Simplified Mechanistic Modelling of Foam Processes for Improved Oil Recovery

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ABSTRACT

This paper describes simplified mechanistic foam modeling using fractional-flow methods and a simulator that incorporates the fixed-limiting-capillary-pressure model for foams. The first part of the paper compares the predictions of fractional-flow methods to those of more-complex foam simulators under conditions of either steady foam flow or alternating injection of liquid and gas (SAG). The fractional-flow approach closely matches the simulator predictions and makes clearer the fundamental mechanisms that dominate foam behavior. Fractional-flow analysis shows that SAG processes depend on foam behavior at extremely low fractional flow of water.

Fractional-flow methods cannot describe two-dimensional displacements or complex reservoir geology. A body of evidence indicates that the collapse of foam at high capillary pressure is the dominating mechanism that controls foam behavior. The second part of this paper describes a simulator based on this mechanism that retains the heart of much-more-complex, fully mechanistic simulators while retaining the simplicity of purely empirical models. Using this simulator, we investigate dimensionless criteria for the foam success in countering gravity override and adverse permeability ratios between geological layers. These criteria focus on the relation between the pressure gradient driving the foam flow and the gravity or capillary driving forces acting normal to the overall direction of flow. Foam success depends critically on the allowable pressure rise at the injection well as well as the dimensions of the reservoir or geological layers.

INTRODUCTION

Foam is used in miscible and steam enhanced-oil-recovery processes to improve sweep efficiency,¹⁻⁴ in well stimulation to divert acid,⁵⁻⁹ and in production-well treatments to improve conformance or to prevent coning.^{10,11} Foams have the advantage that usually they are stronger in high-permeability layers than in low-permeability layers and thus improve the vertical sweep of the reservoir.^{4,6,12}

Predicting foam performance is complicated by the complex behavior of foams in rock. The term "foam mobility" is misleading; there is only the effect of foam on the separate mobilities of the gas and liquid phases. Foam has no direct impact on water mobility.¹³⁻¹⁷ Therefore, knowing the water relative-permeability function in the absence of foam and the water saturation in the presence of foam, one can determine the pressure gradient with foam:

$$\nabla P = u_w \frac{\mu_w}{k k_{rw}(S_w)} \quad (1)$$

where u_w is the water volumetric flux, μ_w water viscosity, k rock permeability and $k_{rw}(S_w)$ the water relative-permeability function. Water saturation S_w in the presence of foam, unfortunately, is not so easily predicted, because it depends on gas mobility.

Gas mobility with foam depends on the texture (bubble size) of foam inside rock,¹⁸ which in turn depends on local pore sizes and heterogeneity, surfactant formulation, flow rates

and, most importantly, capillary pressure.^{19,20} The higher the capillary pressure, the less stable are the foam lamellae that separate gas bubbles, the coarser the foam texture, the higher the gas mobility, and the higher is S_w at steady state.

Due to this complexity, modeling foam requires simplifying assumptions. Depending on these assumptions, there are several approaches to modeling available:

The simplest approach is to fit coreflood data to an empirical relation for foam mobility as a function of flow rates and other variables.²¹⁻²⁵ This approach can be risky when the relations are extrapolated beyond the range of the data.

A second approach is the "population balance," which aims to quantify the relation between foam mobility and texture and all the mechanisms of generation and destruction of lamellae that govern foam texture.^{13,18,26-28} Although this approach provides in principle a framework for a complete description of foam, each individual application is limited by the simplifying assumptions made. For instance, the various published examples^{13,18,26-28} differ in the mechanisms included. None of them yet quantify gas trapping as a function of pressure gradient, and all of them focus on only one of several mechanisms of lamella creation, repeated snap-off in pore throats.

