E36

REPRESENTATION OF FRACTURED WELLS TO NUMERICAL MODELING POST-FRACTURING PRODUCTION FROM TIGHT RESERVOIRS

TORSTEN FRIEDEL, ALEXANDER NEKRASSOV¹, ARON BEHR,
GEORGE MTCHEDLISHVILI and FRIEDER HAEFNER
*Institute of Drilling Engineering and Fluid Mining,
Freiberg University of Mining and Technology,
09596 Freiberg, Germany
¹ now with Scandpower*

Abstract

The work presents a comparison study of different approaches to numerical modeling fractured wells in tight reservoirs. The simulation results were compared against the analytical solutions for an infinite and finite conductivity vertical fracture. The own method which combines elements of known approaches has been verified as advantageous in terms of compromise between the simulation accuracy and number of needed gridblocks.

Introduction

The current trend in the world gas industry is to involve low-permeability reservoirs into the exploitation and development. This implies the application of advanced well stimulation techniques, such as hydraulic fracturing.

The benefits of a precise analysis of the transient flow in the fracture vicinity for improving well performance have been recognized. However, a very small width of the fracture combined with a high flow capacity hampers the handling a fractured well in conventional reservoir simulation (fracture is modeled by a row of gridblocks which reflect its geometry, porous volume and transmissibility). Within the framework of a rectangular (structured) grid, the refinement necessary for an accurate and stable solution results in an unacceptable CPU time and computer memory requirement. A further problem is how to represent the actual relation between the fracture width and the well radius in the model, to correctly reproduce the flow in the immediate vicinity of the well. In this regard, the use of unstructured grids, such as Voronoi grids, is of obvious advantage, [1,2].

Another important point is that the specific processes relating to the hydraulic fracture (e.g., water block in damaged zone; deposition and erosion of the filter cake; stress-dependent behaviour of proppant) can be represented without essential simplifications. With this aim in view, in special approaches, the fracture is considered separately from the reservoir model. Coupling the fracture flow with reservoir flow can be realized in two fundamentally different ways which predetermine basic features of the approaches.

The first of them is based on the source/sink concept. Flow equations in the fracture and the reservoir are discretized by separate grids and coupled through a mass exchange term that is determined by the corresponding pressure in the fracture model and the pressures in the adjacent reservoir gridblocks. The flow into the fracture can be assumed to be linear, as in a typical finite-

difference scheme, [3] or, elliptical, as in Muskat's solution for the steady-state single phase flow into the infinite-conductivity fracture, [4,5]. It is pertinent to note the analytical solution for the case of finite-conductivity elliptical fracture which is also proposed to use in the similar numerical algorithms, [6].

According to the second approach, the presence of the fracture is considered by modification of the reservoir transmissibilities of the blocks containing the fracture, [7]. A reservoir model can communicate with a fracture model via an interface module which performs the transformation of the time-dependent characteristics of the fracture and its vicinity into the values of modified transmissibilities in the reservoir grid. In the context of this approach, there are no differences between the fracture block pressure and fracture pressure. For the well block, the problem of well representation in numerical simulation by finite differences remains to be solved. One way of solving this problem is proposed in this paper.

To realize a comparison of different fracture modeling techniques, a variety of options were implemented in the in-house code (3D black oil and compositional simulator based on the Finite Volume Method). Also, the results obtained with a commercial reservoir simulator [8] using structured and unstructured grids were involved into the study.

Formulation of numerical models to be compared

Rectangular grid

We consider the 2D reservoir single-phase flow around the 1D fracture in an isotropic homogeneous medium. Fig. 1 illustrates the gridblock system with a producing fractured well at the center of block 0, which is also the center of the fracture.

