On the Importance of Correct Inclusion of Capillary Pressure in Reservoir Simulation

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ABSTRACT

Capillary pressure is frequently omitted from large scale reservoir simulation, rationalized by arguments that may not always be valid. A number of papers have shown that correct modelling of capillary pressure may be as important for the simulation result as properly accounting for relative permeabilities. Especially in strongly heterogeneous models it is important to study the sensitivity to all petrophysical parameters before simplifications are introduced in the simulation model.

In a previous paper from Norsk Hydro [1] it was demonstrated and explained why stochastic oil/water relative permeabilities (varying from cell to cell) in many cases can be replaced with one pair of "average" relative permeabilities without significantly changing the simulation results. Our current study indicates that a similar procedure is not always valid for capillary pressure.

A number of simulations exemplifies the effects of stochastic capillary pressure in various displacement regimes in a realistic geostatistical reservoir model:

- Gravity dominated water/oil displacement
- Viscous dominated water/oil displacement
- Varying water/oil mobility ratios
- Including/excluding capillary pressure
- Stochastic vs. constant capillary pressure
- Grid sensitivities

Even if the simulation results can be explained by well understood physical theory, our study emphasizes that the net effect of capillary pressure on the oil production curve is in general difficult to predict without numerical simulation on a fine grid: Front smearing tends to give earlier water breakthrough while suppressing of channelling/fingering will have the opposite effect. These true physical effects can be obscured by the numerical diffusion introduced by the finite difference scheme applied in the reservoir simulator.

INTRODUCTION

Water/oil flow in a reservoir under production is governed by the interaction of viscous, gravity, and capillary forces. Under the assumption of incompressible, isothermal, immiscible water-oil displacement; the volume conservation equation related to the location $\mathbf{x}$ can be expressed as

$$\phi \frac{\partial S_w}{\partial t} + \nabla \cdot \{ f(S_w) \mathbf{u}(\mathbf{x}) \} + \nabla \cdot \mathbf{K} \lambda_o f(S_w) \{ (\rho_w - \rho_o) g \mathbf{k} + \nabla P_c \} = q(\mathbf{x}) \quad (1)$$

where the source/sink term $q(\mathbf{x})$ is equal to zero everywhere outside the wells. The second, third, and fourth term represent the flow caused by the viscous, gravity, and capillary pressure, respectively.

A necessary condition for obtaining a contribution from the capillary term is that $\nabla P_c = \frac{dP_c}{dS_w} \nabla S_w \neq 0$, which implies that water saturation must vary in space and capillary pressure must vary with saturation. There are three situations where water (or oil) saturation can change abruptly if no capillary forces are present:

- Across a viscous generated shock front, described by Buckley-Leverett theory
- Gravity segregation of phases with different density
- Heterogeneity induced separation of phases

In these cases capillary pressure tend to dampen the saturation shocks, leading to "transition zones" with smoothly varying saturation. However, if capillary pressure is the dominating force, the limiting equilibrium situation is constant capillary pressure throughout the reservoir. Since capillary pressure correlates strongly with permeability and porosity, regions of different rock properties may then have different saturations.

In a completely homogeneous reservoir, where all the rock and fluid parameters are independent of position, it is possible to introduce a set of dimensionless

numbers which completely determines the two phase flow behavior. This problem was solved already by Rapoport [2] in 1955.

In this study we shall restrict ourselves to a two-dimensional rectangular reservoir with horizontal length L and vertical thickness H . The reservoir has an injection well in the leftmost column and a producing well in the rightmost column, and is produced with a constant (reservoir) injection/production rate q , which corresponds to a constant horizontal total Darcy velocity u_x . Then, four dimensionless numbers essentially determines the flow:

$$\begin{aligned} \text{Shape factor:} \quad R_l &= \frac{L}{H} \sqrt{\frac{k_x}{k_z}} \\ \text{Gravity number:} \quad N_g &= \frac{(\rho_w - \rho_o)gHk_x}{Lu_x\mu_o} = \frac{\Delta\rho gH}{\Delta p} \\ \text{Capillary number:} \quad N_c &= \frac{p_e^* k_x}{Lu_x\mu_o} = \frac{p_e^*}{\Delta p} \\ \text{Endpoint mobility ratio:} \quad M &= \frac{k_{rw}(S_{orw})/\mu_w}{k_{ro}(S_{wr})/\mu_o} \end{aligned}$$

While Shape factor (Aspect ratio) and Gravity number characterize single phase flow, Capillary number and Endpoint mobility ratio include two phase flow effects.