A third approach is less complex than the population-balance method, but captures the determining mechanisms of foam mobility. The "fixed-limiting-capillary-pressure" or "fixed- P_c^* " model relies on the relation between capillary pressure, foam texture and foam mobility. It is a local-equilibrium version of the population-balance in which capillary pressure is assumed to dominate the texture and mobility of strong foams.²⁹ The transition between strong foam and foam collapse occurs over a narrow range of water saturation centered on the "limiting saturation" S_w^* , corresponding to the "limiting capillary pressure" P_c^* .^{19,29-31} The specific value of P_c^* depends on rock permeability, surfactant formulation and other factors. There is evidence that the less permeable the rock, the higher is P_c^* .²⁹

Fractional-flow methods can model any foam displacement in which phase saturations control mobilities, as in the fixed- P_c^* model.³²⁻³⁴ Fractional-flow methods predict the displacement of "saturation waves" through the rock from the fractional-flow curve for the process. A saturation wave propagates with velocity equal to the slope of the fractional-flow curve at that saturation. Wherever faster waves overtake slower waves in a displacement process, they form a shock front, or jump in saturation. "Chemical shocks," jumps in surfactant concentration, are derived from a mass balance

on surfactant. Fractional-flow methods are limited to one-dimensional (1D) displacements, assume local equilibrium and incompressible phases, and cannot model 2D phenomena like gravity override or fingering. A fuller description of the approach and its limitations can be found elsewhere.^{29,32-34}

In order to represent 2D or 3D displacements with gravity override, fingering or other complications, one must use numerical simulation. Therefore, we adapted the equation-of-state compositional simulator UTCOMP³⁵ for foam simulation based on the fixed- P_c^* model.³⁶ With this simulator, we have begun to investigate the factors controlling foam success in overcoming gravity override and diverting flow between layers with limited crossflow.

In the following, we illustrate the power of the fractional-flow approach applied to foams by comparing its predictions to those of much-more-complex foam simulators. The approach offers particular insights into the successful design of surfactant-alternating-gas (SAG) processes. We further describe initial investigations, using foam simulation, of dimensionless criteria for foam success in overcoming gravity override in homogeneous reservoirs and adverse permeability ratios in layered media.

APPLYING FRACTIONAL-FLOW METHODS TO DYNAMIC FOAM DISPLACEMENTS

Steady Foam Injection

Kovscek *et al.* co-injected gas and surfactant solution into linear Berea cores saturated with either brine or surfactant solution and then simulated these experiments using the population-balance model.²⁶⁻²⁸ One can obtain substantially the same fits with fractional-flow methods and the much-simpler fixed- P_c^* model. Figure 1 shows schematically the construction of the fractional-flow solution. Although the population-balance model has many foam parameters, one needs only one parameter for the fixed- P_c^* model in this case, in which adsorption is insignificant: S_w^* , which governs the liquid mobility behind the foam front. One can determine S_w^* from measured steady-state VP using equation (1). The liquid-relative-permeability function and phase viscosities are given in Table 1. The core was initially filled with surfactant ($S_w = 1$), and then surfactant and gas were injected in a volume ratio of 1:9.

Figure 2 compares the fits of the fixed- P_c^* and population-balance models to one linear coreflood. The fractional-flow results are adjusted for the gas compressibility using the ideal gas law. The fractional-flow approach with

the fixed- P_c^* model matches the data well except within the first 12 cm of the core; there the fractional-flow approach errs in assuming instantaneous attainment of steady-state mobilities. However, the fit of the much-more-complex population-balance model is less than quantitative there as well. Neither model fits at all the delay in foam generation past 0.23 PV injection.

Kovscek *et al.*²⁶⁻²⁸ plot foam texture as a function of position and time derived from the population-balance fit of these data. If desired, one could use the same equations to back-calculate foam texture from $\bar{V}_P$ obtained by the fractional-flow method in Figure 2. However, there are no direct measurements of foam texture to compare with these predictions; moreover, the value of tracking foam texture in this case is moot if it does not materially improve the prediction of foam mobility in the coreflood.

Kovscek *et al.*²⁶⁻²⁸ extrapolate these results to radial flow on a reservoir scale; some parameter values are given in Table 1. Figure 3 compares the predictions of population-balance and fractional-flow approaches. Again the fractional-flow results are adjusted for gas compressibility. There are no data for comparison in this case. The fractional-flow approach matches the population balance prediction with two exceptions: The difference in pressure near the well, corresponding to that near the inlet in Figure 2, would presumably be smaller upon refinement of the simulation grid there. The small deviation along the foam bank corresponds to less than one grid block in the position of the foam front in the simulation. This deviation may well be due to the uneven advance of foam that results from grid discretization in foam simulation.³⁶

Surfactant-Alternating-Gas (SAG) Processes

Fixed- P_c^* Model

Fractional-flow methods show that SAG processes depend critically on foam behavior at extremely low water fractional flow. Two examples illustrate this point.