- **Conventional reservoir model.** The middle row of width Δy_0 should be very narrow to be representative of the fracture. At unchanged well radius, it can not take the actual size because of Peaceman's condition. Taking into account the fictive width of the fracture, the permeability and porosity of the fracture blocks are reduced in order to maintain the transmissibility and porous volume of the fracture:

$$k_f^* = k_f \frac{w_f}{w_f^*}; \quad \phi_f^* = \phi_f \frac{w_f}{w_f^*} \quad (1)$$

- **Source/sink fracture representation.** The model consists of flow equations for the reservoir and the fracture which are coupled through the source/sink term. The fracture model considers actual geometric and hydraulic characteristics and can be gridded independently from the reservoir grid. On the reservoir grid, the finite-difference approximation of the source/sink term (flow into the fracture) can be calculated, using block 5 in Fig.1 as an example, either as a linear flow

$$q_5 = \frac{k_R \Delta x_5 h}{\eta} \left[\frac{p_4 - p_{f5}}{\Delta y_{1/2}} + \frac{p_6 - p_{f5}}{\Delta y_{-1/2}} \right] \quad (2)$$

or as an elliptical flow

$$q_5 = \frac{k_R h}{\eta} (\psi_{5-1/2} - \psi_{5+1/2}) \left[\frac{p_4 - p_{f5}}{\ln[(a_4 + b_4)/x_f]} + \frac{p_6 - p_{f5}}{\ln[(a_6 + b_6)/x_f]} \right] \quad (3)$$

where

$$\psi_{5\pm 1/2} = \cos^{-1} \left(\frac{x_{5\pm 1/2}}{x_f} \right);$$

a_i , b_i are the major and minor semi-axes of an ellipse with half the focal length x_f passing through the center of gridblock i .

- **Model with increased transmissibilities.** Modified transmissibilities in the reservoir model present the combined effect of the conductivity of the reservoir and fracture. According to the grid system in Fig.1, the modified x-direction transmissibility between two adjacent reservoir blocks, whose centers are Δx distant from each other, is

$$T_x = T_x^R + T_x^f = \frac{h}{\eta \Delta x} [(\Delta y_f - w_f)k_R + w_f k_f] \quad (4)$$

Within the framework of this simplified fracture presentation, the y-direction transmissibility remains unchanged. In a more general case, its variation can be used to model, as an example, the damage zone around the fracture.

In this approach, the flow equation is not included in the fracture model. All the fracture parameters that affect the fracture transmissibility can be calculated in numerical procedure as a function of corresponding state variables of the reservoir grid such as block pressure. It is implicit that the fracture pressure is believed to be equal to corresponding reservoir block pressure. It also means that there is no way for such interpretation of well block pressure which could be adapted for the well representation in the finite-difference model.

- **Combined model.** From the above, the advisability of such models might be assumed, when the modified transmissibilities are applied together with a special handling the well block.

It is thought, the well block can be treated as if it represented the unfractured well in the anisotropic porous medium. In doing so, Peaceman's equation [9] can be used with x- and y-direction permeabilities in form of

$$k_x = \frac{1}{\Delta y_f} [(\Delta y_f - w_f)k_R + w_f k_f]; \quad k_y = k_R \quad (5)$$

For a more rigorous treatment, we describe the finite-difference approximation of the mass continuity equation for steady-state flow, regarding the well block, by the mass transfer in a control volume

$$\frac{h}{\eta} \sum_i a_i (p_i - p_0) = Q; \quad i = 1, 3, 5, 7; \quad a_{1,5} = \frac{k_x \Delta y_0}{\Delta x_{-1/2, 1/2}}; \quad a_{3,7} = \frac{k_y \Delta x_0}{\Delta y_{1/2, -1/2}} \quad (6)$$

Here, the symbol Q means not a flow rate to the fracture over the length of block, as the symbol q in the source/sink approach, but a rate of the well inflow which, for the most part, is accumulated over the whole fracture. Considering that the fracture spans a large number of gridblocks, it is, in general case, unacceptable to "localize" the known analytical solutions for flow around the fracture for deriving the relationship between the flow rate of a fractured well, the well pressure and well block pressure.

