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MANAGING UNCERTAINTIES IN RESOURCES EVALUATION AND FIELD DEVELOPMENT PLANNING

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Abstract

Application of improved oil recovery methods requires considerable resources which advocates for sound evaluation of field development scenarios. Uncertainty in reservoir characterization makes an overall estimation more subjective and, therefore, less reliable. A general trend towards the development of small and marginal fields complicates a rigorous analysis and makes it more risky. At early stages of field evaluation and development planning the probabilistic output of decision analysis technique does not help much: sparse data does not allow to reduce subjectivism in probability assessment, thus leading to pitfalls and motivational biases. In such cases a fuzzy systems theory has shown to be an excellent tool for handling uncertainties. Its predictive power in a scattered environment is in the ability to predict the possibility of outcome based on limited information, i.e. in situations where statistics is not available and probabilistic methods can fail. Another advantage of fuzzy technique is that it directly links uncertainty of input data to the reliability estimation of the final decision.

Decline curve analysis (DCA) and material balance equation (MBE) are powerful tools for evaluating resources of oil and gas. Application of fuzzy mathematics to DCA and MBE allows to estimate uncertainty in such evaluations and obtain proven, probable and possible resources in a most natural way, i.e. without assessing subjective probabilities to different categories of resources.

Methods of fuzzy mathematics provide an excellent tool for the decision under uncertainty. Decision tree analysis technique and Bayes' theorem can be generalized and fruitfully applied in fuzzy environment leading to pessimistic, most likely and optimistic realizations of possible outcomes.

Direct handling of uncertainties by means of fuzzy mathematics enables to directly incorporate the input data uncertainty in analysis, reduce the total number of justified development scenarios and risk of subjective judgment. The paper presents an overview on a new approach to the decision analysis process and focus on some practical oilfield applications. Specific examples include results of DCA and MBE evaluations, and the evaluation of a real case development planning.

Introduction

Fuzzy sets were introduced by L. Zade to describe a so-called hard-to-formulate problems¹. The main difference from crisp sets and Boolean logic was an introduction of a so-called *membership function* describing how well data fits into a certain (fuzzy) set. It allowes to avoid simple answers like «yes» or «no» to many technical and non-technical questions and replace them when needed, with more suitable evasive statements like «may be» with different degrees of confidence. Another advantage of this new approach is that it has shown how uncertainty and confidence of a decision relate to each other. Literatrure on fuzzy systems theory has since been flourishing with its promising applications in the areas of fuzzy classification, clustering and ranking, optimization in fuzzy environment and many other areas^{2-4,14-18,21}. However, due mainly to a sofisticated non-conventional mathematics engineering applications of the new approach have been lagging far behind.

A second wave of publications began in late '70s with the introduction of fuzzy numbers which could be considered as a special case of fuzzy sets, and fuzzy arithmetics^{2-5,8-12}. In the following sections we will limit our discussion to a special case of fuzzy numbers namely, triangular fuzzy numbers (TFN).

As shown in Fig. 1 TFN can be defined (1) by its membership function $\mu_A(h)$, (2) by the interval of confidence $[a^1(h), a^3(h)] = [a_1 + (a_2 - a_1) \cdot h, a_3 - (a_3 - a_2) \cdot h]$, (3) by three main values (a_1, a_2, a_3) , and (4) by the most likely value and two spreads $(a, \delta a_1, \delta a_2)^3$.

Arithmetics of fuzzy numbers originates from the classical operations with the interval of confidence':

$$\begin{bmatrix} a_1, a_3 \end{bmatrix} + \begin{bmatrix} b_1, b_3 \end{bmatrix} = \begin{bmatrix} a_1 + b_1, a_3 + b_3 \end{bmatrix}$$

$$\begin{bmatrix} a_1, a_3 \end{bmatrix} - \begin{bmatrix} b_1, b_3 \end{bmatrix} = \begin{bmatrix} a_1 - b_3, a_3 - b_1 \end{bmatrix}$$

$$\begin{bmatrix} a_1, a_3 \end{bmatrix} \cdot \begin{bmatrix} b_1, b_3 \end{bmatrix} = \begin{bmatrix} a_1 \cdot b_1, a_3 \cdot b_3 \end{bmatrix}$$

$$\begin{bmatrix} a_1, a_3 \end{bmatrix} \cdot \begin{bmatrix} b_1, b_3 \end{bmatrix} = \begin{bmatrix} a_1: b_3, a_3: b_1 \end{bmatrix}$$

$$\begin{bmatrix} a_1, a_3 \end{bmatrix} \cdot \begin{bmatrix} b_1, b_3 \end{bmatrix} = \begin{bmatrix} a_1: b_3, a_3: b_1 \end{bmatrix}$$