The first example is based on a fractional-flow curve extrapolated from coreflood data of Persoff *et al.*³¹ using the fixed- P_c^* model. Details are given elsewhere.³³ Figure 4 illustrates construction of the fractional-flow solution. At the start of gas injection, surfactant solution fully saturates the porous medium. Due to the shape of the fractional-flow curve, the displacement is a shock from the initial state I to the point of complete foam collapse predicted by the fixed- P_c^* model. As a result, foam generation and collapse is confined to a shock front between the initial state I ahead of foam and dried-out, collapsed foam near the well.

TABLE 1
Parameter Values for Modeling Floods of Kovscek *et al.* (Ref. 26-28)

Linear Flood:

S_w^*	0.304
u_w	0.046 m/d
u_g	0.43 m/d
Core Length	0.6 m

Radial Flood

Well radius	0.1 m
Outer Boundary	25 m
S_w^*	0.296
Water injection rate	0.165 m ³ /d
Gas injection rate	3.14 m ³ /d
Reservoir radius	25 m

Parameters Common to Both Cases

Permeability	1.3 d
Porosity	0.25
Water viscosity	0.001 Pa s
Gas viscosity	1.8×10^{-5} Pa s
Back-pressure	4.8 MPa

$$k_{rw} \text{ function} \quad 0.7 \left(\frac{S_w - 0.25}{1 - 0.25} \right)^3$$

Figure 1. Construction of fractional-flow fit to data of Kovscek *et al.* (Ref. 28) and construction of waves for injection of surfactant and gas into surfactant-filled medium.

Figure 2. Fit of fractional-flow method and population-balance model to coreflood data of Kovscek *et al.* (Ref. 28). Parameter values are in Table 1. Data: circles. Population-balance model: dashed lines. Fractional-flow method: solid lines.

Figure 3. Fit of fractional-flow method to population-balance model of Kovscek *et al.* (Ref. 28) for radial flow of foam. Parameter values are in Table 1. Population-balance model: dashed lines. Fractional-flow model: solid lines.

Ironically, mobility is reduced somewhat more in lower-permeability layers, resulting in reverse diversion, from low-permeability to high-permeability layers.³³ This result depends on extrapolating Persoff *et al.*'s coreflood data with the fixed- P_c^* model; a marginally more successful process is consistent with these data extrapolated using other assumptions.³³

Model of Fisher *et al.*

Fisher *et al.*²⁴ discuss the evidence for and against the fixed- P_c^* model and propose an alternate expression for the gas mobility based on data from Lee and Heller¹² in effect, they replace the vertical portion of the fractional-flow curve in Figure 4 with an S-shaped curve whose steepness depends on surfactant concentration, as shown in Figure 5. In Fisher *et al.*'s model gas mobility with foam is also a weak function of the gas velocity,²⁴ which we ignore here. Figure 6 expands Figure 5 in the region of low water fractional-flow f_w crucial to SAG processes. Unlike the fixed- P_c^* model this curve is rounded at extremely low values of f_w , even at high surfactant concentration. There are, however, no data in this range; the model was fit to data collected between $0.1 < f_w < 0.44$.

Fisher *et al.* simulate injection of gas into a core fully saturated with a solution of 0.5% surfactant in water.²⁴ As in Figure 4, there is a shock front from the initial condition in the upper right corner of the fractional-flow diagram tangent to the fractional-flow curve at low f_w . Figure 6 shows the point of tangency and the spreading wave at low values of f_w . The wave has low mobility near the point of tangency and only gradually approaches the very high mobilities associated with complete foam collapse. Figure

TABLE 2
Parameter Values for Modeling Flood of Fisher *et al.* (Ref. 24)

Permeability	151 md
μ_g	2.83×10^{-6} m/s
Water viscosity	0.001 Pa s
k_{rw} function	$0.72 \left(\frac{S_w - 0.15}{1 - 0.15} \right)^2$
Core length	0.61 m

Figure 4. Fractional-flow solution for SAG process based on data of Persoff *et al.* (Ref. 31).

