E37**EVALUATION OF THE PERFORMANCE OF COMPLEX WELLS BY BOUNDARY INTEGRAL METHODS**V. Moumas¹, Y. Ding¹, J. Giroire² and T. Ha-Duong²¹ Institut Français du Pétrole, 1 et 4 av. de Bois Préau, 92852 Rueil Malmaison, France² Université de Technologie de Compiègne, 60200 Compiègne, France**Abstract**

Classical numerical methods like finite differences, finite volumes or finite elements do not provide sufficient precise results around the well. This is a consequence of the scale difference between the size of the reservoir (several kilometers) and the radius of the well (several centimeters). Hybrid meshes with grid refinement around the well can improve the precision, but they are difficult to apply for modelling complex wells. To overcome this scale problem between the reservoir size and the wellbore radius, boundary integral methods can be used. In this paper, a stratified reservoir is considered with an arbitrary geometry. The permeability is supposed homogeneous in each reservoir layer. The performance of complex wells is evaluated in such a reservoir by boundary integral methods. The boundary integral equation is developed on the reservoir boundary, on the interfaces between the different layers as well as on the wellbore boundary. Since we are only interested in the performance of the well, we can restrict the problem to a stationary one. The discretization of the integral equations is performed by a Galerkin method. The integral equation on the well can be reduced to a 1D equation. Some numerical examples will be presented to show the efficiency of the method.

1 Introduction

Horizontal and multi-lateral wells are more and more used in petroleum engineering to improve oil recovery. Modelling of well production is a very important issue for petroleum engineers. A numerical model for the evaluation of well performances can allow them to optimize the productivity of the reservoir according to the architecture or the location of the complex wells.

In this paper, evaluation of well performances will be treated by boundary integral methods, well known in domains like acoustics, electromagnetism, or mechanics (see [2]), but recently applied in petroleum engineering. Until now, applications of boundary integral methods in petroleum engineering were generally limited in 2D, and the wells were modelled using sink / source terms. In this paper, we will extend the application of boundary integral methods in 3D for a stratified reservoir with arbitrary reservoir geometry.

The steady-state flow problem is considered for flow modelling. The well is not modelled using a line sink / source, and the real well geometry is considered.

Boundary integral equations are established on the reservoir outer boundaries, at the interfaces between the layers, and on the well boundary. A Galerkin approach is used to solve the boundary integral equations. Numerical integration on the well boundary can be reduced to a 1D problem.

Numerical results show the efficiency of our method.

2 Mathematical model

In this paper, we consider a 3D stratified reservoir with an arbitrary geometry, a well is placed in the reservoir. Let Ω denote the petroleum reservoir which does not include the domain occupied by the well. The reservoir is divided into homogeneous layers Ω_i , $i = 1, \dots, N_R$, with boundary Γ_i . An integral equation is established on the reservoir boundary Γ which is composed by the outer boundaries $\Gamma_D \cup \Gamma_N$, by the well boundary Γ_w , and by the interfaces between the layers $\Gamma_{ij} = \Gamma_i \cap \Gamma_j$. To simplify, we first present the model for a homogeneous reservoir (see figure (a)), and then we will generalize the model to a stratified medium.

2.1 Homogeneous reservoir

Let $x = (x_1, x_2, x_3)$ denote a point in a Cartesian coordinate system and t denote time in the interval $[0, T]$.

If gravity is neglected, the single-phase flow is governed by a diffusion equation:

$$\operatorname{div} \left(\overline{\overline{K}} \cdot \nabla p(x, t) \right) = (\phi \mu c_f) \frac{\partial p(x, t)}{\partial t} \quad x \in \Omega, \quad t \in]0, T] \quad (1)$$

where p is the pressure of the fluid in the porous medium, the tensor $\overline{\overline{K}}$ stands for the reservoir permeability, and ϕ and μ refer respectively to the porosity of the medium and the viscosity of the fluid. Initial condition is given by: $p(x, 0) = p_0, x \in \Omega$.