Zhou, Fayers, and Orr [3] among others have proposed a modification of the gravity and capillary scaling groups:

$$N_{gv} = N_g R_l^2 = \frac{T_h}{T_{gv}}$$

$$N_{cv} = N_c R_l^2 = \frac{T_h}{T_{cv}}$$

Here, T_h is the time for oil (one-phase) to flow across the reservoir in the horizontal direction, caused by the viscous force (horizontal pressure drop). T_{gv} and T_{cv} are the times for oil to flow across the reservoir in the vertical direction caused by the gravity and capillary force, respectively.

Condition for viscous dominated flow:

$$N_{gv}, N_{cv} \ll 1.0$$

Condition for gravity dominated flow:

$$N_{gv} \gg 1.0 \gg N_{cv}$$

Condition for capillary dominated flow:

$$N_{cv} \gg 1.0 \gg N_{gv}$$

During the last 15 years a number of papers have extended the dimensionless or inspectional analysis to heterogeneous reservoirs. The first simple approximation is to substitute the constant horizontal and vertical permeabilities (k_x , k_z) with their averaged counterparts; calculated by analytical formulas or simulated (- as explained by Grindheim and Aasen[4]). However, this approach may lead to wrong conclusions in cases with strong local variations in the flow properties.

Yokohama and Lake [5] proved that capillary crossflow in a layered reservoir (with permeability contrasts and communication between layers) is correlated with the number N_{cv} ; the larger the value of N_{cv} , the less is the tendency for fast channeling through the high permeable layers and the longer is the oil production plateau. But capillary pressure may also have an opposite effect. Smearing of the main saturation shock front may lead to faster water breakthrough. For a constant aspect ratio, large values of N_c enhance this dispersive effect.

In a realistic geologic heterogeneity model there are mixtures of more or less spatial correlated heterogeneities on different scales, leading to a very tortuous flow. Then, there is no more a constant horizontal Darcy velocity u_x and formulas based on average properties may break down. Li and Lake [6] have generalized the previous work to stochastic reservoir models with permeability variation and spatial correlation on three scales; local, zonal and global. Each scale has its own flow behavior. Local and global heterogeneities cause fingering and dispersive flow, while zonal heterogeneity causes channeling.

Langlo and Espedal [7] have studied the interplay between the capillary pressure and heterogeneity generated saturation fingers. If the scale of capillary diffusion is smaller than the scale of the heterogeneity, capillary diffusion is insignificant compared with the heterogeneous fingering. If the capillary diffusion is on the same scale as the heterogeneity, the heterogeneous fingering is reduced. For even larger scale capillary diffusion the heterogeneous fingering can be neglected.

A very important important capillary effect is highlighted in a number of papers from Heriot-Watt University [8, 9, 10]: In certain small scale (mm/cm/dm) periodic heterogeneity patterns, significant amounts of mobile oil may be trapped in the high permeability parts, leading to a much higher remaining oil saturation than what is predicted from viscous dominated core floods. This mechanism has been demonstrated both with fine grid numerical simulations and experimental work. The implication for field scale numerical simulation is that small scale capillary trapping must be included in the (pseudo) saturation dependent functions applied in the large grid. This is an example of a general problem in numerical simulation: Physical effects on a scale smaller than the grid block dimensions can only be accounted for by pseudo parameters.

In this work we concentrate on demonstrating/quantifying capillary effects in simulation models with grid block dimensions 4 - 20 m in the x-directions and 0.4 - 2 m in the z-direction. Angert and Begg [11] and Fitzmorris, Kelsey and Pande [12] have performed similar studies with capillary pressure included in their models. Both papers conclude that capillary pressure may have significant effects both on water breakthrough and total oil recovery. The production

profile can be positively or negatively affected, depending on the heterogeneity model and the balance of forces.