7 compares the fractional-flow prediction with the simulation of Fisher *et al.*²⁴ Again, the fractional-flow prediction has been adjusted for gas compression.³⁶ The two predictions are remarkably close. Comparing this result to that based on Persoff *et al.*'s data in Figure 4 makes clear that the success of the SAG processes depends on the foam behavior at very low f_w - below the range of the data on which either case was based.

If the success of foam SAG processes depends on having a curved fractional-flow curve at low f_w , then Figure 5 suggests that a process with a lower surfactant concentration might be even more successful. Figure 8 shows the fractional-flow prediction for a SAG process with a 0.02% surfactant concentration in the water. Surprisingly, the lower surfactant concentration performs better than the higher surfactant concentration (note the differences in scale in Figures 7 and 8). Figure 5 indicates that the higher surfactant concentration would give a stronger foam in conventional steady-state foam corefloods at fixed f_w . Yet in a SAG process, the lower surfactant concentration gives a higher, longer-lasting pressure build-up than the higher surfactant concentration. This reflects a slower drying out and collapse of the foam at lower surfactant concentration. The benefits of lower surfactant concentration in Figure 8 could of course also be obtained by a choice of a different surfactant formulation, one that gives a "weaker" foam in steady-state corefloods.

Others have suggested using weaker foams in the field in some foam processes to avoid problems caused by excessive foam strength.^{37,38} Our point is different: A foam considered weaker based on steady-state foam corefloods may actually give a stronger foam *in situ* in a SAG process, due to the shape of its fractional-flow curve and the rate at which it dries out and collapses.

Some published laboratory SAG coreflood data^{39,40} indicate long-lasting foam effectiveness. This may reflect a fractional-flow curve like that for the low surfactant concentration in Figure 5. However, laboratory corefloods can be affected by artifacts like slow foam generation (on the time scale of the coreflood) and the capillary end effect. Care is needed in extrapolating laboratory foam SAG coreflood data to field application.

Discussion

Fractional-flow methods can provide valuable insights into the mechanisms that control foam performance. For that reason, they should supplement any attempt to understand complex foam displacements modeled by simulation. A fractional-flow curve can be derived from any foam-mobility model in which mobilities depend

Figure 5. Foam fractional-flow curves based on model of Fisher *et al.* (Ref. 24).

Figure 6. Detail of Figure 5 at low water fractional flow and 0.5% surfactant concentration.

Figure 7. Fit of fractional-flow method to simulation of Fisher *et al.* (Ref. 24) for example in Figure 6.

Figure 8. Prediction of fractional-flow method for gas injection into core filled with 0.02% surfactant solution using model of Fisher *et al.* (Ref. 24). Parameter values are in Table 2.

phase saturations, though some simplifying assumptions (such as local equilibrium and absence of shear-thinning effects) may be required. In the examples above, the fractional-flow approach shows that a single mechanism, foam collapse at S_w^* , controls all the others in the complex population-balance-model simulation of the given coreflood; in other words, in at least this case, the other foam parameters in the population-balance model are unnecessary, and some may have large uncertainty. The fractional-flow approach also reveals that SAG

foam processes hinge on foam behavior under conditions of extremely low water fractional-flow, a range in which there may be few or no data. Fractional-flow methods quantitatively fit the predictions of much-more-complex simulators in some cases.

SIMPLIFIED MECHANISTIC FOAM SIMULATOR

Fractional-flow methods cannot describe processes where gas compressibility, shear thinning rheology, gravity segregation, limited crossflow or complex reservoir heterogeneity play a significant role. To explore some of these phenomena while retaining maximum simplicity, we have developed a foam simulator based on the fixed- P_c^* model.

We have modified the equation-of-state compositional simulator UTCOMP³⁵ for a foam that obeys the fixed- P_c^* model. UTCOMP allows for tracers in the aqueous phase that can partition into other phases and adsorb on rock. UTCOMP is transformed into a foam simulator by designating one aqueous-phase tracer "surfactant" and making gas mobility a function of water saturation and concentration of surfactant.

