E35 COUPLED MODELLING OF CROSSFLOW AND PRESSURE DROPS EFFECT FOR SIMULATION OF ADVANCED WELLS

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Abstract

Heterogeneities in reservoirs may induce wellbore crossflow with significant impacts on the production of the wells. The standard well models do not represent crossflow effects and may lead to serious errors on pressures and on production forecasts. This is especially true for extended horizontal wells where both friction pressure losses and reservoir-well exchanges need an accurate representation.

This article presents an improved well model which has been integrated in an all-purpose reservoir simulator. The method intends to accurately represent exchanges between the reservoir and the wells and to include friction pressure losses in the wellbore. It is available for black oil and compositional models in a fully implicit scheme and treats vertical, horizontal and multi-branch wells. Both physical and numerical aspects of the model are focused. The wells are meshed along their axis and pressures, saturations and compositions are computed in the wellbore. The model is based on the mass and energy conservative equations. An implicit model of friction pressure losses is integrated and supports a flow regime identification from stratified to turbulent with a liquid-gas phase representation. The global simulation has also been optimized using predictive computations for the wells prior to the Newton resolution. These calculations are based on an optimisation algorithm to determine the well pseudo-steady state ensuring production constraints. This predictive step determines flow directions and compositions using iterative flash and pressure computations.

Comparisons are made with a conventional well model and show different flowing conditions in the wells. Results on a case study with both long horizontal and multilateral wells in a highly heterogeneous reservoir are discussed and point out the accuracy of the new well model with a reasonable overhead simulation time.

Introduction

Wellbore crossflow occurs when fluid flows from the reservoir to the well at some levels and from the well to the reservoir at other ones¹. Reservoir heterogeneities mainly create this phenomenon and induce pressure distortions between the well and the reservoir. Crossflow may also be caused by gas-liquid fractions different between the well and the reservoir. It is critical to accurately represent crossflow effects because re-injected fluids may be seen as losses for production¹.

Nowadays, horizontal wells are commonly used in the oil industry. Reservoir properties may greatly vary through the well and may induce crossflow effects. Horizontal wells have generally

a longer interface with the reservoir than vertical ones and pressure friction drops have greater effects.

Standard simulators generally neglect crossflow effects by closing the crossflowing zones which may induce significant errors on production forecast. Some models have been proposed to evaluate these crossflow effects and are based on a mixed well composition² or on a more detailed well composition^{3,4}.

This paper displays a coupled model of crossflow and friction pressure drop effects in the wells which has been integrated in a multi-purpose oil reservoir simulator^{5,6}. In order to accurately represent exchanges between the wells and the reservoir, pressures and compositions are computed at different levels based on a discretization of the well along its axis. A hydrodynamic model determines the flowing conditions and deduces the friction pressure drop⁷. A way to reduce the overhead due to crossflow simulation is displayed using both an exact well-reservoir decoupling method and well prior calculations.

Model assumptions and equations are first described. The global optimization process is also discussed. Then, the model is tested and validated on examples based on both synthetic and real case studies.

Model description

In standard models, wells are commonly strongly coupled to the reservoir using implicit formulations^{8,9}. The only unknown computed in the well is the bottom hole pressure at a given depth. The exchanges between the well and the reservoir are modelled by a productivity index.

In order to represent crossflow, more variables have to be determined in the wells. Thus, the wells are discretized along their axis and pressures and compositions are computed using a finite volume scheme at the center of each well cell (Figure 1). Fluid flow is also determined at the interface between two well cells. The PI formulation is still used for the well-reservoir exchanges. Due to the scale of a typical reservoir simulation (several years of production, time steps of several days), transient effects are neglected in the wells and steady-state conditions are supposed to be reached immediately. This last assumption is used to write both the mass and energy conservation equations, which have to be checked in the well.

The variables to be computed in the wells are the same as in the reservoir, i.e. in a Black Oil simulation the main unknowns are the pressure in a reference phase (generally the oil phase) (P), the water and gas saturations (S_w , S_g), and the molar fraction of the hydrocarbon volatile component in the oil phase (X_v). For compositional models, the molar fractions of each component in the oil and gas phase (X_i , Y_i) are also unknown. Fluid flow in the wellbore does not use the Darcy's law but a hydrodynamic model where both the liquid and gas velocities have to be determined.