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However, it is important to remember that fuzzy numbers do not possess the group properties and, therefore, the following equations:

$$(x, \delta x) = \frac{(b, \delta b)}{(a, \delta a)}$$
 (2)

$$(a,\delta a) \cdot (y,\delta y) = (b,\delta b)$$
 (3)

have different solutions, i.e.

The last result is a solution to the following optimization problem²:

$$F(x,\delta x) = w_1(ax-b)^2 + w_2(a\cdot\delta x + x\cdot\delta a - \delta b)^2 \to \min_{x,\delta x} \qquad (6)$$

Here w_1 and w_2 - are the weighting coefficients indicating contribution (importance) of the most likely values and spreads of fuzzy numbers to the final solution.

Only in one particular case when A is a non-fuzzy number, i.e. A = (a, 0, 0) all these solutions coincide^{*}.

Fuzzy Approach to Typical Reservoir Engineering Problems

Volumetric Reserve Estimates

It is well known that the main difficulty in reservoir characterization is related to the transfer of the information retrieved from few locations (well sites) by direct (core and PVT analysis, etc.) or/and indirect (seismics, well logging, well test analysis) measurements for the entire reservoir, horizon or field. Deterministic methods of reservoir characterization based mainly on the interpolation technique result in smoothed spatial distribution of reservoir parameters critical for volumetric reserve estimate. Accuracy or uncertainty of reservoir parameters is often not estimated or simply ignored, and the final maps do not acknowledge the quantity and quality of the data.

The oil in place is usually evaluated according to the following equations:

$$V_{oi} = A_i \cdot h_i \cdot (N / G)_i \phi_i \cdot S_{oi} \cdot \beta_{oi}; \qquad (7)$$

^{*} This fact should be taken into consideration while solving different problems related to fuzzy environment.

$$STOOIP = \sum_{i=1}^{n} V_{oi} \qquad (8)$$

where n - number of zones on which a reservoir (horizon, field) were divided. Result of such evaluation is a single value of reserves.

Methods of stochastic modeling^{6,13} result in statistic interpretation of reserves and, in addition to a most likely value, give their probabilistic properties. One of the most popular probabilistic methods - *Monte Carlo* simulation, is also based on a volumetric reserve estimate according to equations analogous to (7) and (8). The simulation consists of thousands of repeated trials where values of reservoir parameters randomly selected from their probability distribution function give thousands of outputs forming a cumulative probability distribution of reserves.

Probabilistic estimate of reserve is more realistic than deterministic one and reflects, to a greater extent, quantity of the data and their quality. The more complete information is available - the narrower are statistical properties and the more confident is the estimate. However, in situations where statistics is not available (new discoveries, marginal fields) application of stochastic methods becomes more risky: sparse data does not allow to reduce subjectivism in probability assessment, thus leading to pitfalls and motivational biases.

In such cases, fuzzy methods have proven to be an excellent tool for handling uncertainties. Fuzzy reserve estimate is based on the idea of representing reservoir parameters of eq. (7) as fuzzy numbers. In fuzzy environment (7) can be written as follows:

Here conventional multiplication is replaced by extended multiplication, \otimes used for operations on fuzzy numbers^{4,9}. It is worth to note that fuzzy numbers were invented to describe incomplete, scattered and «fuzzy» information, and that uncertainty in data represented by fuzzy numbers is transfered from the input to the output unbiased.

Table 1. Input data for reserve estimate

Parameter	PV	MLV	ov
Arear A, 10^6 m^2	9.1	10.0	10.8
Net thickness $h(N/G)$, m	19.5	20.0	20.5
Porosity ϕ , %	16.7	17.0	17.2
Init. water saturation S _{wi} , %	20.0	22.0	24.0
Oil shrinkage factor, β_{oi}	.833	.113	.053

Example below illustrates the application of fuzzy methods to the volumetric reserve estimate. Table 1 contains the input data into eq. (9) represented by triangular fuzzy numbers.