For simplicity, we represent the effect of foam on gas mobility as a reduction in gas relative permeability k_{rg} below its foam-free value k_{rg}^0 . The transition between the high-mobility gas and the low-mobility foam occurs over a narrow range in water saturation near the limiting water saturation $S_w^*(P_c^*)$ as follows:

For $S_w < (S_w^* - \epsilon)$ or $C_s < C_s^0$,

$$k_{rg} = k_{rg}^0(S_w) \quad (2)$$

For $(S_w^* - \epsilon) \leq S_w \leq (S_w^* + \epsilon)$ and $C_s \geq C_s^0$

$$k_{rg} = \left(\frac{k_{rg}^0(S_w)}{\left(1 + \frac{(R-1)(S_w - S_w^* + \epsilon)}{2\epsilon} \right)} \right) \quad (3)$$

For $S_w > (S_w^* + \epsilon)$ and $C_s \geq C_s^0$,

$$k_{rg} = k_{rg}^0 / R \quad (4)$$

Here C_s^0 is a threshold surfactant concentration for foam formation and R and ϵ are model parameters. We used the simplistic representation of foam strength as a function of surfactant concentration above to minimize the effects of numerical dispersion in our initial studies.³⁶ We further assume that the surfactant is not soluble in other phases. R is *not* foam "resistance factor" as conventionally defined. In

fact, the particular value of the R is not important as long as it is sufficiently large (e.g. 16000); then virtually all foams fall into the transition regime represented by equation (3). As a result, conventional "resistance factor" depends more on the value of S_w^* than on R : the lower the value of S_w^* , the stronger the foam. For the strict fixed- P_c^* model, parameter ϵ is zero, but numerical problems become greater as ϵ approaches zero. Persoff *et al.*'s original data justify a value of ϵ as large as about 0.01, and we have used that value in most cases. The simulator matches exact solutions using fractional-flow methods for several cases, except for deviations due to finite grid size, which can be serious for SAG displacements.³⁶

Gravity Override

Fractional-flow methods can model gravity override only in the limit of capillary/gravity equilibrium. If this case applies, Khatib *et al.* argue,¹⁹ foams can correct override problems only in relatively thin reservoirs.

As an initial step towards a complete solution, we examine override in a 2D homogeneous reservoir. Our goal is to describe of the process in terms of dimensionless groups that apply regardless of the details of the particular foam formulation and behavior.

Figure 9. Water saturation after 1 PV of gas and brine injection into a brine-filled homogeneous reservoir. Volumetric flux is 0.1905 m/d. $N_g = 3.05$.

Figure 10. Water saturation in the reservoir after 1.56 PV injection of strong foam ($S_w^* = 0.37$). Volumetric flux = 0.1905 m/d. $N_g = 0.024$.

Dimensional analysis^{41,42} suggests that one dimensionless parameter controlling behavior is the gravity number N_g , the ratio of gravitational potential to horizontal pressure gradient behind the foam front $(\nabla P)^f$:

$$N_g \equiv \frac{\Delta \rho g}{(\nabla P)^f} \quad (5)$$

where $\Delta \rho$ is difference between water and gas densities and g is the gravitational constant. For foams that obey the fixed- P_c^* model, $(\nabla P)^f$ can be calculated from equation (1) given S_w^* . Large values of N_g mean that the gravitational driving force for the segregation is large compared to the lateral pressure gradient. Other forms of the gravity number substitute flow rate and endpoint mobilities for the pressure gradient.⁴¹

The key to the overcoming gravity override is that VP be sufficient to push water far into the override zone, while staying within the allowable pressure limits for the well and the formation. This depends in turn on the value of N_g , reservoir dimensions, and possibly other factors.

TABLE 3

Parameter Values for Simulations of 2D Foam Floods

Permeability	1.3 d
Figures 13,14	1.3 d, 130 md
Number of gridblocks (x,y,z)	
Figures 9,10	1,50,10
Figures 11,12	1,20,10
Figure 13	10,10,1
Figure 14	10,50,1
Dimensions (x,y,z)	
Figures 9,10,11,12	1x61x10 m
Figure 13	1x30x1 m
Figure 14	10x46x1 m
Limiting water saturation S_w^*	
Figures 9,10	0.37
Figures 11,12	0.5
Figures 13,14	0.37,0.71

Parameters Common to All Simulations:

Porosity	0.25
k_{rw} function	$0.2 \left(\frac{S_w - 0.2}{0.6} \right)^{4.2}$
R	15850
Initial Pressure	13.8 MPa (2000 psi)
Injected water fractional flux at 13.8 MPa (2000psi)	0.2 m/d
Boundary condition at the production well:	
constant pressure	13.8 MPa (2000 psi)
Water density	1000 kg/m ³
Gas density at 13.8 MPa	160 kg/m ³
Injected water fractional flow	0.2

Figure 11. Water saturation after injection of 1 PV of weaker foam ($S_w^* = 0.5$).