System of equations

Mass conservation equations in the wells

Mass conservation equations are written for each component (water and hydrocarbons).

$$\frac{\partial}{\partial t}(\rho_w S_w) + \text{div}(\rho_w v_w) = f_w \dots\dots\dots(1)$$

$$\frac{\partial}{\partial t}(\rho_o S_o X_i + \rho_g S_g Y_i) + \text{div}(\rho_o X_i v_o + \rho_g Y_i v_g) = f_i \dots\dots\dots(2)$$

Energy conservation equations in the well

The energy equations are written for each phase (liquid (water + oil) and gas). In these equations, the transient terms are neglected.

$$\text{div}(\rho_L v_L \otimes v_L - p_L I - \rho_L g z) = f_{A,L} + f_{F,L} \dots \dots \dots (3)$$

$$\text{div}(\rho_g v_g \otimes v_g - p_g I - \rho_g g z) = f_{A,g} + f_{F,g} \dots \dots \dots (4)$$

Capillary pressures are neglected in the well i.e. $p_L = p_g$. $f_{A,L} = \frac{q_L v_L}{A}$ expresses the acceleration pressure drop. Fluids are supposed entering into the well with a transversal zero velocity and are accelerated to the velocity of the flow at the end of the considered well cell. $f_{F,L}$ is the friction pressure drop and depends on flow conditions.

This system of equations is closed by initial and boundary conditions. Generally, the reservoir is supposed closed and is initially at a vertical equilibrium. Well conditions are pressure or rate constraints at surface or bottom conditions.

This set of equations is discretized using a finite volume scheme. A fully implicit formulation is chosen and all the transport terms are expressed at the time step $n+1$. The exchange between the reservoir cell j and the well cell n is given by a PI formulation,

$$q_{p,j,n} = I p_n \frac{k r_p \rho_p}{\mu_p} (p_j - p_n + h_{j,n}) \dots \dots \dots (7)$$

In this expression, the fluid mobility $\frac{k r_p}{\mu_p}$ has to be defined. If the fluid flows from the reservoir to the well, it is given by Kr curves used in the reservoir cells. If the fluid flows from the well to the reservoir, a mixture is re-injected in the reservoir layers and the fluid mobility is defined similarly to an injector well and is the total fluid mobility weighted by the saturation of the considered phase p in the well².

$$\frac{k r_p}{\mu_p} = \left(\frac{k r_w^*}{\mu_w} + \frac{k r_o^*}{\mu_o} + \frac{k r_g^*}{\mu_g} \right) S_p \dots \dots \dots (8)$$

Hydrodynamic model for the computation of the friction pressure drop

The discretization of the energy conservation equation on the well cell i gives the following equation for the liquid phase,

$$\frac{\rho_{L,i} v_{L,i}^2}{\Delta x} - \frac{\rho_{L,i-1} v_{L,i-1}^2}{\Delta x} + \frac{(P_{i+1} - P_i)}{\Delta x} + \rho_{L,i} g = \frac{q_{L,i} v_{L,i}}{A_i \Delta x} + A_i \left(\frac{\Delta P}{\Delta X} \right)_{L,i} \dots \dots \dots (9)$$

The friction pressure drop $\left(\frac{\Delta P}{\Delta X} \right)_{L,i}$ depends on phase velocities and gas-liquid proportion in the wells which determine flow conditions. Three different flow conditions may appear along the well.

Monophasic flow

Only gas or liquid is flowing in the well. Pressure drop expresses the friction between the fluid and the casing.

$$\left(\frac{\Delta P}{\Delta X}\right)_L = \tau_L \frac{\Omega}{A} \dots\dots\dots(10)$$

Stratified flow

Gas and liquid are flowing in the well. Fluids are supposed to be flowing in two separated phases (gas-liquid). Pressure drop takes into account the friction of each phase against the casing and the friction at the phase interface.

$$\left(\frac{\Delta P}{\Delta X}\right)_L = \tau_L \frac{\Omega_L}{(1-S_g)A} - \tau_I \frac{\Omega_I}{(1-S_g)A} \dots\dots\dots(11)$$

$$\left(\frac{\Delta P}{\Delta X}\right)_g = \tau_g \frac{\Omega_g}{S_g A} + \tau_I \frac{\Omega_I}{S_g A} \dots\dots\dots(12)$$