Therefore, we propose to describe the inflow to the well considering the pressures in the blocks adjacent to the well block. In doing so, the flow in the well vicinity is modeled as a combination of two linear flows: 1) the continuously distributed flux from the matrix to the fracture and 2) the flow in the fracture towards the well. The derivation of the equation for

the well flow rate for the simplified grid system, shown in Fig.1, together with the five-point discretization schema is given in Appendix A.

$$Q = \frac{h}{\eta} \sum_i b_i (p_i - p_w) ; \quad i = 1, 3, 5, 7 ; \quad b_{1,5} = \frac{k_f w_f}{\Delta x_{-1/2, 1/2}} ; \quad b_{3,7} = \frac{k_R (\Delta x_{1/2} + \Delta x_{-1/2})}{2 \Delta y_{1/2, -1/2}} \quad (7)$$

A more precise expression for Q can be produced when introducing the pressures in the diagonal adjacent blocks (nine-point schema) and using a piece-wise linear pressure distribution in x -direction.

It should be noted, that the well rate equation in form of Eq. (7) allows the direct calculation of the well pressure, without the intermediate computation of the well block pressure, just like in the theory of the equivalent interblock distance to simulate unfractured wells, [10]. In essence, the combined method embodies a switch-over from the macroscopic pattern of flow obtained by the modification of transmissibilities to the more detailed local picture produced in the similar way as a source/sink model. The use of derived approximation is of special interest for modeling low-conductivity fractures. If $k_f w_f \gg k_R \Delta y_0$, the modified transmissibilities are also very large, so that a typical period of transient flow in the well gridblock becomes very short. Beyond this time, there are no difference between Eqs (6) and (7) with $p_0 = p_w$ (if $\Delta x_{1/2} = \Delta x_{-1/2}$).

Unstructured grid

The fracture simulation on the unstructured grids is realized in the framework of the conventional reservoir model. Fig.2 shows a unstructured grid around the fractured well automatically produced by the well test analysis package [11]. Here, the radial grid around the well and fracture tips and rectangular grid along the fracture are gradually transformed into the bulk-grid. Such grids can be adapted to the structure of flow. They are flexible in the design of local grid refinement in the fracture vicinity with a smooth transition of block sizes. So, it becomes possible to represent explicit both the actual well and fracture geometry.

Modeling high-conductivity fractures

The first set of computations was done for the case of a high-permeability fracture, at a dimensionless conductivity $k_f w_f / k_R x_f = 1000$. The reservoir and production data are given in Table 1. Under the given conditions, Gringarten's analytical solution for the vertical infinite-conductivity fracture in the unbounded reservoir can be considered as an exact solution. The results, in the form of Cartesian and semi-log plots for the well pressure, are presented in Fig.3.

The idealized flow under study does not contain any features making difficult to simulate within the framework of conventional reservoir model. Therefore, it is not surprising that, by a proper grid refinement, a very high accuracy can be obtained with such a simulation approach. We described the fracture by a series of blocks of size 0.1m in y -direction (fictive width). As expected, using unstructured grids significantly reduces the number of required blocks. For structured grids an additional grid refinement was necessary to capture the flow around the fracture tips.

In the source/sink approach, the fracture was modeled with its true geometry and was coupled with the reservoir model consisting of square blocks 10 m on a side. The numerical results are in close agreement with an exact solution, except for the initial stage. The reason of deviation is the large truncation error in the linear approximation of the flow into the fracture at a period, when a strong non-linear pressure distribution exists in the fracture vicinity. As another conclusion, a

distinctly higher solution accuracy was obtained by employing the elliptical source/sink term, Eq. (3) than that by using the concept of linear flow, Eq. (2).

