by

$$(4) \quad \sigma = b K_f$$

It is assumed that the fractures oppose no resistance to flow normal to their plane. Hence, the pressures P_i , P_c and the potentials Φ_i are continuous across the fractures.

Conservation equations similar to (2) must be written for the fractures, which include exchange terms with the surrounding matrix. This procedure is detailed in [9].

Constitutive equations

Constitutive equations are required for the closure of the set of transport equations in the previous subsection. Note that their choice is partly arbitrary, and different models may be suited for various types of rock matrix or fractures. The ones implemented here are among the most common, and they could easily be substituted with others to address specific situations.

First, the stress balance at the fluid interface at the microscopic scale has to be taken into account. Due to interfacial tension, a pressure jump P_c takes place across the interface, which is called the capillary pressure

$$(5) \quad P_c = P_n - P_w = \Phi_n - \Phi_w + \Delta \rho g z$$

The most widely used model is the one proposed by [10]

$$(6) \quad S_w = \left[1 + \left(\frac{P_c}{P_0} \right)^n \right]^{\frac{1}{n}-1}$$

where $P_0 = P_n - P_w$ is the capillary pressure. In general, the parameters P_0 and n are different in the rock matrix and in the fractures. On physical grounds, one expects that

$$(7) \quad \frac{P_{o,f}}{P_{o,m}} \approx \sqrt{\frac{K_m}{K_f}} = \kappa$$

On the other hand, the relative permeabilities appearing in the generalized Darcy's equations (1) and (3) also depend on the fluid saturations. Many models have been considered for porous media (see [6]). Again, the most widely used model for the wetting phase relative permeability was proposed by [10]

$$(8) \quad K_{r,i} = S_i^{1/2} \left[1 - \left(1 - S_i^{\frac{n}{n-1}} \right)^{\frac{n-1}{n}} \right]^2$$

Note that residual saturations were not considered in the present simulations, but could easily be included. The relative permeability for the non-wetting phase is also sometimes modeled according to (8), with $K_{r,w}$ and S_w replaced by $K_{r,n}$ and S_n , respectively. However, the relative permeability curves for the two phases are generally not mirror images of one another. Thus, a different model was used here, which is discussed below.

Two-phase flows in fractures have given rise to comparatively less experimental studies than three-dimensional porous media, but a few references can be found in the literature. They are reviewed, for instance, by [11]. In the present simulations, we used a simple model for $\sigma_{r,n}$

$$(9) \quad \sigma_{r,n} = S_n^q$$

with the exponent q equal to 2. This model was also applied for $K_{r,n}$ in the rock matrix. Moreover, $\sigma_{r,n}$ was described by an equation of the type of (8).

In summary, the capillary pressure P_c , the relative permeabilities for the wetting fluid $K_{r,w}$ and $\sigma_{r,w}$ and the relative permeabilities for the non-wetting fluid $K_{r,n}$ and $\sigma_{r,n}$ are described both in the matrix and in the fractures by (5), (8) and (9), respectively.

Dimensionless formulation

Dimensionless variables, denoted by primes, can be introduced by using characteristic quantities of the problem. A possible choice is based on the typical values of the matrix porosity and permeability, $\bar{\epsilon}_m$ and $\bar{K}_m$, on the pressure $P_{0,m}$, on the wetting fluid viscosity μ_m and on a characteristic length scale L

$$(10) \quad \begin{aligned} \mathbf{r}' &= \mathbf{r}/L, & \nabla' &= L \nabla, & \epsilon'_m &= \epsilon_m / \bar{\epsilon}_m \\ P'_i &= P_i / P_{0,m}, & \Phi'_i &= \Phi_i / P_{0,m}, & \rho'_i &= gL \rho_i / P_{0,m} \\ K' &= K / \bar{K}_m, & \sigma' &= \sigma / (L \bar{K}_m), & \Lambda'_i &= \mu_w \Lambda_i \\ t' &= \bar{K}_m P_{0,m} t / (\bar{\epsilon}_m \mu_w L^2), & \mathbf{v}'_i &= \mu_w L \mathbf{v}_i / (\bar{K}_m P_{0,m}), & \mathbf{j}'_i &= \mu_w \mathbf{j}_i / (\bar{K}_m P_{0,m}) \end{aligned}$$

Numerical model

Three-dimensional meshing of fractured porous media

A literature review was done by [9]. The meshing technique was also extensively described in this paper; therefore, it is briefly schematized here. The fracture network is triangulated first as described by [12]. Then the space between the fractures is paved by an unstructured boundary-constrained tetrahedral mesh, according to an advancing front technique. Typical performances and computational requirements are discussed by [9].

