E50 STOCHASTIC APPROACH TO OPTIMIZATION OF WELL SPACING IN LENTICULAR RESERVOIR

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Abstract

The new approach to simulation lenticular reservoirs using Boolean sets is presented. The probabilistic description of the lens system includes the shape and the sizes of the area, the shape of lenses, probability densities of lens location and of their sizes, total number of lenses in the domain. Connections between the system of lenses and the system of wells are studied.

The proposed approach has been used to analyze the uncertainty and risk of well spacing and to estimate the effect of either horizontal well drilling or hydraulic fracturing in such reservoirs. The new formula for probability of the event that each well will be connected with certain of the lenses was obtained. This result has been applied to the inverse problem solution. The number of lenses as well as the probability density parameters for lens sizes and for their location has been estimated by maximum likelihood method using results of exploratory wells.

The proposed approach is illustrated by its application to the assessment of oil field parameters. As an example Priobskoe field (Western Siberia) is taken.

Introduction

A geological model concept for a heterogeneous lenticular reservoir always includes uncertainty to account for geometry and distribution of sandstone lenses in interwell space. Probability theory provides a more adequate framework for risk assessment in planning of well spacing into such reservoirs either by estimation of both reservoir properties and probabilities of appearance of different connections between wells and lenses [1,2], or by direct simulation using Monte-Carlo methods [3-5]. The second approach is today the most commonly used. The internal heterogeneity of reservoirs is simulated using various probabilistic models, such as Gaussian models [3,6], neural networks [7], Markov models [3], indicator approximations [8], Boolean sets [1], etc.

The lithology modeling is usually based on concept of the discrete random field, which represents the spatial distribution of the facies [1,6,9]. The sandstone lens sizes, their form and orientation can be determined either by relationship between lens thickness and lens width for analogous reservoirs, or using well-to-well correlation, if wells are closely spaced in the area, or by seismic data, or again on the base of genetic method which incorporates the geological knowledge on the sedimentary structure [9-11].

This paper presents the new approach to simulation heterogeneous reservoirs consisting of sandstone lenses in shale matrix. The lenses of predetermined shape and random size are statistically distributed in the domain under study. Connections between the system of lenses and

the system of wells have been studied and probabilities of the events induced by the structure of connections in the system "lenses-wells" have been obtained.

The main stages of research are following. In the first stage mathematical models are developed and formulae for probabilities of different events in general form are derived (direct problem solution). These relationships contain the characteristics of the lens system that are unknown numerically yet. In the second stage the unknown characteristics are estimated by maximum likelihood method using results of exploratory wells (inverse problem solution). And at last, in the third stage, these parameters are used to calculate the probabilities of events being investigated.

From the obtained equations practically important results have been deduced. Both the probability of making an empty hole and the probability of missing a sand body by a group of exploratory wells were estimated. For hydraulic fractures and horizontal holes of different length, the formula for probability of the new lens exposing as a function of length was obtained.

Idealization of the reservoir system and formulation of the problem

A heterogeneous reservoir consisting of sandstone lenses in shale matrix is considered. A pseudo 3D model is analyzed. It is assumed that the lenses of predetermined shape and random size are statistically distributed in the domain under study. They are independent. The domain is characterized by 2D stochastic field defined as a random number of lenses located under each point of the surface. The probabilistic description of the lens system includes the shape and the sizes of the area, the shape of lenses, the probability densities of lens location and of their sizes, total number of lenses in the domain.

It is supposed that lenses are contained in rectangles with the sides parallel to axis X and Y . For each lens the minimum rectangle is considered. The lens shape is determined by coefficient β equal to the ratio of the lens area to the rectangle area. For example, ellipses are given by $\beta = \pi/4$. Rectangles are characterized by random coordinates of centers (ξ, η) and random lengths of sides $2l, 2s$. These four random variables are independent in the aggregate. Their probability distributions are prescribed, but they depend on parameters whose numerical values can be unknown. Number of lenses is n . Consequently the lens system is determined by the set of $4n$ random variables independent in the aggregate. For example, random variables can be characterized by normal distribution:

$$\xi \sim N(0, \sigma_\xi^2), \quad \eta \sim N(0, \sigma_\eta^2), \quad l \sim N(m_l, \sigma_l^2), \quad s \sim N(m_s, \sigma_s^2) \quad (1)$$

Here, $N(m, \sigma^2)$ is the normal distribution with mean value m and standard deviation σ . Thus, in this example the origin of coordinates corresponds to the maximum density of lens location. Characteristics σ_ξ, σ_η must conform to area sizes. It is assumed that $m_l > 2\sigma_l$, $m_s > 2\sigma_s$ and the lenses with the sizes close to zero are excluded from consideration with high probability.

