Application of Control Theory to Optimize Horizontal Well Location Producing from a Thin Oil Zone

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Abstract

Production from reservoirs consisting of an oil layer sandwiched between a gas cap and bottom water has always been a challenging task for petroleum engineers because of the coning problems. The present paper addresses the problem of determining optimal location of horizontal wells used to produce from such reservoirs. An optimization procedure based on optimal control theory is outlined, and implemented to determine optimal location for horizontal production wells. The optimization technique is based on

- 2D three-phase cross-sectional model
- adjoint equations
- linear programming

and is general enough so that the optimization can be carried out for a variety of reservoir and control parameters. It is demonstrated that the optimal well location strongly depends on the development term, and on the heterogeneity of the reservoir. We present several numerical examples with different development terms and different types of heterogeneities where placing of a producing horizontal well in oil, water, or gas zone is optimal from the point of view of final oil recovery.

Introduction

One of the main factors strongly influencing oil production from thin oil rims is gas and water coning. To avoid unwanted fluids the strategy used in practice has been to establish the producer in the oil zone as far away from the unwanted fluids as possible and keeping production rates below critical values. The critical production rates depend on reservoir and fluid parameters and they often are economically ineffective. To some extent the recovery from reservoirs with thin oil zones between a gas cap and bottom water can be improved by horizontal well drainage due to reduced coning. In several recent publications practical experience with horizontal wells is reported [2], [6], [8], [9], [10], [11].

Haug et al. [7] suggested another solution for the Troll field. By simulations they showed that improved recovery may be obtained by completing the well below the water/oil contact. Well location in the water zone will reduce the effect of gas coning. The method relies on a rapid down-coning of oil into the well through the water zone, the "inverse coning" process.

Another physical phenomenon taking place in reservoirs containing gas, oil, and water is the movement of oil bank into the gas cap. If certain conditions are met (on of which is a limited size of the gas cap preventing large amount of oil to be dispersed and become immobile) this effect may be used to enhance oil recovery. In this case it may be beneficial to locate a horizontal well in the gas cap. A strategy with a horizontal well location in the gas cap was found to be optimal for homogeneous reservoirs from point of view of the maximal final oil recovery in our previous publication [15].

A field experience when a vertical well perforated in the gas cap was used to produce reservoirs with thin oil rims, active aquifer, and a gas cap of a fairly limited size is reported by Clovin et al.[3].

The position of a horizontal well will strongly influence oil recovery process during a long time, at the same time information on optimal well location is crucial before starting drilling a horizontal well. An eventual benefit associated with choosing a place to locate a horizontal well is associated with rise, because the expected oil recovery increase depends not only on the above mentioned condition that the gas
This solution should maximize the final oil recovery, on optimal control theory is applied to determine the optimal location of a horizontal well. Two heterogeneous reservoirs of the reservoir may influence the optimal location of optimization algorithms which are able to account for the specific reservoir and fluids properties.

In this paper we demonstrate how heterogeneity of the reservoir may influence the optimal location of a horizontal well. Two heterogeneous reservoirs are considered: with permeability monotonously increasing and decreasing upwards. In both cases the reservoir consists of an oil zone between a gas cap and bottom water. One injection well is located in the water zone. The reservoir is drained with two horizontal production wells which are located at a specified distance from the injector, their vertical position is unknown. An optimization procedure based on optimal control theory is applied to determine the vertical position of the producers. For a specified development term and fixed injection rate, the position of the production wells and their rates are obtained as a solution of the optimal control problem. This solution should maximize the final oil recovery, which is the objective function for the problem. The optimal well configuration appears to be strongly influenced by both the heterogeneity and the development term.

Since the optimization problem is solved on a coarse grid the obtained solutions are then tested using refined grid and an industry standard simulator ECLIPSE 100 [4].