The diffusion equation (1) is usually used for modelling well-testing. Here we are only interested in the performance of the well, so that we proceed with the study of the stationary problem:

$$\operatorname{div} \left(\frac{\overline{\overline{K}}}{\mu} \cdot \nabla p(x) \right) = 0 \quad x \in \Omega. \quad (2)$$

We suppose the tensor $\overline{\overline{K}}$ diagonal with diagonal elements (k_1, k_2, k_3) .

Unlike in the work in the literature (Koh and Tiab [8], Pecher and Stanislav [12], Mohamad and Numbere [10], or Oguztoreli and Wong [13]), equation (2) does not include any sink or source term. The contribution of the well is incorporated to the problem by means of the following boundary condition:

$$Q = \frac{\tilde{k}}{\mu} \int_{\Gamma_w} \frac{\partial p(x)}{\partial (\overline{\overline{K}} n)} d\gamma(x) \quad (3)$$

where Q is the prescribed total flow rate, n is the outward normal vector to Γ_w ,

and $\frac{\partial p}{\partial (\overline{\overline{K}} n)} = \frac{1}{\tilde{k}} \overline{\overline{K}} \nabla p \cdot n$, where $\tilde{k} = (k_1 k_2 k_3)^{1/3}$.

Now let us write the boundary conditions.

2.1.1 Reservoir outer boundary conditions

At the outer boundary, two types of conditions are considered:

$$p(x) = p_D(x) \quad x \in \Gamma_D \quad (4)$$

$$\frac{\partial p(x)}{\partial (\overline{\overline{K}} n)} = g(x) \quad x \in \Gamma_N \quad (5)$$

The Dirichlet condition on Γ_D (constant pressure) corresponds for instance to the presence of a large capacity aquifer.

The Neumann condition imposes a fluid flow; this happens for example when we have an impermeable barrier: the flux is therefore null ($g \equiv 0$ on Γ_N).

2.1.2 Well boundary conditions

For simplicity of notation, we consider a single horizontal well with radius r_w and length L .

We can have two different conditions on Γ_w : infinite conductivity condition or finite conductivity condition, according to whether we take into account the effects of pressure drop or not.

Assuming the pressure p constant in each cross-section of the well, we can write p as a function of the axis variable $l \in [0, L]$ along the well.

Infinite conductivity If we do not take into account the effects of pressure drop, we assume the pressure constant along the well:

$$p(l) = p_w, \quad l \in [0, L]. \quad (6)$$

This is known as infinite conductivity condition in petroleum engineering, and is widely accepted.

Finite conductivity If we take into account the effects of pressure drop, we have a finite conductivity condition. The corresponding equation can be written in the following form:

$$\frac{\partial p(l)}{\partial l} = C_w q_w^2(l), \quad l \in [0, L] \quad (7)$$

$$\text{with } q_w(l) = \int_{\Gamma_{w,l}} \frac{\partial p(x)}{\partial(\overline{\overline{K}}n)} d\gamma(x), \quad l \in [0, L], \quad (8)$$

where C_w is a constant, and n is the outward normal vector to $\Gamma_{w,l}$.

$\Gamma_{w,l}$ denotes the part of the well boundary Γ_w from which the flow comes at the section S_l , as shown here for a complex well in figure (b).

The infinite conductivity condition (6) is a particular case of the finite conductivity condition (take $C_w = 0$).

Equations (2-5) and (7-8) constitute the governing equations of the model with finite conductivity condition in the case of a homogeneous reservoir.

In the next section, we give additional conditions to generalize this model to a stratified reservoir.

2.2 Stratified reservoir

To obtain the model for a stratified reservoir, we begin by writing the previous model (2-5) and (7-8) indexed by i for each layer Ω_i (which is supposed homogeneous as seen previously) and then we complete the model with the following interface conditions.