For the liquid and gas phases, friction constraints are written below,

$$\tau_L = \frac{1}{2} f_L \rho_L \left(\frac{v_L}{1-S_g} \right)^2 \dots\dots\dots(13) \quad \tau_g = \frac{1}{2} f_g \rho_g \left(\frac{v_g}{S_g} \right)^2 \dots\dots\dots(14)$$

The expression of the friction coefficients f depends on the Reynolds' number of the phase flow. The friction at the gas-liquid interface also depends on the relative velocity $v_g - v_L$.

Intermittent or turbulent flow

If the interface between the gas-liquid phases is unstable, flow conditions evolve quickly from stratified to intermittent flow. This flow is supposed to be under permanent conditions and is composed of successive stratified flow zones and mixed liquid with gas bubbles areas⁷. Pressure drop is calculated on an averaged configuration and α_p gives the proportion of the stratified configuration in the total flow. The total pressure drop is the sum of the pressure drop in each zone. For the liquid phase, we write,

$$\left(\frac{\Delta P}{\Delta X}\right)_L = \alpha_p \left(\frac{\Delta P}{\Delta X}\right)_{L,P} + (1-\alpha_p) \left(\frac{\Delta P}{\Delta X}\right)_B \dots\dots\dots(15)$$

$$\left(\frac{\Delta P}{\Delta X}\right)_{L,P} = \tau_L \frac{\Omega_L}{(1-S_g)A} - \tau_I \frac{\Omega_I}{(1-S_g)A} \dots\dots\dots(16)$$

$$\left(\frac{\Delta P}{\Delta X}\right)_B = \frac{2 f_{MB} \rho_{MB} v_{MB}^2}{D} \dots\dots\dots(17)$$

$$\alpha_p = \frac{S_g - R_{gB}}{R_{gP} - R_{gB}} \dots\dots\dots(18)$$

The gas proportion in each configuration R_{gP} , R_{gB} is computed using an iterative process and correlations on the pressure drop in the mixed zone.

System resolution

Non linear equations are solved using a Newton's method. The Jacobian matrix may be divided between the reservoir and the well blocks (Figure 2). The linear system is then written as,

$$A_{RR}\delta X_R + \sum_{i=1}^{n_{well}} C_{wi}\delta X_{wi} = R \dots\dots\dots(19)$$

$$B_{wi}\delta X + D_{wi}\delta X_{wi} = R_{wi} \dots\dots\dots(20)$$

which can take the following forms,

$$\delta X_{wi} = D_{wi}^{-1}(R_{wi} - B_{wi}\delta X) \dots\dots\dots(21)$$

$$\left(A_{RR} - \sum_{i=1}^{n_{well}} C_{wi}D_{wi}^{-1}B_{wi} \right) \delta X = R - \sum_{i=1}^{n_{well}} C_{wi}D_{wi}^{-1}R_{wi} \dots\dots\dots(21)$$

Reservoir unknowns are first solved then well unknowns are computed. It can be noticed that this decoupling method remains exact and reduces the size of the system.

Prior well calculations

The decoupling reservoir-well method is insufficient to ensure time steps of several days. The well cells are very small compared to the reservoir ones and variations in these cells are important. Permanent flow is supposed to be reached immediately in the wells. Global simulation may be optimized if the Newton resolution is started from this state, because the unknown variations in the wells are already taken into account and convergence is accelerated. The computation of this state may be seen as an optimization process⁶. Given production constraint, well variables have to be calculated and the reservoir ones are fixed at this step. Starting from a pressure at the bottom of the well, entering flow in the bottom well cell is computed. From flash computations, saturations in the cell are defined. Then, the hydrodynamic model is used to calculate the friction drop pressure and to deduce the pressure in the upper well cell. This method is iterated until the top well cell. The computed rate is compared to the constraint and with a gradient method, the pressure in the bottom of the well is corrected. Iterations are made until convergence.