It is assumed that m wells are located in the area under study in points $M_1(x_1, y_1), M_2(x_2, y_2), \dots, M_m(x_m, y_m)$. Each lens α ($\alpha = 1, \dots, n$) can be associated with the number sequence $(\pi_{\alpha 1}, \pi_{\alpha 2}, \dots, \pi_{\alpha m})$, that is determined as follows: $\pi_{\alpha i} = 1$, if i -th well throws into the lens α , and $\pi_{\alpha i} = 0$ otherwise. The event $(\pi_{\alpha 1}, \pi_{\alpha 2}, \dots, \pi_{\alpha m})$ is that the lens α is crossed only by wells for which $\pi_{\alpha i} = 1$. $P(\pi_{\alpha 1}, \pi_{\alpha 2}, \dots, \pi_{\alpha m})$ is the probability of the event $(\pi_{\alpha 1}, \pi_{\alpha 2}, \dots, \pi_{\alpha m})$.

Structural matrix $\pi = (\pi_{\alpha i})$ specifies the structure of connections in the "lenses-wells" system. It is of $m \times n$ dimension. Rows correspond to lenses and columns correspond to wells. Every

structural matrix π generates an event in the respective probabilistic space. Presence of null rows in the matrix π means that corresponding lenses are not crossed by any well. If $\pi = 0$, the matrix π incorporates only zeros. $P(\pi)$ is the probability of the matrix π . In practice, a number of lenses can be unknown as well as other characteristics of the lens system, but the estimation method is presented in this paper.

Probability estimations for the “lenses-wells” system

Let us assume that m wells are located in the area in points $M_1(x_1, y_1)$, $M_2(x_2, y_2)$, ..., $M_m(x_m, y_m)$ and the structural matrix π is predetermined. Estimate the probability $P(\pi)$ of the event π .

For arbitrary distributed random values ξ , η , l , s the joint distribution densities of variables $\xi + l$, $\xi - l$ and $\eta + s$, $\eta - s$ characterizing lens boundaries, are as follows [12]:

$$f_{\xi+l, \xi-l}(u, v) = 0.5 f_{\xi} \left(\frac{u+v}{2} \right) f_l \left(\frac{u-v}{2} \right), \quad g_{\eta+s, \eta-s}(u, v) = 0.5 g_{\eta} \left(\frac{u+v}{2} \right) g_s \left(\frac{u-v}{2} \right) \quad (2)$$

Here $f_l, g_s, f_{\xi}, g_{\eta}$ are probability densities for respective values l, s, ξ and η .

Specifically, for normally distributed values (1) the expressions (2) take the form:

$$f_{\xi+l, \xi-l}(u, v) = (4\pi\sigma_{\xi}\sigma_l)^{-1} \exp \left[-\frac{\sigma_{\xi}^{-2}(u+v)^2 + \sigma_l^{-2}(u-v-2m_l)^2}{8} \right] \quad (3)$$

$$g_{\eta+s, \eta-s}(u, v) = (4\pi\sigma_{\eta}\sigma_s)^{-1} \exp \left[-\frac{\sigma_{\eta}^{-2}(u+v)^2 + \sigma_s^{-2}(u-v-2m_s)^2}{8} \right]$$

If coordinates of lens centers ξ and η are uniformly distributed in the rectangle $\xi_1 \leq \xi \leq \xi_2$, $\eta_1 \leq \eta \leq \eta_2$, the equations (2) give:

$$f_{\xi+l, \xi-l}(u, v) = \frac{0.5}{\xi_2 - \xi_1} \mathbf{I}_{[\xi_1, \xi_2]} \left(\frac{u+v}{2} \right) f_l \left(\frac{u-v}{2} \right), \quad g_{\eta+s, \eta-s}(u, v) = \frac{0.5}{\eta_2 - \eta_1} \mathbf{I}_{[\eta_1, \eta_2]} \left(\frac{u+v}{2} \right) g_s \left(\frac{u-v}{2} \right)$$

$$\mathbf{I}_{[a, b]}(w) = \begin{cases} 1, & w \in [a, b] \\ 0, & w \notin [a, b] \end{cases} \quad (4)$$