Pressure and flux continuity conditions must be satisfied at the zonal interfaces Γ_{ij} :

$$\begin{aligned} p_i &= p_j & \text{on } \Gamma_{ij} \\ \tilde{k}_i \frac{\partial p_i}{\partial(\overline{\overline{K}}_i n_i)} &= -\tilde{k}_j \frac{\partial p_j}{\partial(\overline{\overline{K}}_j n_j)} & \text{on } \Gamma_{ij} \end{aligned}$$

3 Integral methods

Two types of methods can be used to solve this problem: volumetric numerical methods (such as control-volume method) or boundary integral methods. Numerical methods can solve complicated problems, for example heterogeneous media or arbitrary geometry reservoirs. However, numerical methods suffer from a lack of precision around the well: it comes from an important scale difference between the size of the reservoir (several kilometers) and the radius of the well (≤ 0.15 cm). Hybrid meshes with grid refinement around the well can improve the precision, but they are difficult to apply for complex wells. To overcome the scale problem between the reservoir size and the wellbore radius, boundary integral methods can be used.

Boundary integral methods are widely used in electromagnetism, mechanics [2]... but their applications in petroleum engineering are somewhat rare. Liggett and Liu [9] in 1979 were the first to use such methods in petroleum engineering, then some examples are the work by Koh and Tiab [8], Pecher and Stanislav [12], Kikani and Horne [7], Oguztoreli and Wong [13], and more recently Ding [4] and [5] for complex well modelling.

The interest of such methods is that the problem can be formulated on the boundaries: the equations are written on the reservoir outer boundaries, on the well boundary, and on the interfaces between the layers. We need to mesh only the boundaries, and no more all the reservoir.

In the next section, we present the integral formulation of our problem and we will focus on the linear model (infinite conductivity condition on the well) for a homogeneous reservoir. Finite conductivity condition, corresponding to a non-linear model, will be treated in future work by a Newton method.

4 Integral representation of the model

Two kinds of integral representations can be written as (see Bonnet [2]):

$$cp = -Vq + Sp, \quad \text{on } \Gamma, \quad (9)$$

or:

$$cq = -Hq + Dp \quad \text{on } \Gamma, \quad (10)$$

where q stands for the normal derivative of the pressure $\frac{\partial p}{\partial(\bar{K}n_x)}$, c is a constant depending on the boundary Γ (if Γ is regular, $c = 1/2$, if not see [12]), and the operators V, S, H, D are defined by :

$$\begin{aligned} Vq &= \int_{\Gamma} G(x-y)q(y) d\gamma(y) & Sp &= \int_{\Gamma} \frac{\partial G(x-y)}{\partial(\bar{K}n_y)} p(y) d\gamma(y) \\ Dp &= \frac{\partial}{\partial n_x} \int_{\Gamma} \frac{\partial G(x-y)}{\partial(\bar{K}n_y)} p(y) d\gamma(y) & Hq &= \int_{\Gamma} \frac{\partial G(x-y)}{\partial(\bar{K}n_x)} q(y) d\gamma(y) \end{aligned}$$

G is the Green function associated to the problem and is given by:

$$G(x-y) = \frac{1}{4\pi} \frac{1}{(a^2(x_1-y_1)^2 + b^2(x_2-y_2)^2 + c^2(x_3-y_3)^2)^{1/2}}$$

$$\text{where } a = \frac{(k_1 k_2 k_3)^{1/6}}{k_1^{1/2}}, \quad b = \frac{(k_1 k_2 k_3)^{1/6}}{k_2^{1/2}}, \quad c = \frac{(k_1 k_2 k_3)^{1/6}}{k_3^{1/2}}.$$

We have to choose a formulation for each boundary. In petroleum engineering, boundary integral equations correspond to the first formulation (9). The second formulation (10) has not been used yet in petroleum engineering whereas its application in acoustics or mechanics has been successful. The interest of (10) is that this formulation is more stable than the first one (9) for Dirichlet boundary conditions (see [11] or first kind integral equations / second kind integral equations in Bonnet [2]). On the Dirichlet boundary Γ_D , we have written the second representation (10), and on the Neumann boundary Γ_N as well as on the well boundary Γ_w , the first formulation (9):

$$cq = -Hq + Dp \quad \text{on } \Gamma_D \quad (11)$$

$$cp = -Vq + Sp \quad \text{on } \Gamma_N \quad (12)$$

$$cp = -Vq + Sp \quad \text{on } \Gamma_w \quad (13)$$

Our results will show the efficiency of this representation.