Three-dimensional views of two triangulated fractured media are shown in Figure 1. In both cases, the fractures are hexagonal, with circumscribed radius R , and the cell size is $L=4R$. Figure 1a,b is the sample which will be used. The cell contains $N_{fr}=16$ fractures, and the mesh contains about 1800 node points, 22000 triangles and 11000 tetrahedra. Figure 1c is one of the samples which will be used, with $N_{fr}=32$. The discretization is slightly finer. The mesh contains about 3700 node points, 46000 triangles and 23000 tetrahedra.

Spatial discretization of the equations

The non-wetting phase potential Φ_n and the capillary pressure P_c are evaluated at the mesh points located at the vertices of the tetrahedra and triangles. Since the fractures are viewed as vanishingly thin, empty or very permeable layers, there is no pressure jump between two points facing each other on the two opposite sides of a fracture. Thus, single values of Φ_n and P_c can be used in the numerical formulation per vertex of fracture element.

A finite volume formulation of the problem is obtained by applying the balance equations to control volumes Ω surrounding each of the mesh points, as shown in Figure 2.

Time discretization and solution algorithm

The strong nonlinearity of the coefficients in the equations requests an implicit time formulation. The one used here is an extension of the modified Picard scheme described by [13] for the solution of equation (2b). The main idea of the method is the linearization of the time derivative in a mass conservative form.

Figure 2. Control volume Ω for the mass balance equation at a node; in this illustration, the node is on a fracture.

The derivative $\partial S_w / \partial t'$ is approximated by the first order difference over a time step $\delta t'$

$$(11) \quad \frac{\partial S_w}{\partial t'} = \frac{1}{\delta t'} (S_w^p - S_w^{p-1})$$

where the superscript p refers to the successive times $t_0' + p \delta t'$. S_w^p is obtained as the asymptotic value of successive estimations $S_w^{p,k}$

$$(12) \quad S_w^p = \lim_{k \rightarrow \infty} S_w^{p,k}$$

$\delta t'$ is automatically recomputed at each time step. Its choice is based on a desired maximum saturation increment ΔS . The new step $\delta t'_{p+1}$ is deduced from the maximum of the instantaneous local time derivative $\partial S_w / \partial t'$. In view of (11), this derivative is exactly the rate of variation over the previous step.

Regular fracture networks

In order to thoroughly test the routine, two regular structures were studied which will be discussed during the presentation. An analytical solution is derived for the global relative permeabilities of a regular array made of infinite parallel plane fractures when the macroscopic pressure gradient is constant and parallel to the fractures. A perfect agreement was obtained between this model and the numerical results. This could be generalized to media containing several families of fractures. The case of a sugar-box reservoir was also studied.

Randomly fractured porous media

Illustrative example

A detailed set of results is presented in this Section, relative to an homogeneous matrix rock containing the fracture networks shown in Figure 1a,c. The fractures are plane regular hexagons, with a circumscribed radius R which is taken as the length unit L in Eq. (10), and they have a constant permeability $\sigma' = 1$ with $\kappa = 10^{-3/2}$. The cell size is $(4R)^3$. The network does not percolate. The constitutive parameters are set as described previously, with $n_m = n_f = q = 2$. The fluid densities are equal and $\mu_m / \mu_w = 10$.

All the frames in Figure 3 are wetting phase saturation maps S_w in the matrix in the horizontal section Π in Figure 1a. The three columns of Figure 2 show the evolution of the saturation field when a macroscopic flow is induced by a pressure gradient $P_{0,m}/R$, starting from the three different initial saturation fields in the top row, with identical global saturation $\bar{S}_w = 0.371$.

The leftmost initial state corresponds to rest equilibrium, with uniform capillary pressure. The two others are arbitrary initial phase distributions, with bands normal or parallel to the applied pressure gradient.

In all cases, an identical steady regime is reached, where saturation is not uniform. S_w ranges from about 0.32 to 0.43, and it is different on the inlet and outlet sides of the fractures. The disturbances in the saturation field introduced by the presence of the fractures during a steady flow with respect to the rest state are due to the different capillary properties of the fractures and rock matrix (see Eq. 7). They are

Figure 3. Wetting phase saturation at various times (top to bottom), in the plane Π marked in Figure 1a. The mean flow is oriented from the left to the right of the figure. The mean saturation is always $\bar{S}_w = 0.371$. Each

column corresponds to the evolution of the saturation which starts from the initial condition shown at the top of the column.

observed in all our simulations, and increase with the fracture permeability σ' and with the mean flow rate (or pressure gradient). This effect will be specifically investigated in a future work.