Joint distribution functions are calculated as follows:

$$F_{\xi+l, \xi-l}(u, v) = \int_{-\infty}^u \int_{-\infty}^v f_{\xi+l, \xi-l}(\lambda, \nu) d\lambda d\nu, \quad G_{\eta+s, \eta-s}(u, v) = \int_{-\infty}^u \int_{-\infty}^v g_{\eta+s, \eta-s}(\lambda, \nu) d\lambda d\nu \quad (5)$$

Let us consider the regular well spacing to start with. Construct the variational series of abscissae and ordinates of points $M_1, \dots, M_m$ and complement them by extreme terms $x_{(0)} = -\infty$, $x_{(m+1)} = +\infty$, $y_{(0)} = -\infty$, $y_{(m+1)} = +\infty$. Thus the extended variational series $x_{(0)}, x_{(1)}, \dots, x_{(m)}, x_{(m+1)}$ and $y_{(0)}, y_{(1)}, \dots, y_{(m)}, y_{(m+1)}$ are obtained.

At first one lens α and an event $(\pi_{\alpha_1}, \pi_{\alpha_2}, \dots, \pi_{\alpha_m})$ are treated and the probability $P(\pi_{\alpha_1}, \pi_{\alpha_2}, \dots, \pi_{\alpha_m})$ is estimated. The case when one or more wells throw into the lens, will be our initial concern: $(\pi_{\alpha_1}, \pi_{\alpha_2}, \dots, \pi_{\alpha_m}) \neq (0, 0, \dots, 0)$. Let R_{α} be a minimum size rectangle with sides parallel to coordinate system axes, containing all wells crossing the lens. The rectangle R_{α} is

defined as the direct product of segments whose ends are coincident with some members of appropriate variational series: $R_\alpha = [x_\alpha^-, x_\alpha^+] \times [y_\alpha^-, y_\alpha^+]$. R_α is a nonempty set for the associated event, however it can be degenerate, if one or two segments are degenerate: $x_\alpha^- = x_\alpha^+$ or (and) $y_\alpha^- = y_\alpha^+$. $R_\alpha^* = [x_\alpha^{--}, x_\alpha^{++}] \times [y_\alpha^{--}, y_\alpha^{++}]$ is a minimum size rectangle with sides parallel to coordinate system axes, for which the predetermined probability of containing the lens is close to one. If $\beta = 1$, the rectangle R_α^* is defined with regard to the following additional constraint: the domain $R_\alpha^* - R_\alpha$ does not contain any well. In this case $x_\alpha^{--}, x_\alpha^{++}, y_\alpha^{--}, y_\alpha^{++}$ can be coincident with some members of extended variational series.

In such a manner the single-valued mapping of the considered event set into the set of rectangle pairs is defined: $(\pi_{\alpha 1}, \pi_{\alpha 2}, \dots, \pi_{\alpha m}) \rightarrow (R_\alpha, R_\alpha^*)$. In view of the fact that $R_\alpha \subset R_\alpha^*$, following inequalities are satisfied:

$$x_\alpha^{--} < x_\alpha^- \leq x_\alpha^+ < x_\alpha^{++}, \quad y_\alpha^{--} < y_\alpha^- \leq y_\alpha^+ < y_\alpha^{++}$$

For each event $(\pi_{\alpha 1}, \pi_{\alpha 2}, \dots, \pi_{\alpha m})$, there is the system of inequalities:

$$x_\alpha^{--} < \xi_\alpha - l_\alpha \leq x_\alpha^-, \quad x_\alpha^+ \leq \xi_\alpha + l_\alpha < x_\alpha^{++}, \quad y_\alpha^{--} < \eta_\alpha - s_\alpha \leq y_\alpha^-, \quad y_\alpha^+ \leq \eta_\alpha + s_\alpha < y_\alpha^{++}$$

By virtue of the assumption that random variables ξ , η , l , s are independent in the aggregate, the probability of the event $(\pi_{\alpha 1}, \pi_{\alpha 2}, \dots, \pi_{\alpha m})$ is given by:

$$P(\pi_{\alpha 1}, \pi_{\alpha 2}, \dots, \pi_{\alpha m}) = \beta^{k_1} (1 - \beta)^{k - k_1} P(x_\alpha^+ \leq \xi_\alpha + l_\alpha < x_\alpha^{++}, x_\alpha^{--} < \xi_\alpha - l_\alpha \leq x_\alpha^-) \times \\ \times P(y_\alpha^+ \leq \eta_\alpha + s_\alpha < y_\alpha^{++}, y_\alpha^{--} < \eta_\alpha - s_\alpha \leq y_\alpha^-) \quad (6)$$