5 Numerical aspects

To solve the boundary integral equations, a Galerkin method has been performed.

We choose as basis functions $(\psi_i^h)_{i=1,\dots,N}$, the finite element functions $\mathbb{P}^1$, N is the total number of nodes in the mesh. We denote by p^h and q^h the approximations of the unknowns p and q , and we decompose p^h and q^h in the basis $(\psi_i^h)_{i=1,\dots,N}$:

$$p^h(x) = \sum_{j=1}^N p_j \psi_j^h(x), \quad q^h(x) = \sum_{j=1}^N q_j \psi_j^h(x).$$

Let Γ_h be an approximation of the boundary Γ . After multiplying the three previous integral equations by a basis function ψ_i^h and then integrated once more over the boundary Γ_h , it is necessary to evaluate the five following terms:

$$\int_{\Gamma_h} \psi_i^h(x) \psi_j^h(x) d\gamma_h(x) \quad (14)$$

$$\int_{\Gamma_h} \int_{\Gamma_h} \psi_i^h(x) \psi_j^h(y) G(x-y) d\gamma_h(y) d\gamma_h(x) \quad (15)$$

$$\int_{\Gamma_h} \int_{\Gamma_h} \psi_i^h(x) \psi_j^h(y) \frac{\partial G(x-y)}{\partial(\overline{\overline{K}}n_y)} d\gamma_h(y) d\gamma_h(x) \quad (16)$$

$$\int_{\Gamma_h} \int_{\Gamma_h} \psi_i^h(x) \psi_j^h(y) \frac{\partial G(x-y)}{\partial(\overline{\overline{K}}n_x)} d\gamma_h(y) d\gamma_h(x) \quad (17)$$

$$\int_{\Gamma_h} \psi_i^h(x) \frac{\partial}{\partial(\overline{\overline{K}}n_x)} \int_{\Gamma_h} \psi_j^h(y) \frac{\partial G(x-y)}{\partial(\overline{\overline{K}}n_y)} d\gamma_h(y) d\gamma_h(x) \quad (18)$$

The first integral (14) comes from the left-hand side of equations (11),(12), (13) and is very easy to calculate, the second integral (15) comes from the operator V appearing in (12) and (13), the third (16) from the operator S in (12) and (13), the fourth (17) from the operator H in (11), and the fifth (18) from the operator D in (11).

5.1 Singularities

Integrals (15), (16) and (17) are weakly singular integrals: the kernels G and $\partial G/\partial(\overline{\overline{K}}n)$ ($n = n_x$ or n_y) present singularities at $x \rightarrow y$. The kernels G and $\partial G/\partial(\overline{\overline{K}}n)$ behave as r^{-1} , where $r = |x - y|$. By taking into account these singularities, we can obtain an analytic formula for the integral with respect to y and then the integral with respect to x is performed by numerical integration. On the wellbore boundary, an analytic formula can be obtained for both x and y integrals along the well (see section (5.2)).

The difficult point here is integral (18). This is a hypersingular integral: the trouble is that the normal derivative $\partial/\partial(\overline{\overline{K}}n_x)$ cannot be included into the integral with respect to y , otherwise this integral would not be defined. By same techniques as in Giroire [6], we can obtain the following formula:

$$\begin{aligned} & \int_{\Gamma_h} \psi_i^h(x) \frac{\partial}{\partial(\overline{\overline{K}}n_x)} \int_{\Gamma_h} \psi_j^h(y) \frac{\partial G(x-y)}{\partial(\overline{\overline{K}}n_y)} d\gamma_h(y) d\gamma_h(x) \\ &= -\frac{1}{4\pi} \int_{\Gamma_h} \int_{\Gamma_h} \frac{\tilde{\text{rot}}_{\Gamma_h} \psi_i^h(x) \cdot \tilde{\text{rot}}_{\Gamma_h} \psi_j^h(y) d\gamma_h(y) d\gamma_h(x)}{(a^2(x_1 - y_1)^2 + b^2(x_2 - y_2)^2 + c^2(x_3 - y_3)^2)^{1/2}} \end{aligned}$$