The approach with modified transmissibilities gives a satisfying accuracy in reproducing the pressure drawdown during the entire simulation time. It is thought that, at the early period, the undercalculated flow into the fracture (because of the truncation error pointed out), which results in an additional well pressure drop at a fixed well rate, is balanced out by increased effective fracture capacity on account of the unreduced porosity of the blocks containing the fracture. Notice also that, due to a very large fracture permeability, the contribution of Eq. (7) to correct the well pressures is negligible.

Modeling low-conductivity fractures

The results of a simulation series at the dimensionless conductivity close to one are shown in Fig. 4. The fracture permeability was three order of magnitude less than in the model with high-conductivity fracture (see Table 1). The basis for comparison was Cinco-Ley's analytical solution for a finite-conductivity vertical fracture in the infinite-acting reservoir.

As for a high-permeability fracture, a rather fine grid is necessary for a conventional reservoir model to achieve a good agreement with an exact solution. With decreasing fracture conductivity the flow characteristics is gradually changing to be radial dominated around the well. Therefore, an additional well refinement was required. Also, the use of a fictive well radius, to satisfy the Peaceman's condition for a given fracture block width, induces a non-ignored error. Vice versa, choosing a fracture block width to maintain the actual well-geometry under Peaceman's condition, e.g., 5 times the well radius for an equilateral well block, can cause an even larger error.

For the early-time flow, the deviation of the solutions obtained by the source/sink method remains noticeable. This is because the mentioned truncation error depends mainly on the conductivity of matrix blocks adjacent to the fracture rather than on the fracture conductivity. At the same time, reducing the fracture permeability results in the disappearance of the distinctions between the curves corresponding to the linear and elliptical source/sink term in the model. The reason is that the deflection of the rectilinear isobars takes place in the fracture tip vicinity, and the contribution of the flow across the fracture tips to the total inflow (over the whole fracture) decreases with reducing the fracture permeability.

In contrast to the source/sink method, the approach with modified transmissibilities overvalues the well pressure during the early period. In this case, a dominant error is caused by the slowed progress of the pressure front along the fracture as a consequence of averaging the permeability. It means delay in the pressure dropping in the fracture blocks neighboring to the well block, and with them in the well block in itself, with maintaining the well flow rate and total transmissibility. The combined method with more detailed consideration of the well inflow, significantly reduces this error. In a range of early times, the corresponding well pressure drawdown is in a closer agreement with the exact solution and manifests a transition from the source/sink method to the method of modified transmissibilities.

The adoption of Peaceman's formula for the anisotropic porous media demonstrates the worst results and is practically unsuited to model the fractured wells. The constant displacement from the solution by increased transmissibilities is too large. This points to the fact that the smearing a flow pattern around the fractured well by introducing the effective anisotropy is incompatible with the well-established interpretation of the well block pressure.

Summary

A rather simplified flow was used as a basis for the performed work, but its results presented for high- and low-permeability fractures are of a more general character, adaptable, e.g., to multiphase flow and a more complex fracture geometry. The conclusions of the analysis can be considered as guidelines for an appropriate choice of simulation model depending on the required accuracy and degree of process detailization.

Nomenclature

h	thickness	$\Delta x_{i+1/2}, \Delta y_{i+1/2}$	distance between centers of blocks i and $i+1$
k	permeability		
p	pressure	ϕ	porosity
q	source/sink term		
Q	well flow rate	Indices:	
T	transmissibility	i	gridblock number
u	specific flow rate	l	from left
w	fracture width	r	from right
x, y	coordinates	R	reservoir
x_f	fracture half-length	f	fracture
η	viscosity	w	well
$\Delta x_i, \Delta y_i$	sizes of gridblock i	*	fictive

