E52 UNCERTAINTY PROPAGATION IN THE WELL LOG INTERPRETATION PROCESS

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Abstract

The log interpretation process is based on data acquired under environmental and thermodynamic conditions which may potentially affect the accuracy and the precision of the measurement system. All the possible sources of errors should be recognized and classified as systematic errors or random errors. Systematic errors should be corrected before interpretation whereas random errors propagation should be accounted for in the calculation process to evaluate the uncertainty associated to the final results.

Uncertainty propagation can be calculated by two different approaches: the analytical method, and the numerical method. Both the analytical and the numerical methods were applied to the deterministic log interpretation process in order to select the most suitable approach to evaluate the uncertainty affecting the reservoir petrophysical characterization.

The results of the study showed that, depending on the rock and fluid properties, the analytical approach may overestimate or underestimate the uncertainties associated to porosity and water saturation values and may lead to results that do not honor the volume balance equations.

Introduction

In the log interpretation process all the available information are combined together to obtain the sought petrophysical parameters, i.e. porosity and water saturation. Each information can be affected by errors of different nature like errors due to the calibration of the measuring instruments, errors associated to an incorrect choice of the interpretation models or to the model parameters, and errors due to the physics of the measurement system. Each source of error has to be characterized, classified, and possibly quantified so that some of the errors associated to input data can be avoided or corrected. The remaining errors represent the uncertainty associated to input data, and their propagation to the results of the interpretation process should be accounted for. This work was focused on evaluating whether the analytical method or the numerical method is the most suitable approach for propagating the uncertainties throughout the log interpretation process.

Overview of the errors affecting the log interpretation process

As a first step errors can be classified in errors associated to the selection of the interpretation models, also termed methodological errors, and errors associated to the input data.

The model selection represents a key issue in the log interpretation process because the models applied to correlate log measurements and core data to the sought reservoir properties were

derived under simplifying hypotheses of the real system, thus potentially introducing and propagating errors to the calculated results. Potential errors associated to an incorrect choice of the interpretation models cannot be quantified because the magnitude of the errors may vary significantly depending on the adequacy of the models to represent the real system. Nevertheless, most methodological errors can be corrected prior to any calculation with the aid of appropriate guidelines drawn on the basis of the applicability limits and constraints underlying each model.

The errors associated to the input parameters can be classified in systematic errors and random errors. A systematic error is a reproducible inaccuracy of the measurement system due to different causes as inadequate calibration, change in borehole environment, etc. (Theys, 1991). Once recognized, systematic errors can be quantified and removed from field or lab measurements. For instance, environmental conditions that can cause a shift, different for each log, between the measured and the actual value of the rock physical property that the tool is supposed to record. This shift, which represents a systematic error, has to be quantified and corrected before beginning the log interpretation process.

The remaining errors affecting input data are random errors, which are mostly imputable to the physics of the measurement system and cannot be corrected because they cannot be reproduced. Therefore, each possible source of random errors needs to be recognized and random errors accounted for in the calculation process (Theys, 1991).

Application of probabilistic approaches for uncertainty evaluation

Estimation of random errors is defined as uncertainty, and it is common practice to describe the uncertainty associated to any measurement or any result of a calculation process as a statistical distribution. Uncertainty propagation can be taken into account by application of a probabilistic approach. In the log interpretation process two probabilistic approaches can be used, the analytical method and the numerical method based on the Monte Carlo simulation.

The analytical approach is based on the general formula of the error propagation developed assuming that the uncertainties associated to both input data and results obey a normal distribution (Flenda et al., 1969). The analytical formula of the error propagation is obtained approximating the Taylor series to the first order and, therefore, assuming that errors are small. If it is possible to describe a parameter y as an analytical function of several parameters x_i :

$$y = f(x_1, \dots, x_i, \dots, x_n) \quad (1)$$

the uncertainty associated to y , σ_y , due to the uncertainty associated to each input parameter can be expressed as:

$$\sigma_y^2 \approx \sum_{i=1}^n \sigma_{x_i}^2 \left(\frac{\partial y}{\partial x_i} \right)^2 \quad (2)$$

where σ_{x_i} is the uncertainty associated to the i -th input datum. Eq. (2) is rigorously true only if eq. (1) is linear:

$$y = a_1 x_1 + \dots + a_i x_i + \dots + a_n x_n \quad (3)$$

The uncertainty associated to the output parameter can be calculated as:

$$\sigma_y^2 = a_1^2 \sigma_1^2 + \dots + a_i^2 \sigma_i^2 + \dots + a_n^2 \sigma_n^2 \quad (4)$$