Optimization problem

Formulation. The three phase flow is simulated using a cross section model. Fig. 3 shows a vertical cross section of the reservoir. The length of the reservoir is 2000 meters, the width is 500 meters and the height 50 meters. The gas/oil contact is located 15 meters below the top of the reservoir and the water/oil contact is located 15 meters up from the bottom. Rock and fluid properties:

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porosity</td>
<td>0.3</td>
</tr>
<tr>
<td>Water viscosity</td>
<td>0.5 cp</td>
</tr>
<tr>
<td>Oil viscosity</td>
<td>1.8 cp</td>
</tr>
<tr>
<td>Gas viscosity</td>
<td>0.018 cp</td>
</tr>
<tr>
<td>Water density</td>
<td>1000 kg/m³</td>
</tr>
<tr>
<td>Oil density</td>
<td>880 kg/m³</td>
</tr>
<tr>
<td>Gas density</td>
<td>120 kg/m³</td>
</tr>
</tbody>
</table>

2 producers in the reservoir. The injection well is located in the water zone, its position and flow rate is fixed. The producers are placed at 1000m, and 2000m away from the injection well. Their horizontal position is fixed and the vertical locations allowed to change in order to obtain maximal recovery. The wells are 500 meters long which equals the total width of the reservoir. The outer boundaries are no flow boundaries. All the three phases are assumed incompressible and immiscible, capillary effects are neglected. (Mathematical formulation of the model is described in the Appendix).

In order to find vertical positions of the two horizontal wells giving the highest possible oil recovery two unknown function are introduced representing distribution of production rate along the vertical axes \( \eta_1(z) \) and \( \eta_2(z) \). These two functions are the control variables of the problem. The unknown functions are independent of time which reflects the fact that the horizontal wells cannot be moved, they should be non-negative, and they must satisfy the following constraint:

\[
\int_0^H [\eta_1(z) + \eta_2(z)] dz = q^{\text{inj}},
\]

which reflects the fact that the overall production and injection rates should be balanced since the reservoir boundaries are impermeable, and fluids are supposed to be incompressible. In addition we specify the constraints of inequality type to avoid injection at the production wells and vice versa.

After the solution is found, i.e., the two production densities \( \eta_1(z) \) and \( \eta_2(z) \) are determined, they are substituted by the two wells. The vertical coordinates of the wells are determined as centers of mass by formula:

\[
z_i = \frac{1}{q_i^{\text{prod}}} \int_0^H \eta_i(z) dz,
\]

\[
q_i^{\text{prod}} = \int_0^H \eta_i(z) dz, \quad i = 1, 2
\]

Solutions. Four cases are considered: two different permeability distributions with two development terms, 5 and 10 years. Let Case 1 be the case of permeability monotonously increasing upwards and 5 years development term, Case 2 the same permeability distribution but 10 years development term, Case 3 and Case 4 are permeability decreasing upwards, and 5 and 10 years development period.

The dimension of the difference grid used is \( N_x = 11, N_z = 6 \), the injector is placed in the left lower corner. The position and flow rate are the same for all cases. The producers 1 and 2 are placed at \( N_x = 6, N_z = 11 \). The resulting flow rate distributions are displayed in Figures 1 and 2. Oil recovery corresponding to the optimal distribution of the production rates for the four cases is as follows:
Table 1: Oil recovery

<table>
<thead>
<tr>
<th>Development term</th>
<th>Permeability increasing</th>
<th>Permeability increasing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>upwards</td>
<td>downwards</td>
</tr>
<tr>
<td>5 years</td>
<td>15.7 %</td>
<td>13.6 %</td>
</tr>
<tr>
<td>10 years</td>
<td>36.3 %</td>
<td>19.6 %</td>
</tr>
</tbody>
</table>

Figure 1: Distribution of production rates. Permeability increases upwards.

Comments. From observation of the Figures 1 and 2 one may see that for the 5 years development term there is no production distributed in the upper half of reservoir, while for the 10 years term the production density corresponding to the first well is concentrated in the upper half of reservoir for the both assessed types of heterogeneity. For permeability increasing upwards case there is a significant oil recovery increase as the development term increases from 5 to 10 years. For permeability decreasing upwards case this oil recovery increase is a much smaller.