References

1. Heinemann, Z.E., Brand, C.W., *Gridding Techniques in Reservoir Simulation*, Proc. First International Forum on Reservoir Simulation, Alpbach, 1988, p. 339-426.
2. Banerjee, R., Gunasekera, D., Fletcher, K.J.C. *Simulation of Hydraulically Fractured Horizontal and Vertical Wells to Well Testing Accuracy Using Unstructured Grids*, Proc. ECMOR VII, 2000, M-21.
3. Settary, A. *Simulation of Hydraulic Fracturing Processes*, SPEJ, Dec. 1980, p. 487-500.
4. Nghiem, L.X. *Modeling Infinite-Conductivity Vertical Fractures With Source and Sink Terms*, SPE, Aug. 1983, p. 633-644.
5. Nghiem, L.X., Forsyth Jr., P.A., Behie, A. *A Fully Implicit Hydraulic Fracture Model*, JPT, July, 1984, p. 1191-1198.
6. Settary, A., Puchyr, P.J., Bachman, R.C. *Partially Decoupled Modeling of Hydraulic Fracturing Processes*, SPE paper 16031, 1987.
7. Kanevskaya, R.D. *Simulation of Oil and Gas Reservoir Engineering Using Hydraulic Fracturing*, Moscow, Nedra, 1999.
8. ECLIPSE 100 User Manual, Schlumberger_GeoQuest, Abingdon, 2000.
9. Peaceman D.W. *Interpretation of Well-Block Pressures in Numerical Reservoir Simulation With Nonsquare Gridblocks and Anisotropic Permeability*, SPEJ, June. 1983, p. 531-543.
10. Ding, Y, Renard, G. *A New Representation of Wells in Numerical Reservoir Simulation*, SPE Res. Eng., May, 1994, p. 140-144.
11. Weltest 200 User Manual, Schlumberger_GeoQuest, Abingdon, 2000.

Appendix A – Derivation of the Approximation of the Well Flow Rate

It is assumed, that the flux into the fracture is uniformly distributed over the segment between centers of the blocks 1 and 5 and is perpendicular to the fracture plane. The flow rate per unit length, from above and from below, is given by

$$u = \frac{k_R h}{\eta} \left(\frac{p_3 - p_w}{\Delta y_{1/2}} + \frac{p_7 - p_w}{\Delta y_{-1/2}} \right) \quad (\text{A-1})$$

Consider, for definiteness, the right part of the fracture segment (from well to center of the block 5). The x -dependent flow in the fracture can be expressed as the whole flow rate into the well from the left less the inflow from the matrix accumulated on the length of x :

$$q_f(x) = Q_r - u x \quad (\text{A-2})$$

On the other hand, the local flow in fracture can be described by the differential equation

$$q_f(x) = \frac{k_f w h}{\eta} \frac{d p_f}{d x} \quad (\text{A-3})$$

Combining Eqs. (A-1), (A-2), and (A-3), and integrating over the corresponding portion of the fracture, we obtain

$$Q_r = \frac{k_f w h}{\eta} \left(\frac{p_5 - p_w}{\Delta x_{1/2}} \right) + \frac{k_R h \Delta x_{1/2}}{2\eta} \left(\frac{p_3 - p_w}{\Delta y_{1/2}} + \frac{p_7 - p_w}{\Delta y_{-1/2}} \right) \quad (\text{A-4})$$

In a like manner, the flow to the well from left Q_l is derived. The summarized well inflow $Q = Q_l + Q_r$ can be represented in the form of Eq. (7)

Table 1: Reservoir and production data:

Formation porosity	0.1	Production time, day	360
Formation permeability, mD	0.1	Production rate, kg/day	300
Formation thickness, m	10	Fracture half-length, m	55
Initial pressure, bar	100	Fracture permeability, D	600 / 0.6
Fluid compressibility, 1/Pas	5E-10	Fracture porosity	0.4
Viscosity, cp	2	Fracture width, m	0.01
Fluid density, kg/m ³	1000	Well radius, m	0.057

Fig. 1 Orthogonal structured grid for reservoir and fracture

Fig. 2 Fracture-presentation by unstructured grid

Fig. 3 Simulation results for high-conductivity fracture: a) Cartesian plot b) semi-log plot

Fig. 4 Simulation results for low-conductivity fracture