Eq. (4) indicates that any shift or change of scale assigned to the position index causes a shift and a change of scale of each point of the distribution, and thus the shape parameter of the shifted

and scaled distribution does not change. If one or more assumptions under which the formula of error propagation was developed are not verified, the shape of the distribution associated to the dependent variable could be different from normal. In general, given a known uncertainty of the input parameters, the analytical approach provides a reasonable but not accurate evaluation of the uncertainty associated to the output parameter.

The numerical approach is based on the application of the Monte Carlo method which provides the possibility to simulate a high number of measurements that cannot be performed in common practice due to economical reasons. In all Monte Carlo simulations it is necessary to draw statistically representative samples from given probability distributions (Spanier et al., 1969). The results of the process are computed for each sampling so that after a reasonable number of sampling a probability distribution of results can be defined. Thus, a suitable number of samples need to be selected in order to obtain stable and reliable results in a reasonable elaboration time. As an example, fig. 1 shows that a minimum number of 500 samples need to be drawn for the mean and standard deviation of water saturation to reach stabilization. A number of 1000 samples was judged satisfactory for all the Monte Carlo simulations of the log interpretation process.

The Monte Carlo simulations were performed in compliance with the volume balance equation of the system, thus imposing that the sum of the components of the system be equal unity as well as the sum of the saturating fluids:

$$\sum V_i + \phi = 1 \quad i = 1 \dots n \quad (5)$$

$$S_g + S_o + S_w = 1 \quad (6)$$

where the following constraints apply:

$$\begin{cases} 0 \leq V_i \leq 1 \\ 0 \leq \phi \leq 1 \\ 0 \leq S_i \leq 1 \end{cases} \quad (7)$$

Therefore, the calculated value of porosity or water saturation was automatically corrected whenever the limits indicated by eq. (7) were not respected. As a consequence, for high water saturation values the uncertainty calculated with the Monte Carlo method is smaller than the uncertainty calculated by the analytical method because all the saturation values exceeding unity are set equal to one, thus the dispersion is strongly reduced.

The analytical and the numerical approaches were applied to the conventional deterministic log interpretation process solely considering the uncertainties associated to log readings. The other parameters, i.e. the Archie's parameters, were assumed to be certain. The uncertainty associated to the log measurements was represented by a normal statistical distribution where the measured value represents the mean value and the standard deviation is equal to the instrumental error. The instrumental error associated to each log type is given in table 1.

Table 1: instrumental errors (Giaccardo, 1999 modified)

Log	GR	CNL	R _t	Sonic Log	LDT
Standard Deviation	±5%	±7%	±10%	±5%	±0.015 (g/cm ³)

Rt modeling

The water saturation determination is often performed based on the deep resistivity array taken as the true resistivity profile. However, resistivity measurements are strongly affected by the layering of the formation, by the presence of a mud cake, and by mud invasion. Therefore, the true formation resistivity should be obtained by suitable modeling of the measured values. The modeled true resistivity is affected by an uncertainty that depends on several parameters, such as the uncertainty associated to log measurements, resistivity and geometry of the invasion zone, and inversion algorithm. Several simulations were performed in order to characterize the uncertainty affecting the true resistivity adopted in the log interpretation process. As a first step, log responses of several synthetic formations were generated by forward modeling. Each resistivity log response was then repeatedly modified to account for uncertainty, and the obtained resistivity profiles were inverted in order to obtain the corresponding true resistivity profiles. Results indicated that the error affecting the calculated water saturation values when adopting the deep resistivity curve as the true resistivity approximately ranges between 3% to 40%, depending on the thickness of the layer and on the resistivity contrast between the hydrocarbon bearing layer and the shoulder beds. Furthermore, the uncertainty related to the true resistivity is not always represented by a normal distribution. A comparison between the discrete distribution associated to the modeled true resistivity and the normal distribution representing the instrumental accuracy is shown in fig. 2. Also, the selection of the invasion model has a strong impact on the calculated resistivity profile.