AIM OF STUDY

An earlier study from Norsk Hydro [1] concerning effects of stochastic relative permeabilities in reservoir simulation, demonstrated that in many cases the oil production profile is not significantly altered if only one pair of average curves is applied throughout the simulation grid. In order to produce a realistic pair of average relative permeability curves one need enough measurements to cover the geological variation; especially it is important to obtain realistic estimates of the mean values of initial water saturation and residual oil saturation since these parameters determine the mobile oil volume. It should be emphasized that the cited result is only valid for simple models covering one depositional unit. The result supports the common praxis of applying one pair of relative permeability curves for each geological unit in fine scale reservoir simulation. However, since pseudo curves are not applied, no effects of heterogeneities on smaller scale than the gridblock dimensions are included.

It was observed that in the experimental dataset there is no significant correlation between absolute permeability and the water/oil shock front velocity. This fact inspired Holden [13] to investigate a generalization of the Buckley-Leverett model to a stochastic 1D porous medium. Essentially he proved that one pair of average relative permeability curves can replace spatially varying relative permeability curves as long as there is no spatial correlation between the curves.

In the previous study capillary pressure was not included in the simulation model. The next step will then be to investigate capillary pressure effects in the simulation results:

- What are the important effects of capillary pressure under various balances of viscous, gravity and capillary forces?
- Capillary pressure is determined by a formula including absolute permeability, porosity, and initial water saturation. What is the implication of substituting varying capillary pressure with one average curve?
- How is capillary pressure affecting the spread in production profiles from several realizations of the same geostatistical model?
- How does grid refinement affect the simulation results?

SIMULATION MODEL

Our work is a continuation of earlier studies from Norsk Hydro [1, 14]. Based on measurements on 40 core plugs from a North Sea well in a Distal Mouthbar

deposit, a geostatistical model has been developed. A simplified 2D representation is a rectangle of dimensions $L = 720\text{m}$ and $H = 96\text{m}$ where the spatial variability of the various petrophysical variables is governed by a simple Gaussian random field model. The mean and the variance is given for each variable, as well as the range in a common spherical variogram. The correlation lengths in the horizontal and vertical directions are set equal to L and $H/4$, respectively. Fig. 1 shows the distribution of horizontal permeability for a given realization.

Also experimentally obtained correlations between petrophysical parameters are respected in the model construction. There is a Pearson correlation of 0.7 between horizontal and vertical permeabilities, with a factor 10 between the means. Tables 1 and 2 summarize the parameter variations and correlations. Even within a fixed depositional environment there are large parameter variations. One observes as expected, strong correlations between $\log k_x$ and S_{wi} , $\log k_x$ and $k_{rw}(S_{orw})$. Most parameter pairs, however, are only weakly correlated.

Capillary pressure is given by a deterministic formula obtained from multivariate analysis between logs and core measurements [15]:

$$P_c(S_w) = 0.0865\phi^{-0.906}k^{-0.109}S_w^{*-0.744} - 0.192 \quad (2)$$

where S_w^* is normalized water saturation defined by

$$S_w^* = \frac{S_w - S_{wi}}{1 - S_{orw} - S_{wi}} \quad (3)$$

When S_w tends towards S_{wi} the capillary pressure tends towards infinity. To obtain a finite value for $P(S_{wi})$ we define (arbitrarily) $P(S_{wi}) = P(S_{wi} + 0.01)$

The measured capillary pressure drainage curves are obtained from porous plate experiments. The imbibition curves may differ substantially from the drainage curves, but in this generic study we have chosen to apply the drainage curves also for water displacing oil.

Some simulator runs are performed with constant relative permeability curves obtained by arithmetical averaging of each rel.perm. parameter over the simulation grid. The corresponding capillary pressure is obtained by applying the formula above with mean values of porosity, permeability and initial water saturation.

We have simulated cases with equal oil and water densities at reservoir conditions, and also cases with significant gravity effects ($\Delta\rho = 0.156 \text{ kg/m}^3$). To study the influence of mobility ratio we have applied $M = 1$ and $M = 10$.

The gravity numbers in these models are $N_{gv} \approx 20$ for $M = 1$ and $N_{gv} \approx 2$ for $M = 10$, indicating strong gravity influenced flow. The corresponding capillary numbers are $N_{cv} \approx 2$ and $N_{cv} \approx 0.2$, indicating substantial effects of capillary pressure.