The Case 4, permeability decreasing upwards and 10 years term, proved to be a difficult task for the optimization program. The unique optimal solution was difficult to find because oil recovery after 10 years was quite insensitive to the control variables. This case requires special consideration since the incremental oil recovery is comparable with the accuracy of simulation.

Simulation results

The simulations are performed using the black oil simulator ECLIPSE 100 [4]. The initial fluid distribution in the reservoir is shown in Fig. 3.

The reservoir injection rate is 2000 $m^3$/day for all runs. The production rates predicted by the optimization program are used to specify production.

The objective of the simulations is to check the validity of the results obtained by the optimization program.

Three different grids were used (see Table 2.)

The coarse $11 \times 6$ grid corresponds to the grid used by the optimization program.

The first step in the simulation procedure consists of selecting the grid to be used for all subsequent simulations. Sensitivity runs show that the difference between the recovery curves for the $40 \times 20$ and $80 \times 40$ grids is small, while the difference between the $11 \times 6$ and $40 \times 20$ is substantial.

For this study the $40 \times 20$ grid is used to obtain simulation results for comparison with results obtained by the optimization program.

Table 2: Difference grids

<table>
<thead>
<tr>
<th>Grid size</th>
<th>$\Delta x$ (meters)</th>
<th>$\Delta y$ (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$11 \times 6$</td>
<td>100, 9×200, 100</td>
<td>5, 4×10, 5</td>
</tr>
<tr>
<td>$40 \times 20$</td>
<td>40×50</td>
<td>20×2.5</td>
</tr>
<tr>
<td>$80 \times 40$</td>
<td>80×25</td>
<td>40×1.25</td>
</tr>
</tbody>
</table>

Figure 2: Distribution of production rates. Permeability decreases upwards.

Figure 3: Initial distribution of oil, water and gas in the reservoir.
Table 3: Permeability increases upwards

<table>
<thead>
<tr>
<th>layers</th>
<th>(k_x) (mD)</th>
<th>(k_y) (mD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 3</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>4 - 6</td>
<td>900</td>
<td>900</td>
</tr>
<tr>
<td>7 - 10</td>
<td>700</td>
<td>700</td>
</tr>
<tr>
<td>11 - 14</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>15 - 17</td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>18 - 20</td>
<td>200</td>
<td>200</td>
</tr>
</tbody>
</table>

Table 4: Permeability decreases upwards

<table>
<thead>
<tr>
<th>layers</th>
<th>(k_x) (mD)</th>
<th>(k_y) (mD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 3</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>4 - 6</td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>7 - 10</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>11 - 14</td>
<td>700</td>
<td>700</td>
</tr>
<tr>
<td>15 - 17</td>
<td>900</td>
<td>900</td>
</tr>
<tr>
<td>18 - 20</td>
<td>1000</td>
<td>1000</td>
</tr>
</tbody>
</table>

The two permeability distributions used are presented in Tables 3 and 4.

The well locations and rates for the producers predicted by the optimization program are given by the Table 5.

For Case 1 and Case 2 the simulation results confirm the outcome of the optimization procedure. As depicted in Figure 4 the recovery curves intersect after 5 years showing that the optimal locations and rates for production wells depend on the development period as forecasted by the optimization program.

The saturation distributions at the end of the development period for Case 1 and Case 2 are shown in Figures 5 and 6. The interfaces between phase regions are taken to be 50 % saturation contour lines.

Figure 7 shows that the recovery after 5 years is higher for Case 3 than for Case 4, which is in agreement with predictions obtained by the optimization program. However, according to the simulation results recovery after 10 years for Case 3 is still higher than for Case 4 while the optimization procedure shows the opposite (see Figure 7). The reason is that the optimization results are obtained using a 11 x 6 grid which turns out to be too coarse for this problem.