Here β is the lens shape coefficient; k is the number of wells ending up in the rectangle R_α^* ; $k_1 \leq k$ is the number of wells throwing into the lens α . When deriving (6), there was used the Bernoulli scheme in which the well hit in the lens is the successful result with probability β . The equations (5), (6) give:

$$P(\pi_{\alpha 1}, \pi_{\alpha 2}, \dots, \pi_{\alpha m}) = \beta^{k_1} (1 - \beta)^{k - k_1} \int_{x_\alpha^+}^{x_\alpha^{++}} \int_{x_\alpha^-}^{x_\alpha^{--}} f_{\xi+l, \xi-l}(\lambda, \nu) d\lambda d\nu \int_{y_\alpha^+}^{y_\alpha^{++}} \int_{y_\alpha^-}^{y_\alpha^{--}} g_{\eta+s, \eta-s}(\lambda, \nu) d\lambda d\nu \quad (7)$$

The expression (7) can be presented as follows:

$$P(\pi_{\alpha 1}, \pi_{\alpha 2}, \dots, \pi_{\alpha m}) = \beta^{k_1} (1 - \beta)^{k - k_1} \times \\ \times [F_{\xi+l, \xi-l}(x_\alpha^{++}, x_\alpha^-) - F_{\xi+l, \xi-l}(x_\alpha^{++}, x_\alpha^{--}) - F_{\xi+l, \xi-l}(x_\alpha^+, x_\alpha^-) + F_{\xi+l, \xi-l}(x_\alpha^+, x_\alpha^{--})] \times \\ \times [G_{\eta+s, \eta-s}(y_\alpha^{++}, y_\alpha^-) - G_{\eta+s, \eta-s}(y_\alpha^{++}, y_\alpha^{--}) - G_{\eta+s, \eta-s}(y_\alpha^+, y_\alpha^-) + G_{\eta+s, \eta-s}(y_\alpha^+, y_\alpha^{--})] \quad (8)$$

The formulae (6)-(8) were obtained for the case $(\pi_{\alpha 1}, \pi_{\alpha 2}, \dots, \pi_{\alpha m}) \neq (0, 0, \dots, 0)$.

For one lens the probability of the event that any well will not throw into this lens, is determined by the formula:

$$P(0, 0, \dots, 0) = 1 - \sum_{(\pi_1, \pi_2, \dots, \pi_m) \neq (0, 0, \dots, 0)} P(\pi_1, \pi_2, \dots, \pi_m) \quad (9)$$

The equation (9) estimates the risk to miss the lens by the group of wells.

Using a polynomial distribution formula and taking into account that all $4n$ random values are independent in the aggregate, it is possible to calculate the probability of the event π :

$$P(\pi) = \frac{n!}{\prod n(\pi_1, \pi_2, \dots, \pi_m)!} \prod_{\alpha=1}^n P(\pi_{\alpha 1}, \pi_{\alpha 2}, \dots, \pi_{\alpha m}) \quad (10)$$

Here $(\pi_1, \pi_2, \dots, \pi_m)$ is arbitrary row of zeros and unities; $n(\pi_1, \pi_2, \dots, \pi_m)$ is the number of appearances of this row in the matrix π ; the product of $n(\pi_1, \pi_2, \dots, \pi_m)!$ is taken for all possible different rows.

The expressions (9), (10) give the probability of the event that a group of wells located in the points $M_1(x_1, y_1), M_2(x_2, y_2), \dots, M_m(x_m, y_m)$ will not cross any lens:

$$P(0) = P(0, 0, \dots, 0)^n \quad (11)$$

The equations (6)-(11) were obtained for regular well spacing. For arbitrary location of wells the solutions of the problems studied are algorithmic in character and they were realized by means of specially developed computer program.

The probability $P_M(0)$ of the event that the well located in the point $M(x, y)$ will not throw into any lens, can be estimated too. After transformation of (8) for the case of one well followed by the application of formulae (9), (11) the expression for the probability $P_M(0)$ was obtained:

$$P_M(0) = \{1 - \beta[F_{\xi-l}(x) - F_{\xi+l}(x)][G_{\eta-s}(y) - G_{\eta+s}(y)]\}^n \quad (12)$$

The equation (12) estimates the risk to make empty hole. For example, for the domain $2 \times 10 \text{ km}^2$ of Priobskoe oil field (AC₁₀ unit), where lens centers are uniformly distributed and their sizes are normally distributed with $m_l = 1.3 \text{ km}$, $m_s = 1.2 \text{ km}$, $\sigma_l = 0.7$, $\sigma_s = 0.65$ [13] and number of lenses is $n=24$, it is: $P_M(0)=0.202$.