Three different cartesian regular grids have been applied: 36×48 , 180×48 , and 36×240 . When comparing runs with fine and coarse grids, absolute permeabilities are always scaled from the fine to the coarse grid by a single phase simulation method.

When comparing several realizations from the geostatistical model the total pore volume has been scaled in order to maintain identical movable pore volumes.

The simulator applied is commercial software (ECLIPSE) with one-point upstream weighting and fully implicit solution of the system of pressure and saturation equations. The simulator is run with constant and equal injection/production rates corresponding to 5% pore volume per year. Tables 3 and 4 gives an overview of the various runs with corresponding cumulative oil production at water breakthrough and after 15 years of production.

RESULTS

The main emphasize is on results with and without capillary pressure and comparison of constant vs. varying capillary pressure. We also consider grid refinement. Table 3 summarizes the different simulation runs and the main production results.

No gravity effects: In order to cancel any gravity effect the oil and water densities (- at reservoir conditions) are assumed identical. These runs are performed on a 180×48 grid, that is, with grid refinement in the horizontal direction in order to allow for detailed modeling of channeling/fingering. Fig. 2 shows that in all cases there are some differences between the oil production profiles with/without capillary pressure for a period just after water breakthrough. However, there is no significant effect on the total cumulative oil production. For unit mobility ratio there is a 2 years difference in water breakthrough times with constant and variable saturation dependent functions.

Fig. 3 illustrates the effect of (varying) capillary pressure in the case with $M = 10$. While there are three pronounced channels without capillary pressure, capillary crossflow "connect" the channels. The effect on the oil production profiles, however, is less than what could be expected from the saturation plots.

Fig. 4 shows saturation plots for the cases with unit mobility ratio, with constant and variable capillary pressure. The plots confirm that after 6 years the water front has not reached the producer when capillary pressure is constant, but with varying capillary pressure there is more dispersion and faster water breakthrough.

Different oil and water densities: In order to obtain a detailed resolution in the vertical direction the grid has 36×240 blocks. Fig. 5 shows that with spatially constant relative permeabilities there is no effect of capillary pressure when $M = 1$. When $M = 10$

constant capillary pressure gives later water breakthrough and significantly higher cumulative oil production than the corresponding case without capillary pressure.

With varying relative permeabilities, water breakthrough arrives first when capillary pressure is included. For $M = 1$ there is also significantly higher cumulative oil production when capillary pressure is not included.

Fig. 6 shows saturation plots for the case with $M = 10$, constant relative permeability, with/without (constant) capillary pressure. Without capillary pressure there is a lot of channeling/fingering and not a very pronounced effect of gravity. Capillary crossflow together with gravity segregation change the picture. Now there is a gravity tongue developing, but still with significant channeling behind the primary water front. In both cases there is a high degree of dispersion.

Different Realizations: 5 realizations with the same geostochastic geological model have been simulated. Fig. 7 shows an example (Constant rel.perm., $\Delta\rho > 0$, $M = 10$) where capillary pressure significantly reduces the variation in the production profiles compared to the similar runs where the capillary pressure is not included. However, this is not a general result. In the case with variable rel.perm., no gravity effects and $M = 1$; fig. 8 shows that the variation in the production profiles has not decreased when capillary pressure is included.

Grid Sensitivity: The 180×48 and 36×240 grids are reduced to a 36×48 grid by compressing five and five blocks in the horizontal, respectively the vertical direction.

Comparing results with fine and coarse grids in the horizontal direction, only minor effects are observed regarding the oil production profiles for constant saturation dependent functions. With variable relative permeability and capillary pressure there is always a significant later water breakthrough in the fine grid compared with the coarse grid case (fig. 9). For grid sensitivity in the vertical direction, in those cases where there are differences between the fine and coarse grid simulation results, the fine grid production profiles always show the fastest water breakthrough.

Another observation is that the difference in production profiles between similar fine grid and coarse grid simulation are reduced when capillary pressure is included. Following the arguments in the paper by Lantz [16]; capillary and numerical diffusion add in the applied numerical scheme (one point upstream weighting, implicit solution). However, while numerical diffusion is proportional with the horizontal grid block dimension, capillary diffusion is larger the smaller the grid block size (since the first order approximation of ∇P_c is increased by reduced Δx)

CONCLUSIONS

The main conclusion is that capillary pressure may significantly affect the flow behavior. The capillary pressure effect can be much stronger in models with heterogeneities on different scales than in homogeneous or layered systems. The net effect on oil and water production depends on the balance between the capillary diffusion and capillary crossflow.