Table 5: Simulated cases

<table>
<thead>
<tr>
<th>location</th>
<th>rate</th>
<th>location</th>
<th>rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>(20,13)</td>
<td>1285</td>
<td>(40,17)</td>
</tr>
<tr>
<td>Case 2</td>
<td>(20,3)</td>
<td>1670</td>
<td>(40,16)</td>
</tr>
<tr>
<td>Case 3</td>
<td>(20,15)</td>
<td>1537</td>
<td>(40,13)</td>
</tr>
<tr>
<td>Case 4</td>
<td>(20,4)</td>
<td>1224</td>
<td>(40,12)</td>
</tr>
</tbody>
</table>

Figure 4: Oil recovery curves for two different well locations. Permeability increases upwards.

Figure 5: Final saturations and wells location for the 5 years development term. Permeability increases upwards.

The saturation distributions at the end of the development period for Case 3 and Case 4 are shown in Figures 8 and 9.

From comparison of Figures 6 and 9 one may see that in both cases oil layer between the injector and the first producer is driven up in the gas cap. For the permeability decreasing upwards case more oil is left undisplaced between the injector and the first producer due to slower water front advance upwards. Water flows mainly through a more permeable lower part and then cones up into the production well.

For permeability decreasing upwards, the final distribution of phases after 10 years with the configuration of wells found optimal for the 5 years development term is shown in Figure 10. It appears to be similar to the one obtained after 5 years for permeability increasing upwards case (Figure 5). The oil recovery values for those two cases are also close, see Figures 7 and 4.

Discussion

As observed from the simulations, two physical phenomena accompany oil production from an oil reservoir with a gas cap and bottom water: movement of oil bank up into the gas cap and coning. These phenomena strongly influence the location of the first
Figure 6: Final saturations and wells location for the 10 years development term. Permeability increases upwards.

Figure 8: Final saturations and wells location for the 5 years development term. Permeability decreases upwards.

Figure 7: Oil recovery curves for two different well locations. Permeability decreases upwards.

Figure 9: Final saturations and wells location for the 10 years development term. Permeability decreases upwards.

producer. The second producer is only influenced by coning since it is not reached by the oil bank.

Location of first producer. As demonstrated earlier [15],[3] in a homogeneous reservoir with a gas cap of a limited size oil production may be strongly enhanced by placing a well into the gas cap, so that the moving oil bank reaches perforations. The effect of oil recovery increase appears to be even more pronounced in case of heterogeneous reservoir with a systematic absolute permeability increase from the bottom to the top.

If the absolute permeability systematically decreases upwards, placing a well in the gas cap does not work. Dramatical increase in water coning is observed. Because of increased water production oil bank movement slows down. For this case higher oil recovery is obtained if the producer is located bellow the initial oil-water contact, and oil is produced due to inverse coning.

As one may see, the oil recovery increase due to optimization of the vertical position of a horizontal well may be quite significant. At the same time an optimal well configuration may dramatically change as the parameters describing reservoir and fluid properties change. The observed optimal solutions are optimal only for the considered cases.

General rules may hardly be derived. From our point of view, the only tool allowing solution of the optimization problem for the particular reservoir is the usage automatic optimization algorithms similar to the one described in the paper.

Conclusions

1. An optimization procedure based on optimal control theory is outlined, and applied to determine optimal location and rates of horizontal wells to develop a reservoir with an oil zone between a gas cap and bottom water.

2. The optimal well configuration depends on the development term and on the reservoir heterogeneity. As demonstrated, it might be beneficial to locate the well in water zone, oil zone, or in gas cap depending on both development term and heterogeneity type.

3. In the assessed examples it was found beneficial to locate a well in the gas cap if absolute permeability increases upwards and the development term is long enough to allow the advancing oil bank to reach the well. If absolute permeability increases downwards, and/or
Figure 10: Final distribution of phases after 10 years with the configuration of wells found optimal for the 5 years development term. Permeability decreases upwards.

the development term is not long enough, then higher oil recovery is achieved by placing the well below the oil-water contact. Oil in this case is produced due to the inverse coning effect.