where $\tilde{\text{rot}}_{\Gamma_h} u = \left(\frac{1}{\overline{\overline{k}}^{1/2}} \overline{\overline{K}}^{1/2} \nabla u \right) \wedge \left(\frac{1}{\overline{\overline{k}}^{1/2}} \overline{\overline{K}}^{1/2} n \right)$, $\overline{\overline{K}}^{1/2}$ is the diagonal matrix with diagonal elements $(k_1^{1/2}, k_2^{1/2}, k_3^{1/2})$, $\wedge$ is the vectorial product, and n is the outward normal to the boundary Γ_h .

The basic idea to obtain this result is to transform the normal derivatives $\partial/\partial(\overline{\overline{K}}n_x)$ and $\partial/\partial(\overline{\overline{K}}n_y)$ into tangential derivatives on the basis functions ψ_i^h and ψ_j^h (for more details see [11]).

Now it is worth pointing out that this formula is very simplified: since the basis functions ψ_i^h ,

$i = 1, \dots, N$, are polynomials of degree 1, the terms $\tilde{\text{rot}}_{\Gamma_h} \psi_i^h(x)$ and $\tilde{\text{rot}}_{\Gamma_h} \psi_j^h(y)$ become constants. Then the crucial advantage of using a Galerkin method lies in the fact that the hypersingularity can be treated easily thanks to the additionnal integration and the basis function.

For boundary discretization, Γ_D^h and Γ_N^h are discretized by triangles.

The well boundary Γ_w^h is discretized by cylinder segments. As mentioned in section 3, the radius r_w of the well is very small with respect to the size of the reservoir, and then with respect to the length L of the well. Under this condition, integration along a well cylindrical segment can be reduced to a line integration, this point will be explained in the next section.

5.2 Special treatment for the well

Our aim is to evaluate a 2D integral (on a cylinder) by a 1D integral (on a line). The difficulty is that the normal derivative is not defined in 1D.

We recall that we have supposed that the unknowns p and its normal derivative q are constant over a well boundary cross-section.

Let h_i be the discretized well segment length. If h_i is sufficiently large with respect to the radius r_w , we can calculate the integral terms analytically.

For simplicity of notation, we consider a horizontal well in an isotropic medium. We will take for example the integral (16) and we will restrict ourselves to the treatment of the part of this integral on $\Gamma_w^h \times \Gamma_w^h$. When discretized, this integral is expressed as:

$$A_{ij} = 2r_w \int_0^L \psi_i^h(l') dl' \int_0^L \psi_j^h(l) dl \int_0^{\pi/2} \frac{2r_w^2 \sin^2 \theta d\theta}{[4r_w^2 \sin^2 \theta + (l - l')^2]^{3/2}}$$

$$A_{ij} = 2r_w \sum_{k=1}^{N-1} \int_{l_k}^{l_{k+1}} \psi_i^h(l') dl' \sum_{m=1}^{N-1} \int_{l_m}^{l_{m+1}} \psi_j^h(l) dl \int_0^{\pi/2} \frac{2r_w^2 \sin^2 \theta d\theta}{[4r_w^2 \sin^2 \theta + (l - l')^2]^{3/2}}$$

If the distance between elements $[l_k, l_{k+1}]$ and $[l_m, l_{m+1}]$ is sufficiently large, we can neglect the denominator term $4r_w^2 \sin^2 \theta$ with respect to $(l - l')^2$, and then the integral has an analytic formulation. If not, we can first integrate analytically with respect to l and l' . By some variable transformations, we can write that:

$$A_{ij} = 2r_w \sum_{k \in \mathcal{I}} \sum_{m \in \mathcal{J}} I_{ikjm}, \text{ where } \mathcal{I} = \{i-1, i\} \text{ if } 2 \leq k \leq N-2, \mathcal{I} = \{i\} \text{ if } k \in \{1, N-1\},$$

$$\mathcal{J} = \{j-1, j\} \text{ if } 2 \leq m \leq N-2, \mathcal{J} = \{j\} \text{ if } m \in \{1, N-1\},$$