Volumetric flux=0.1905 m/d. $N_g = 0.26$.

Figure 12. Water saturation after injection of 1.58 PV of weaker foam ($S_w^* = 0.5$).

Volumetric flux=1.905 m/d. $N_g = 0.026$.

Figure 13. Water saturation after 3.73 PV injection of foam into brine-filled layers in capillary contact.

Permeability contrast is 10. Volumetric flux=0.01905 m/d. $N_{ce} = 0.14$.

Figure 14. Water saturation after

1.8 PV injection of foam into brine-filled layers in capillary contact.

Permeability contrast is 10. Volumetric flux=0.1905 m/d. $N_{ce} = 0.0014$.

Some examples suggest a possibility of unifying description of foam processes in terms of N_g . Figure 9 shows water saturation after one pore volume (PV) of gas and brine injection into a reservoir initially saturated with brine. Parameters are listed in Table 3. The gas rises quickly to the top of the reservoir and bypasses most of the reservoir. For this case $N_g = 3.05$.

Figure 10 shows water saturation in the same reservoir after subsequent injection of 1.56 PV of strong foam ($S_w^* = 0.37$). In this case, N_g is reduced (0.024) and there is no gravity override within the foam bank. The pressure drop of 5.65 MPa (820 psi) across the foam region might be excessive for field application, however. Figure 11 shows the same process with a weaker foam ($S_w^* = 0.5$) after 1 PV injection; $N_g = 0.26$. Gravity override occurs in spite of the presence of foam, due to the large value of N_g . (In Figure 10, segregation of gas occurs ahead of the foam, but in Figure 11 segregation occurs well within the region penetrated by surfactant, in which foam is present.) Figure 12 shows that the same weaker foam is effective at a higher injection rate (1.905 m/d), with a corresponding reduction in N_g (0.026) to a value close to that of Figure 10. In this case there is no segregation in the region swept by surfactant, which is smaller than in Figure 10 due to smaller foam slug and higher S_w behind the foam front.

These results suggest several avenues for further investigation. First, foam success is ultimately limited by attainable injection-well pressure and injection rate, since they control the attainable value of N_g through equations (1) and (5). This further suggests an optimal, if idealized, foam design strategy: Design a foam that gives the maximum allowable well pressure at the maximum allowable injection rate at the desired depth of foam penetration. This might not be the strongest foam available (although, considering the difficulty of obtaining strong foams in field application, it may be). Any stronger foam than this would require reducing injection rate with no increase in N_g ; any weaker foam would reduce the value of N_g with no increase in injection rate.

Capillary Cross-Flow

Fractional-flow methods can predict flow diversion between the layers only in the limits of either capillary equilibrium or capillary isolation.^{29,33} In the case of capillary isolation, foam usually at least partially diverts flow into the lower-permeability layer. In the case of capillary equilibrium, however, foam can divert flow into the higher-permeability layer, depending on the relative values of P_c^* in the two layers.^{27,29,33} Clearly it is important to

establish under what conditions each limit applies.

Again we seek a dimensionless analysis that could unify description of foam processes. The nature of foam diversion between layers depends on two factors. First, if capillary equilibrium applies, then the nature of diversion depends on the value of P_c^* in the layers: If P_c^* is higher in the low-permeability layer, then the foam is stronger there, diverting flow into the high-permeability layer. If P_c^* is higher in the high-permeability layer, then the foam is stronger there and flow is diverted into the low-permeability layer. This factor is explored in more detail elsewhere.^{33,36}

Just as N_g governs whether gravity segregation occurs, a "capillary equilibrium number" N_{ce} governs the extent of capillary equilibration:

$$N_{ce} \equiv \frac{\Delta P_c^*}{(\nabla P)^f X} \quad (7)$$

where X is the width of each of the two layers and ΔP_c^* is the difference between P_c^* in the two layers. Small values of N_{ce} correspond to capillary equilibrium, and large values to capillary isolation. N_{ce} is not the conventional "capillary number" that governs mobilization of residual phases. It governs the extent of capillary crossflow between layers.