Observations from our simulation study can be summarized as follows:

- * Capillary pressure may have significant effects on simulated oil production profiles even with grid-block dimensions in the 10m range.
- * Large differences in local saturation distributions do not always show up in the oil production profiles.
- * Dimensionless capillary and gravity numbers give "rules of thumb" for the importance of capillary pressure in field scale simulations. Only in the gravity dominated case ($N_{gv} \approx 20$) there are no observable effects of capillary pressure on the oil production profile. In all other cases that have been investigated there are differences between runs with/without inclusion of capillary pressure.
- * There are significant differences between runs with constant and variable relative capillary pressure, implying that previously obtained results regarding relative permeability variation cannot be extended to capillary pressure.
- * Examples show that capillary pressure can both increase and reduce the variation in oil production profiles for a number of realizations from the same geostatistical model.
- * The sensitivity to grid coarsening is less with than without capillary pressure included in the simulation model.

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NOMENCLATURE

f	fractional flow
g	gravity acceleration
k	absolute permeability
k_r	relative permeability
n	Corey exponent
o	oil
orw	residual oil
w	water
w_i	initial water
q	source/sink term
u	scalar velocity

x	horizontal direction
z	vertical direction
H	reservoir thickness
L	reservoir length
M	mobility ratio
N	dimensionless number
P_c	capillary pressure
S	saturation
k	vertical unity vector
K	permeability tensor
u	total velocity vector
x	location vector
μ	viscosity
λ	mobility
ρ	density

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Table 1 Parameter statistics

Variable	N	Mean	St. dev.	Min.	Max.	Norm.*
ϕ	41	0.214	0.022	0.151	0.248	no
k_x	40	319	458	12	2823	no
$\log_e k_x$	40	5.26	0.99	2.51	7.94	
S_{wi}	41	0.221	0.074	0.070	0.488	
S_{orw}	41	0.266	0.049	0.200	0.392	no
n_o	40	3.14	0.63	2.24	5.33	no
$\log_e n_o$	40	1.11	0.19	0.81	1.67	
n_w	40	2.61	1.05	0.69	6.78	no
$\log_e n_w$	40	0.89	0.37	-0.37	1.91	
$k_{rw}(S_{or})$	41	0.39	0.15	0.06	0.69	

* "No" denotes significant deviation from normal distr.

Table 2 Correlation coefficients

	$\log_e k_H$	S_{wi}	S_{or}	$\log_e n_o$	$\log_e n_w$	$k_{rw}(S_{or})$
ϕ	0.44	-0.42	-0.02	-0.30	0.14	0.17
$\log_e k_H$		-0.77	0.19	0.52	-0.59	0.81
S_{wi}			-0.53	-0.37	0.58	-0.63
S_{orw}				0.21	-0.35	0.32
$\log_e n_o$					-0.73	0.67
$\log_e n_w$						-0.57

bold face indicates strong correlation between variables.

Table 3 Run summaries

Production Characteristics	Water Breakthrough Years		Total Oil Production $10^6 Sm^3$	
	Grid 180x48	Grid 36x48	Grid 180x48	Grid 36x48
Const.rel.perm.:				
$\Delta\rho = 0, w/P_c, M=1$	5.51	4.74	2.745	2.708
$\Delta\rho = 0, w/P_c, M=1$	5.61	4.96	2.802	2.781
$\Delta\rho = 0, w/P_c, M=10$	0.93	0.65	1.820	1.786
$\Delta\rho = 0, w/P_c, M=10$	0.95	0.66	1.853	1.837
Var.rel.perm.:				
$\Delta\rho = 0, w/P_c, M=1$	5.12	2.82	2.340	2.353
$\Delta\rho = 0, w/P_c, M=1$	3.59	2.60	2.363	2.362
$\Delta\rho = 0, w/P_c, M=10$	1.05	0.27	1.646	1.585
$\Delta\rho = 0, w/P_c, M=10$	0.99	0.84	1.671	1.613