4. The results obtained by the optimization procedure are confirmed using fine grid simulations.

Acknowledgement

We acknowledge the support from RF-Rogaland Research Institute and Rogaland University Centre (Norway), and Stanford University (USA).

Nomenclature

\( e \) fractional flow at injection

\( f \) fractional flow function

\( J \) oil recovery

\( g \) absolute permeability

\( H \) thickness of reservoir

\( N_x, N_z \) numbers of grid blocks in horizontal and vertical directions

\( p \) pressure

\( S \) saturation

\( t \) time

\( T \) development term

\( T \) transposition

\( q \) production (injection) rate

\( w \) total Darcy velocity

\( w_i \) phase velocity

\( x_1, x_2 \) co-ordinates of plane

\( \gamma \) specific gravity

\( \varphi, \psi \) adjoint variables

\( \phi \) porosity

\( \lambda_p \) phase mobility

\( \lambda_t \) total mobility

\( \mu_i \) phase viscosity

\( \xi \) injection density

\( \eta \) production density

\( \theta_1, \theta_2 \) angles between the vertical direction and the coordinate axes

\( \Omega \) flow domain

References


Conjugate equations.

The optimal control problem (1)-(5) and (7) is solved by the successive linearization method [5], [12], [13], [14]. We use the standard technique of differentiating functionals, applied in optimal control theory [5]. Introducing conjugate variables \( p(z, t), i_1(x, t), i_2(x, t), \) we write conjugate equations:

\[
\text{div } g \cdot \nabla V - \text{div } g V_{i, b} = 0, \\
\frac{1}{V_0} \int_{\Omega} \eta f_1 dx dt, \quad V_0 = \int_{\Omega} \phi S_0^0 dx \tag{5}
\]

Let us consider an optimization problem for the multi-phase flow described by equations (1)-(5): the distribution of sources and sinks should be chosen in such a manner that the oil recovery is maximized. In order to avoid trivial solutions one must specify formal constraints for well flow rates. This can be done in several manners. For instance:

- Total produced volume is limited
  \[ \int_{0}^{T} \int_{\Omega} \eta dx = Q_0 \tag{6} \]

- Overall production rate is fixed
  \[ \int_{\Omega} \eta dx = q_0(t) \tag{7} \]

- Maximal injection and production densities are specified
  \[ 0 \leq \xi \leq \xi^+(x, t), 0 \leq \eta \leq \eta^+(x, t), \tag{8} \]

In other words the task is to find the optimal controls \( \xi(x, t), \eta(x, t) \). To the optimal control signals there must correspond a solution of the initial system, which maximizes functional (5) under the specified constraints.

Conjugate equations. The optimal control problem (1)-(5) and (7) is solved by the successive linearization method [5], [12], [13], [14].

We use the standard technique of differentiating functionals, applied in optimal control theory [5]. Introducing conjugate variables \( \varphi(x, t), \psi_1(x, t), \psi_2(x, t) \), we write conjugate equations:

\[
\text{div } g \lambda_1 \nabla \varphi - \text{div } g \lambda^T \nabla \psi = 0, \tag{9}
\]

\[
\sum_{i=1}^{3} K_{ij} \lambda_1 (\nabla \psi_i, w_i) + \phi \frac{\partial \psi_j}{\partial t} - \eta \sum_{i=1}^{2} F_{ij} \psi_i \tag{10}
\]

\[
-(\nabla \varphi, w) A_{ij} \lambda_1 - (\nabla \varphi, v) C_j - \eta F_{ij} = 0, \tag{11}
\]

\[
v = gbc \tag{12}
\]

\[
C_j = \frac{A_{ij} B_j}{\lambda_1} \tag{13}
\]

If gravity is neglected,

\[
w_i = w f_i, i = 1, 2, v = 0. \tag{14}
\]

For this case we obtain the following equations for \( \psi_1(x, t), \psi_2(x, t) \).
\[ \frac{K^T}{\lambda_t} (\text{div} \; w \psi + \psi (\eta - \xi)) - \eta F^T \psi - A (\nabla \varphi, w) + \frac{\partial \psi}{\partial t} = \frac{\partial f_j}{\partial S} \]

\[ v(x, t) = (\psi_1(x, t), \psi_2(x, t)), \quad F = F_{ij}, \quad K = K_{ij} \]

If one takes into account \( \lambda_t F = K - f \lambda T \) it is clear that this equation coincides with the equation derived in [13].