Tests and discussion

Validation case study

The purpose of this test is to validate the friction pressure computation on a monophasic flow in a horizontal well. The results are compared with those given by a hydrodynamic model⁷. Reservoir initial pressure is 2290 Psi at a depth of 3600 feet (reservoir top). The well has an oil-rate constraint of 100000 bbl/j with a bottom hole pressure limit of 2000 Psi. The casing has a 4.5-inch diameter and a relative rugosity of 0.001. The pressure drop along the well is represented on the Figure 3 using different models. The explicit formulation shows important oscillations on the pressure results. This method has a CFL condition, which is not suited for reservoir simulation where time steps are important. The crossflow model and the semi-implicit formulation option give similar results. The model ensures good numerical results and this test shows the importance to implicitly represent the pressure drop.

Application to a real case study

The second example consisted of a real field case study of a Black-Oil fractured reservoir with 35000 active cells (Figure 4). A double permeability / double porosity model is used. The reservoir is highly heterogeneous with horizontal permeabilities varying from 10 to 20 mD in the matrix medium and from 100 to 500 mD in the fractures. The reservoir is produced by about 30 producers (including 15 horizontal and 3 multi-branch) and water injection is made in the lower side. Producers are surface oil-rate constraint by a global field oil rate. We study a 2500 days production and the crossflow model is compared with a standard one. Both crossflow and friction pressure drop are modeled in all the producers and pressure drop tables are used between the last perforated segment and the surface. During the simulation, crossflow occurs in many wells and we focus on the well P407 which has two branches.

Both models give the same oil rate forecast. Perforation oil rate gives great differences between the two models (Figure 5). During the simulation, the perforations of the main branch are closed using a standard model. Pressure computation shows that the pressure in the main branch is greater than in the reservoir. Neglecting crossflow effects induce the closure of the branch. The new model does not close the branch and computes an oil re-injection which amount up to 20-25 % of the oil rate of the well. Bottom-hole pressure also shows differences once crossflow occurs (Figure 6). Pressure in the standard model is more important because the branch is closed and we have a pressure build up toward the pressure reservoir. Using the crossflow model, pressure is weaker because the total production required from the well is more important (in-situ production thus reservoir-well pressure gradient has to be bigger to take into account both the oil production constraint and the re-injections).

The global CPU overhead (16 %) is reasonable given the better physical results obtained by the new model. The additional cost mainly comes from the resolution of the linear system due to a fuller Jacobian matrix.

Conclusions

The new well model accurately compute the exchanges between the reservoir and the wells. Standard simulation gives significant errors when crossflow appears. The model adequately simulates this phenomenon and may be used for both Black Oil and compositional oil representation. It may be applied to real case studies with reasonable CPU overhead and gives better quality of production forecasts than those obtained with standard models. Perspectives to this job lay in parallelization algorithm for a further optimization and in the representation of others physics such as thermal effects in the wells.

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Nomenclature

ρ , molar density (mol.m^{-3}).

S , saturation.

v , velocity (m.s^{-1}).

ϕ , porosity.

X_i , molar fraction of the i component in the oil phase.

Y_i , molar fraction of the i component in the gas phase.

P , pressure (Pa).

kr , relative permeability.

h , hydrostatic gradient between the well and the reservoir (Pa).

$q_{p,j,n}$, molar flow rate of the phase p between the reservoir cell j and the well cell n (mol.s^{-1}).

A , well segment area (m^2).

g , gravity acceleration (m.s^{-2}).

m , molar mass (kg.mol^{-1}).

μ , viscosity (Pa.s).

PI , productivity index of the well (given by the Peaceman formulation for example).

n , normal vector to the surface.

α , proportion of the stratified zone.

τ , friction constraint.

f , friction coefficient.

Ω , perimeter wet on the casing (m).

Subscripts

p , phase.

w , water phase.

o , oil phase.

g , gas phase.

S , surface conditions.

L , liquid phase.

B , mixed zone (turbulent flow).

P , stratified zone.

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Figures

Figure 1 : Well discretization

Figure 2 : Structure of the Jacobian matrix

Figure 3 : Pressure drop in the well

Figure 4 : Real case study

Figure 5 : Comparison of the oil rate of the main branch for the well P407

Figure 6 : Comparison of the bottom hole pressure for the well P407