At first glance, equation (7) appears ambiguous: in which layer is $(\nabla P)^f$ computed, and at what flow rate? Just as one defines N_g assuming a homogeneous foam bank, i.e., before any gravity segregation occurs, we define N_{ce} in the limit of capillary isolation, before any crossflow occurs. In this case $(\nabla P)^f$ is the same in both layers, though calculating flow rate in each layer and $(\nabla P)^f$ requires trial and error.³³

Figures 13 and 14 illustrate the effects of N_{ce} and reservoir geometry on attainment of capillary equilibrium. Foam is injected into two layers with a permeability contrast of 10. The capillary-pressure functions are the same as those used by Kovscek *et al.*²⁶⁻²⁸ The values of S_w^* for the high- and low-permeability layer are 0.37 and 0.71, respectively, which implies that P_c^* is higher in the low-permeability layer. In other words, under capillary isolation, flow would be diverted into the low-permeability layer, and at capillary equilibrium flow would be diverted into the high-permeability layer.²⁹⁻³³

Figure 13 shows water saturation in the two layers after injection of 3.73 PV of foam with $N_{ce} = 0.14$. The system is virtually at steady state; the gas front does not advance in the low-permeability layer upon further injection. (The water-saturated zone near the outlet of both layers evidently reflects the capillary end effect.)

This represents a case characterized by capillary equilibrium, i.e. nearly complete blockage of low-permeability layer. Figure 14 on the other hand, with its higher axial flow rate ($N_{ce} = 0.0014$) and wider layers, represents an intermediate case between capillary isolation and capillary equilibrium. The frontal advance rates in the two layers are nearly equal, reflecting flow diversion into the low-permeability layer. However, this diversion is not as complete as in capillary isolation, which for this case corresponds to tenfold higher flow rate in the low-permeability layer.³³

Achieving this result in Figure 14 required increasing flow rate and widening the layers. Overcoming either gravity override or adverse permeability ratios between layers is expected to depend both on N_g or N_{ce} and on the reservoir dimensions.

CONCLUSIONS

1. Two simple approaches to foam modeling, the fixed- P_c^* model for foam mobility and fractional-flow methods, represent a variety of foam processes nearly as well as much-more-complex foam simulators. These approaches can highlight the key mechanisms in more-complex representations of foam behavior and should therefore supplement attempts to simulate complex foam displacements mechanistically.
2. The success of foam processes that alternate injection of surfactant and gas (SAG processes) depends on foam behavior at extremely low fractional flow of water. A foam formulation that would conventionally be called "weaker" based on steady-state coreflood data can form a stronger, longer-lived foam *in situ* in a SAG process, due to slower water drainage and collapse of foam.
3. One can incorporate the fixed- P_c^* model for strong foams into a simple simulator for foam process that incorporates the key foam mechanisms. Initial studies with such a simulator suggest that foam success in overcoming gravity override depends on the dimensionless ratio of lateral pressure gradient with foam to vertical potential gradient driving segregation. Foam success in overcoming adverse permeability ratios between layers depends on a similar ratio of lateral pressure gradient with foam to the capillary-pressure gradient driving crossflow.

SYMBOLS

C_s = surfactant concentration in water phase

C_s^0	= threshold surfactant concentration for foam
f_w	= water fractional flow
g	= gravitational acceleration
k	= permeability
k_{rg}	= relative permeability to gas
k_{rg}^0	= relative permeability to gas at same S_w , without foam
k_{rw}	= relative permeability to water
N_{ce}	= "capillary equilibrium" number
N_g	= "gravity" number
∇P	= pressure gradient
P_c	= capillary pressure
P_c^*	= limiting capillary pressure
R	= parameter in model for gas mobility
S_w	= water saturation
S_w^*	= limiting water saturation
u_w	= water volumetric flux, 'Darcy' velocity
X	= width of the layer
ϵ	= parameter in model for gas mobility with foam
μ_w	= water viscosity
$\Delta\rho$	= density difference liquid/gas

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