Estimation of fracturing effect in a lenticular reservoir

The probabilistic approach makes possible the estimation of the effect of hydraulic fracture length on the recovery of a lenticular reservoir. Estimations of this sort can also be made for a horizontal hole. A symmetric vertical hydraulic fracture of half-length ρ is considered. The fracture orientation is prescribed by the angle $\alpha \in (-\pi/2, \pi/2]$ made with the x -axis. Thus the fracture is simulated by the interval in the plane xy

$$x_{\pm} = x_0 \pm \rho t \cos \alpha, \quad y_{\pm} = y_0 \pm \rho t \sin \alpha, \quad 0 \leq t \leq 1 \quad (13)$$

Here the point $M(x_0, y_0)$ is appropriate to the fractured well location. The plus sign corresponds to the fracture wing where $x > x_0$, and the minus sign to the opposite. Let us evaluate the probability $P_0(M, \rho, \alpha)$ of the event that the fracture will not cross any new lens when propagating.

The case of the infinitesimal ρ and only one lens will be our initial concern. The probability $P_1(M, \rho, \alpha)$ that the fracture will intersect a boundary of only one new lens when propagating is determined to an accuracy of a quantity infinitesimal with respect to ρ as follows:

$$P_1(M, \rho, \alpha) = \beta \times \\ [P(x_0 < \xi - l < x_0 + \rho \cos \alpha, \eta - s < y_0 < \eta + s) + P(y_0 < \eta - s < y_0 + \rho \sin \alpha, \xi - l < x_0 < \xi + l) + \\ + P(x_0 - \rho \cos \alpha < \xi + l < x_0, \eta - s < y_0 < \eta + s) + P(y_0 - \rho \sin \alpha < \eta + s < y_0, \xi - l < x_0 < \xi + l)]$$

On the right of this expression the probabilities of the incompatible events are contained in brackets and in consequence, the sum of these probabilities is used to derive $P_1(M, \rho, \alpha)$. The random variables ξ , η , l , s are independent in the aggregate. So it is evident that:

$$P_1(M, \rho, \alpha) = \beta \mu \rho, \quad \mu = \mu(x_0, y_0, \alpha) \quad (14)$$

$$\mu(x_{\pm}, y_{\pm}, \alpha) = \\ = \{f_{\xi-l}(x_+) [G_{\eta-s}(y_+) - G_{\eta+s}(y_+)] + f_{\xi+l}(x_-) [G_{\eta-s}(y_-) - G_{\eta+s}(y_-)]\} \cos \alpha + \\ + \{g_{\eta-s}(y_+) [F_{\xi-l}(x_+) - F_{\xi+l}(x_+)] + g_{\eta+s}(y_-) [F_{\xi-l}(x_-) - F_{\xi+l}(x_-)]\} \sin \alpha \quad (15)$$

In the expression (15) the notations (13) are used. The function $\mu(x_{\pm}, y_{\pm}, \alpha)$ is fully determined by ξ , η , l , s distribution characteristics.

The equation (14) is deduced for the case of the infinitesimal ρ on the assumption that there is only one lens. If there are n lenses, the probability $P_1(M, \rho, \alpha)$ is determined as follows:

$$P_1(M, \rho, \alpha) = n \beta \mu \rho (1 - \beta \mu \rho)^{n-1} \approx n \beta \mu \rho \quad (16)$$

To eliminate the assumption that ρ is infinitesimal, a discrete non-decreasing stochastic process $\zeta(t)$ is considered. $\zeta(t)$ takes integer values, equal to the number of lenses whose boundaries are intersected by the fracture when propagating or when parameter t increases from 0 to 1 in the expression (13). Let us suppose that ρ is sufficiently small when compared to the lens size.