and I_{ikjm} is defined by:

$$I_{ikjm} = \frac{1}{|b_{ik} b_{jm}|} \int_0^{\pi/2} \int_0^{b_{ik}} \int_0^{b_{jm}} \frac{2r_w^2 \sin^2 \theta s s' ds ds' d\theta}{[4r_w^2 \sin^2 \theta + (s - s' + c_{ikjm})^2]^{3/2}}$$

where $b_{ik} = l_i - (\delta_{ik} l_{k+1} + \delta_{i,k+1} l_k)$, $b_{jm} = l_j - (\delta_{jm} l_{m+1} + \delta_{j,m+1} l_m)$,

$c_{ikjm} = (\delta_{jm} l_{m+1} + \delta_{j,m+1} l_m) - (\delta_{ik} l_{k+1} + \delta_{i,k+1} l_k)$, and δ is the Dirac function.

After integrating with respect to s and s' , we can obtain:

$$I_{ikjm} = \frac{1}{|b_{ik} b_{jm}|} \int_0^{\pi/2} \{f(\theta, b_{jm} - b_{ik} + c_{ikjm}) - f(\theta, b_{jm} + c_{ikjm}) - f(\theta, -b_{ik} + c_{ikjm}) + f(\theta, c_{ikjm})\} d\theta$$

where

$$f(\theta, t) = c_{ikjm} r_w^2 \sin^2 \theta \ln \left(t + \sqrt{4r_w^2 \sin^2 \theta + t^2} \right) - \frac{1}{6} (4r_w^2 \sin^2 \theta + t^2)^{3/2} + \lambda(t) (4r_w^2 \sin^2 \theta + t^2)^{1/2},$$

and $\lambda(t)$ is a constant depending on t and on b_{ik}, b_{jm}, c_{ikjm} (for more details see [11]).

We can notice that t is depending on h_i and h_j . We can show that if h_i, h_j are sufficiently large with respect to r_w , we have $|t| \gg r_w$ or $t = 0$.

Then we can neglect $4r_w^2 \sin^2 \theta$ in the last two terms if $t \neq 0$. For the first term, if $t > 0$, we can also neglect $4r_w^2 \sin^2 \theta$ inside the logarithm, which is then reduced to $\ln(2|t|)$; if $t < 0$, we multiply the terms

inside the logarithm by their conjugate expression, and then we can make the same simplification. Thanks to this method, we can obtain an analytic formula. If $t = 0$, standard integration gives an analytic formula.

With this method, we have seen that we can obtain an analytic formula for all matrix elements.

6 First results and validations

The first validation test is a horizontal well in an infinite reservoir. We assume the pressure to be 0 at infinity. An infinite conductivity condition is taken. Then the integral representation of the model consists of equation (13).

The well has length $L = 1000m$, and radius $r_w = 0.108m$. The pressure along the well is $p_w = 1$ bar. The reservoir is isotropic with permeability $1mD$.

The well is discretized into $N = 8$ non uniform segments. We have taken as reference solution a solution computed by an integral method discretized with a collocation method (see [2]). Figure (c) shows the flux along the well axis.

Figure (d) shows the convergence of the Galerkin method, the well is successively discretized into $N = 8$ segments, $N = 10$ segments, $N = 20$ segments and $N = 50$ segments. Here the mesh is uniform.

These two results show that the formulation chosen on the well is a good formulation.

7 Conclusion

Boundary integral methods have been applied to treat a 3D steady-state problem for advanced well modelling in petroleum engineering. The boundary integral equations have been discretized with a Galerkin method, and the integrals on the well have been simplified to model the well as a line by a nonusual method. The preliminary tests give promising results.

Future work will be to extend these results to a stratified reservoir with arbitrary geometry, with finite conductivity condition. Then thanks to Laplace transform, this will be generalized to the transient problem.

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8 Figures

(a) Homogeneous reservoir

(b) Part $\Gamma_{w,l}$ of the well boundary Γ_w

(c) -: Galerkin method o: Collocation method

(d) +: N=8 o: N=10 *:N=20 x: N=50