Shale volume calculation

A shale indicator is a conventional log or log combination whose response equations can incorporate a shale fraction term. It is a common practice to determine several estimates of V_{sh} and then to select the lowest value as the best estimate at a particular level (Worthington, 1985). The most common shale indicators are derived from GR log. A comparison between the results provided by the analytical approach and the numerical approach applied for three GR shale indicators (Western Atlas, 1985) is shown in fig. 3. The standard deviations calculated by the two approaches are in good agreement except than for high shale volume values where the Monte Carlo approach truncates the probability distribution according to eq. (7). When several shale indicators are applied, the calculated uncertainty is comprised between the uncertainty associated to each selected shale indicator (fig. 4).

Total and effective porosity calculation

The total porosity is typically computed by solving a linear system of equations based on the clean matrix and fluid log readings. It is common practice to compute the effective porosity from a density-neutron Crossplot selecting a clean matrix point, a fluid point, and a shale point (La Vigne, 1994). Both the total and effective porosity models are linear relations, and show the same structure of eq. (3). Therefore, the results provided by the numerical and analytical approaches are generally in good agreement except for very low porosity values. When porosity approaches zero the Monte Carlo method modifies the distributions in order to obey eq. (7) and, therefore, the estimated uncertainty is more accurate with respect to the analytical method. In fact, the uncertainty calculated with the analytical method is overestimated (Verga et al., 2002).

Water saturation calculation

Since resistivity logs have a markedly nonlinear response, the condition of linearity expressed by eq. (3) is not rigorously respected by any water saturation model (Doveton, 1994).

According to the analytical method the uncertainty associated to any shaly-sand water saturation model due to the uncertainty affecting porosity, resistivity, and shale volume is:

$$\sigma_{S_w} = \sqrt{\left(\frac{\partial S_w}{\partial \phi}\right)^2 \sigma_{\phi}^2 + \left(\frac{\partial S_w}{\partial R_T}\right)^2 \sigma_{R_T}^2 + \left(\frac{\partial S_w}{\partial V_{sh}}\right)^2 \sigma_{V_{sh}}^2} \quad (8)$$

Assuming that R_T keeps constant over a wide porosity range, the relationship between porosity and water saturation is represented by a branch of hyperbola having porosity as the only unknown ($S_w = f(\phi)$) (fig. 5). Considering two increments $\Delta\phi^-$ and $\Delta\phi^+$, with the same amplitude, taken respectively to the left and to the right of the porosity value:

$$\Delta\phi^+ = \Delta\phi^- \quad (9)$$

the correspondent water saturation values are:

$$S_w + \Delta S_w^+ = f(\phi - \Delta\phi^-) \quad (10)$$

$$S_w - \Delta S_w^- = f(\phi + \Delta\phi^+) \quad (11)$$

where, as it is shown in fig. 5:

$$\Delta S_w^- \neq \Delta S_w^+ \quad (12)$$

If the uncertainty is small, the relationship between porosity and water saturation can be approximated with a straight line and the results provided by the analytical method and the numerical method are in good agreement:

$$\Delta S_w^- \approx \Delta S_w^+ \quad (13)$$

Similar considerations apply to the shale volume, and to the true formation resistivity. Nevertheless, it rarely occurs that the uncertainties affecting each input quantity showing in the water saturation models are small at the same time, and therefore eq. (7) cannot be taken as a general rule. A comparison between the results provided by the analytical method and the numerical method for Archie's equation, assuming that porosity is the only uncertain parameter, is shown in fig. 6. The results provided by the two methods are in good agreement for high porosity values, i.e. when eq. (13) is verified, and are strongly different for low porosity values where statistical distributions deviate from a normal shape and need to be truncated in order to respect eq. (7). The simulations were performed assuming that the Archie's parameters were not affected by uncertainty. In the future developments of this work also the parameter uncertainties will be taken into account considering the correlation between the parameters. However, some sensitivity analyses were performed in order to evaluate the impact of the Archie's parameters on water saturation. It was shown that the water saturation exponent has the effect of modifying the curvature of the hyperbolic branch and therefore, of enhancing or reducing the deviation from the normal shape of the statistical distributions associated to water saturation. Similar consideration can be carried out for the porosity exponent (fig. 7) which mostly influences the low porosity zones. The impact of the saturation exponent n is higher for the low water saturation values/high porosity values (fig. 8).

Application of the methodology

Both the analytical method and the numerical approach were applied to the petrophysical interpretation of several shaly-sand formations. Fig. 9 and 10 show an example of the results provided by application of the two methods. The color scale of both pictures ranges between blue, corresponding to a zero probability value and red, corresponding to the maximum probability value. Although the results provided by the two approaches seem similar, it is possible to note that the left tails of the distributions provided by the analytical method are longer than the left tails of the distributions provided by the numerical method. This is due to the

not-Gaussian shape of the statistical distributions characterizing the water saturation and, in turn, to the non-linearity of the model. Moreover, the uncertainty associated to the water saturation values approaching unity is very high according to the analytical method and is about zero according to the numerical method. Fig. 11 shows the results provided by the application of the Monte Carlo simulation to a formation, characterized by high porosity values, for which Archie's parameter were estimated equal to $a = 0.81$, $n = 2$ and $m = 2$. Fig. 12 shows the results provided by the application of the Monte Carlo method to a low porosity formation for which Archie's parameters were $a = 1$, $m = 2$, and $n = 1.22$. A comparison between results confirmed that uncertainty is greater in low porosity formations than in high porosity formations.

Conclusions

Uncertainty evaluation requires that all data be corrected for any source of systematic error and methodological errors. Subsequently, the evaluation of the uncertainty can be performed adopting two different probabilistic approaches, the Monte Carlo and the analytical methods. The analytical method was developed under several simplifying hypotheses, whereas the Monte Carlo method does not assume any simplification and therefore provides the rigorous solution. A systematic and accurate comparison between the results provided by the two methodologies was performed. Although in some circumstances, i.e.: for particular combinations of the petrophysical parameters, the results provided by the two approaches are in good agreement, the approximations introduced by the analytical method can lead to an overestimation or underestimation of the uncertainty affecting the results of the log interpretation process. It was demonstrated that the error associated to the true resistivity does not always have a normal distribution according to the results provided by the resistivity modeling. Application of the statistical approach to several shaly-sand formations showed that the uncertainty associated to the calculated water saturation is strongly affected by formation petrophysical properties like the porosity range, resistivity, and Archie's parameters, and needs to be carefully assessed by application of a statistical approach.

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Fig. 1: evaluation of the minimum number of samples for the Monte Carlo simulations of water saturation

Fig. 2: comparison between instrumental error and uncertainty for true the resistivity

Fig. 3: calculated standard deviations for the GR shale volume indicators

Fig. 4: Monte Carlo method: standard deviation associated to the shale volume

Fig. 5: relationship between water saturation and porosity

Fig. 6: water saturation standard deviation as a function of porosity

Fig. 7: impact of the porosity exponent in the porosity - saturation relationship

Fig. 8: impact of the saturation exponent in the porosity - saturation relationship

Figure 9: probability associated to a water saturation profile as provided by the analytical method

Figure 10: probability associated to a water saturation profile as provided by the numerical method

Fig. 11: uncertainty associated to a water saturation profile of a high porosity formation

Fig. 12: uncertainty associated to a water saturation profile of a low porosity formation