Production Characteristics	Water Breakthrough Years		Total Oil Production $10^6 Sm^3$	
	Grid 36x240	Grid 36x48	Grid 36x240	Grid 36x48
Const.rel.perm.:				
$\Delta\rho \neq 0, w/P_c, M=1$	3.11	3.03	2.865	2.860
$\Delta\rho \neq 0, w/P_c, M=1$	3.21	3.21	2.860	2.857
$\Delta\rho \neq 0, w/P_c, M=10$	1.48	1.27	1.738	1.897
$\Delta\rho \neq 0, w/P_c, M=10$	1.60	1.22	1.896	1.899
Var.rel.perm.:				
$\Delta\rho \neq 0, w/P_c, M=1$	2.95	2.96	2.688	2.683
$\Delta\rho \neq 0, w/P_c, M=1$	1.83	2.39	2.514	2.549
$\Delta\rho \neq 0, w/P_c, M=10$	1.51	1.17	1.622	1.738
$\Delta\rho \neq 0, w/P_c, M=10$	0.84	1.22	1.586	1.655

Fig.1 Horizontal permeabilities, Grid 36 × 48

Fig.2 Effect of cap.pres. on oil production profile; no gravity; grid 180 × 48 (no cap.pres. —, with cap.press. ...)

Fig. 2a Constant $k_r, P_c; M = 1$

Fig. 2b Constant k_r, P_c ; $M = 10$

Fig. 2c Variable k_r, P_c ; $M = 1$

Fig. 2d Variable k_r, P_c ; $M = 10$

Fig. 3 Effect of cap. pres. on saturation distribution; no gravity; grid 180×48 ; variable k_r ; $M = 10$

Fig. 3a Saturation after 2 years; $P_c = 0$

Fig. 3b Saturation after 6 years; $P_c = 0$

Fig. 3c Saturation after 2 years; $P_c > 0$

Fig. 3d Saturation after 6 years; $P_c > 0$

Fig. 4 Effect of const./var. cap. pres on saturation distribution; no gravity; grid 180×48 ; $M = 1$

Fig. 4a Saturation after 6 years; constant k_r, P_c

Fig. 4b Saturation after 6 years; variable k_r, P_c

Fig. 5 Effect of cap. pres. on oil production profile; with gravity; grid 36×240 (no cap. pres. —, with cap. press. ...)

Fig. 5a Constant $k_r, P_c; M = 1$

Fig. 5b Variable $k_r, P_c; M = 1$

Fig. 5c Constant $k_r, P_c; M = 10$

Fig. 5d Variable $k_r, P_c; M = 10$

Fig. 6 Effect of no/const. cap. pres on saturation distribution; with gravity; grid $36 \times 240; M = 10$

SWAT at step 1 731. DAYS
CONTOUR INTERVAL 0.1000

Fig. 6a Saturation after 2 years; $P_c = 0$

SWAT at step 2 2192. DAYS
CONTOUR INTERVAL 0.1000

Fig. 6b Saturation after 6 years; $P_c = 0$

SWAT at step 1 731. DAYS
CONTOUR INTERVAL 0.1000

Fig. 6c Saturation after 2 years; constant k_r , P_c

SWAT at step 2 2192. DAYS
CONTOUR INTERVAL 0.1000

Fig. 6d Saturation after 6 years; constant k_r , P_c

Fig. 7 Five realizations; with gravity; grid 36×240 ; $M = 10$

Fig. 7a Constant k_r ; $P_c = 0$

Fig. 7b Constant k_r ; $P_c > 0$

Fig. 8 Five realizations; no gravity; grid 180×48 ; $M = 1$

Fig. 8a Variable k_r ; $P_c = 0$

Fig. 8b Variable k_r ; $P_c > 0$

Fig.9 Comparison of fine and coarse grid in the x-direction; no gravity; variable k_r , (grid 180×48 — , grid 36×48 ...)

Fig. 9a Variable k_r ; $P_c = 0$; $M = 1$

Fig. 9b Variable k_r ; $P_c = 0$; $M = 10$

Fig. 9c Variable k_r , P_c ; $M = 10$

Fig. 9d Variable k_r , P_c ; $M = 10$