According to (16), for an infinitesimal ρ the probability of the event that the fracture will intersect the boundary of one lens only is proportional to ρ . It can be shown that the probability of the event that the fracture will cross the boundaries of several lenses is infinitesimal with respect to ρ . It follows that $\zeta(t)$ is a Poisson process [12] and the formula for the probability of the event $\zeta(t) = 0$, that the fracture will not intersect the boundary of any new lens, looks like:

$$P(\zeta(t) = 0) = e^{-n \beta \mu \rho t} \quad (17)$$

For $t = 1$ the expression (17) is reduced to the form:

$$P_1(M, \rho, \alpha) = e^{-n \beta \mu \rho} \quad (18)$$

The probability $P_M(0)$ of the event that the well located in the point M , will not throw into any lens, is estimated by equation (12). This event and the event that the fracture will not intersect the boundary of any new lens when propagating are independent. Thus in view of (12), (18), the probability $P_{f0}(M, \rho, \alpha)$ of the event that the fracture will not cross any lens is defined as follows:

$$P_{f0}(M, \rho, \alpha) = P_M(0) e^{-n \beta \mu \rho} \quad (19)$$

Equations (18), (19) are valid when the variation of $\mu(x_{\pm}, y_{\pm}, \alpha)$ for $0 \leq t \leq 1$ is neglected.

Otherwise the average value $\mu = \int_0^1 \mu(x_{\pm}, y_{\pm}, \alpha) dt$ must be taken into account.

According to (19), the probability of the event that the fracture will not cross any lens exponentially decreases with the increasing of both the fracture length and the number of lenses; it also decreases as the lens sizes increase. The probability of the event that the fracture will cross one or more lenses is defined from (19):

$$P_{f>0}(M, \rho, \alpha) = 1 - P_M(0) e^{-n\beta\mu\rho}$$

The formula for the probability that the fracture will strike a new lens when propagating follows from (18):

$$P_{>0}(M, \rho, \alpha) = 1 - e^{-n\beta\mu\rho}$$

For the above-described example of the Priobskoe field area, the probability that the fracture will strike a new lens depends on the fracture length and the lens shape as presented in the table:

Fracture length, m	50	100	150	200	250	300
Elliptical lenses	0.02	0.05	0.07	0.09	0.11	0.14
Rectangular lenses	0.03	0.06	0.09	0.12	0.14	0.17

Estimation of unknown characteristics for lens system

The probabilistic description of the lens system includes the shape and the sizes of the area, the shape of lenses, the probability densities of lens location and of their sizes, total number of lenses in the domain. These characteristics can be unknown quantitatively. For their estimation it is proposed to use the results of the experimental game for realization of the event π , that is determined by the structural matrix π constructed for m exploratory wells located in the area in points $M_1(x_1, y_1)$, $M_2(x_2, y_2)$, ..., $M_m(x_m, y_m)$. This inverse problem can be solved by the maximum likelihood method. This is the most convenient method of mathematical statistics and in most cases it gives estimations with the best statistic properties [14]. Substitution of the particular numerical values specified by well coordinates and respective π matrix in the above-described formulae and in the algorithms for $P(\pi)$ probability calculation enables a likelihood function depending on all the unknown parameters to be obtained. To estimate the unknown characteristics, there should be taken such numerical values of them that the likelihood function is maximal. These values can be found as a result of solving the optimization problem.

If all characteristics with the exception of the total number n of lenses in the area are known, the likelihood estimation of the number of lenses can be found as a result of optimization of the expression (10) for the probability $P(\pi)$ by n :

$$n = \left[\frac{n_1}{1 - P(0, 0, \dots, 0)} \right] \quad (20)$$

Here n_1 is the number of lenses crossed by currently available wells; $P(0, 0, \dots, 0)$ is determined by (9); the square brackets specify the integer part of the number.

For the above-described Priobskoe field domain, only 11 lenses were found by 4 exploratory wells located in this area. But the estimation (20) of the total number of lenses is $n=24$.

Conclusions

1. Some direct and inverse problems of the "lenses-wells" system characterization are solved. Both the probability of making an empty hole and the probability of missing a sand body by a group of exploratory wells are estimated. For hydraulic fractures of different length, the formula for probability that the fracture will strike a new lens was obtained. The probability of the event that the fracture will not cross any lens exponentially decreases with increasing of both the fracture length and the number of lenses; it also decreases as the lens sizes increase.
2. Maximal likelihood estimates of unknown characteristics of lens system can be used in Monte-Carlo simulation of lenticular reservoirs for the recovery prediction.
3. The obtained results can be used as a basis for developing of algorithms of sequential optimization of well spacing. While realizing the step-by-step strategy of new exploratory well spacing, it is necessary to maximize the amount of information about the statistical characteristics of the lens system. When spacing production wells, the probability of missing a lens must be taken into account.

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