Steady state relative permeabilities as functions of mean saturation

The previous example has shown that identical saturation fields and phase flow rates are eventually reached when a fractured medium is submitted to a macroscopic pressure gradient, starting from very different initial phase distributions. It is therefore possible to define steady state macroscopic phase relative permeabilities for this medium at a given mean saturation $\bar{S}_w$. These relative permeabilities $K_{r,i}$ are intrinsic in the sense that they do not depend on the initial conditions; it was shown on a few examples that they do not depend either on the magnitude of the applied macroscopic pressure gradient, nor on the viscosity ratio, at least in a reasonable range.

The results are shown in Figure 4 for 16- and 32-fracture samples. The symbols correspond to the statistical averages over 27 calculations, and the error bars to the full range of variation of the individual data. The solid lines are the relative permeabilities for the fractures and for the rock matrix, which are identical functions of the saturation, in the present case.

In spite of the difference in percolation probability between the two cases, the general aspects of the results are similar. The presence of the fractures increases the relative permeability for the non-wetting phase and decreases the relative permeability for the wetting phase, with respect to the intact matrix material. However, the amplitude of these variations is larger for the denser fracture networks.

The strongest effects are observed for the largest saturations, and for the non-wetting fluid permeability $\overline{K}_{r,n}$. This is a consequence of the different capillary functions of the fractures and rock matrix (see Eq. 7). For the same value of P_c , the non-wetting phase saturation is much larger in the fractures than in the surrounding matrix, and the relative permeability $\sigma_{r,n}$ is larger than $K_{r,n}$. Thus, the fractures are preferential paths for the non-wetting phase.

Conversely, $\overline{K}_{r,w}$ is smaller than $K_{r,w}$ in the rock matrix, but this is mostly a consequence of the increase of the absolute permeability induced by the presence of the fractures, $\overline{K} > K_m$. The products $\overline{K} \overline{K}_{r,w}$ and $K_m K_{r,w}$ are identical, which means that the fractures do not significantly affect the wetting phase flow rate, with respect to the intact rock.

Figure 4. Macroscopic relative permeabilities $\overline{K}_{r,i}$ as functions of the mean saturation $\overline{S}_w$. Data are for samples containing 16 (a) or 32 (b) hexagonal fractures. The cell size is $L=4R$. The fractures have a permeability $\sigma'=1$, and $\kappa=10^{-3/2}$. The fluids have equal densities. The symbols are the averages of $\overline{K}_{r,n}$ () and $\overline{K}_{r,w}$ (o) over 27 calculations, conducted in $N_r=9$ random realizations, with a pressure gradient with magnitude $P_{0,m}/R$ set along the x -, y - and z -axis. The horizontal lines show the full variation range of the individual data. The solid lines are the relative permeabilities for the fractures and for the rock matrix, with $n_m=n_f=q=2$. The broken line in (b) is a prediction for infinite plane fractures with the same characteristics and the same global intrinsic permeability.

Influence of the other parameters

The influence of various parameters, namely the fracture permeability σ' , the magnitude G of the driving pressure gradient, the viscosity ratio, and the exponents n_m and n_f , was briefly tested by varying a single parameter at a time with respect to the base case considered previously. Calculations were run for a few values of the mean saturation $\overline{S}_w$, on fractured media containing $N_{fr}=16$ or 32 fractures.

Comparison with a capillarity dominated model

Underlying the macroscopic description of steady two-phase flows in terms of relative permeabilities independent of the driving pressure gradient magnitude is the assumption that the spatial phase distribution is not significantly influenced by the flow, with respect to the equilibrium rest state. This corresponds to a small value of the capillary number.

In this approximation, the local distribution of the phases in the pore volume is assumed to be determined by capillary forces only, and each phase flows through its own system of channels as if the other phase were immobile. Thus, the two flows are actually decoupled, and can be treated as two single phase flows in a medium where the local permeability is determined by saturation, via the local relative permeabilities.

These considerations, together with the additional feature that the fractures are nearly saturated with the non-wetting fluid, can be applied to devise a simple model for the prediction of the global relative permeabilities which is in good agreement with the full numerical results.

Concluding remarks

We presented in this paper a numerical tool for the simulation of two-phase flows in fractured porous media, together with a set of applications that demonstrated its ability to handle steady or transient flows in complex random media.

Owing to the large number of physical parameters, the scope of the simulations was necessarily restricted to a few representative situations. Still, a systematic study of the macroscopic flow properties was conducted as a function of the global mean saturation, for two types of media containing percolating or non percolating fracture networks, and the influence of many other parameters was briefly addressed.

Among the main results is the demonstration that the large scale steady state flow properties can be estimated from the rest state phase distribution for slow flows. However, this still requires the solution of the flow equations in the actual fractured medium complex geometry, since neither the absolute nor the relative permeabilities are accurately predicted by an infinite plane model for small or moderate fracture density. For faster flows, the phase distribution is influenced by the fluid motion, and a full solution of the coupled two-phase flow equations cannot be avoided.

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