For the general case we have

\[ \text{div} \; g \chi \nabla \varphi - \text{div} \; g \lambda^T \nabla \psi = 0, \quad (12) \]

\[ \sum_{i=1}^{2} K_{ij} (\text{div} \; \psi_i w_i - \psi_i \text{div} \; w_i) + \frac{\partial \psi_j}{\partial t} - \eta \sum_{i=1}^{2} F_{ij} \psi_i \]

\[ - (\nabla \varphi, ((w + v) A_j + v B_j)) = \eta F_{ij} \]

\[ \varphi \mid_{B_2} = \psi^T f, \quad g \mid_{B_2} = 0, \quad \psi \mid_{\lambda = T} = 0. \]

\[ F_{ij} = \frac{\partial f_i}{\partial S_j}, \quad A_j = \frac{\partial \lambda_t}{\partial S_j} \]

It is necessary to take into account that varying the controls in the general case leads to change of the boundary \( B_3 \), and the values of saturations remain constant only on the part of the permeable boundary. Moreover, it is assumed, that the permeable boundary, if it exists, is located far enough from sources of perturbations, such that boundary saturations are constant on it, and initial saturations are constant in a sufficiently large neighbourhood of \( B_2 \). Hence, the variation of saturations on the permeable boundary is equal to zero, though the direction of flow across the boundary can change. In this case the following expression for the variation of the functional is valid:

\[ \delta J = \frac{1}{V_0} \int_0^T \int_{\Gamma} [\delta \xi (\varphi - \xi^T \psi) + \delta \eta (f_1 - \varphi + f^T \psi)] dx dt \]

**Numerical solution.** The numerical solution of the initial equations is received using the IMPES method. The difference schemes applied for the conjugate equations are consistent with the schemes used for the initial equations.

Both for the space-time discretisation introduced for receiving difference equations, and calculation of the functional and its gradient, it is necessary to introduce a discrete form of the control signals. The controls are assumed to be piece-wise constant functions of space and time, and the time step for the controls is set constant.

\[ \xi(x, t_n) = \sum \xi_k^n d_k(x), \quad \eta(x, t_n) = \sum \eta_k^n d_k(x) \]

\[ d_k(x) = \begin{cases} 1 & \text{if } k \in U \\ 0 & \text{otherwise} \end{cases} \]

The introduced form of the controls is sufficiently general. It includes both the case of distributed controls (when \( U \) contains all numbers of grid blocks), and the case when the controls are concentrated in a fixed number of points, i.e. the co-ordinates of possible location of wells are fixed. Note that fixing the set \( U \) substantially effects the solution of the problem. However, this operation can not be made completely formally, because it is necessary to take into the consideration an information about the structure of the solution. Further, we implement the following heuristic rule: on the first stage a rich enough set of grid blocks \( U \) is fixed. Then after the analysis of the received solution we go to the second stage with the limited set of grid blocks where the location of wells is allowed.

In the first stage of the horizontal well optimization problem \( U \) is specified as a set of straight lines in vertical direction, thus allowing the flow rate densities to be functions of the vertical coordinate while the set of horizontal co-ordinates is fixed. The number of time steps for the controls is 1, which reflects the fact that a horizontal well may not be moved. Therefore, the vertical distribution of the flow rate can not change in time. After receiving the solution, the same optimization program is run for a second time. Now the positions of the wells are totally fixed, but the flow rates are allowed to change in time.