Successive application of extended multiplication to the input data results in the following fuzzy estimate of reserve:

$$\tilde{N} = (22.10, 6.49, 4.56) \times 10^{\circ} \text{ Sm}^{3}$$

Procedure of fuzzy evaluation of reserve is illustrated in Fig. 2, while Fig. 3 shows the output represented as *cumulative possibility curve*. Here evaluation of reserve is carried out for 0%- and 50%-level of confidence. The third (dashed) line represents evaluation of reserves carried out by *interval analysis*. In

	Reserves, 10 ⁶ Sm ³			
Type of estimate	Proven	Proven+ probable	Proven+ probable+ possible	
TFN, 0% level				
of confidence	19.3	21.7	23.5	
Interval numbers	17.9	21.5	24.7	
TFN, 50% level				
of confidence	16.7	21.0	25.0	

Table 2. Volumetric fuzzy reserve estimates

the latter case, expert has an access to a very limited information when reservoir parameters are represented by the range of their variation. Results of evaluation are summarised in Table 2 where the following important values are emphasized ⁷: *proven* reserves corresponding to a P_{90} (P_{80}) level of the cumulative possibility curve (CPC); *proven+probable* (most likely) reserves at P_{50} level of CPC; and *proven+probable+possible* reserves corresponding to P_{10} (P_{20}) level of CPC.

Material Balance Equation

Material balance equation (MBE)¹⁴ is one of the most powerful and accurate methods of the evaluation of hydrocarbon reserves. However, in overwhelming majority of cases behavior of a reservoir deviates from that predicted by the theory reflecting complex in-situ processes of multiphase flow, multiple drive mechanisms and a complex reservoir structure. Standard application of MBE enables to obtain a mean or most likely value of reserves. Fuzzy systems theory gives an opportunity to describe MBE in fuzzy environment thus, incorporating uncertainty of input data and imperfection of our knowledge directly into material balance equations. It results in the output data that can be classified in terms of categories used in volumetric reserve estimates, i.e. in categories of proven, probable and possible reserves.

Example discussed below shows the difference in reserve estimates obtained by classical method of Havlena and Odeh¹⁴ applied in deterministic and fuzzy environment.

Assume for the sake of simplicity that an oil reservoir without an aquifer and a gas cap is developed by depletion (see Table 3). Neglecting compressibility of connate water and rock grains one can write the following linearised form of MBE:

$$F = NE_a \qquad (10)$$

where $F = N_p \Big[B_o + (R - R_s) B_g \Big]$ - cumulative hydrocarbon production expresses at prevailing reservoir conditions; $E_o = (B_o - B_{oi}) + (R_{si} - R_s) B_g$ - term accounting for oil and gas expansion; N - STOOIP; N_p cumulative oil production at standard conditions; B_o , B_g - oil and gas formation volume factors, respectively; R, R_s - GOR and solution GOR; *i* - index corresponding to initial reservoir conditions. Using data in Table 3 and solving eq. (10) by the least square method one can obtain the following

deterministic estimate of oil reserve: $N = 21.2 \times 10^6$ Sm³.

<i>p</i> , MPa	<i>Np</i> , MMSm ³	$GOR, m^3/m^3$	<i>Bo</i> , rm ³ /Sm ³	Rs, m ³ /m ³	Bg, rm ³ /Sm ³	
23.0	-	-	1.2511	90.8	0.00489	
21.7	0.185	187	1.2353	84.9	0.00517	
21.4	0.243	188	1.2300	82.5	0.00525	
20.7	0.352	190	1.2222	80.1	0.00539	
20.2	0.429	198	1.2172	78.2	0.00553	
20.0	0.459	202	1.2147	76.9	0.00560	
19.7	0.492	206	1.2122	75.65	0.00568	

Table 3. Oil and gas production data

Fuzzy evaluation of reserve can be obtained by expanded form of eq. (10):

$$F = \tilde{N} \cdot E_{a} \quad \dots \quad \dots \quad (11)$$

where $\tilde{N} = (N, a, a)$ is a simmetrical fuzzy number that can be found by solving a *linear* possibility regression model²:

$$\min_{N,a} J(a) = a$$

$$F_j \leq E_{oj} \cdot N + (1-h)E_{oj} \cdot a \qquad \dots \quad (12)$$

$$F_j \geq E_{oj} \cdot N - (1-h)E_{oj} \cdot a$$

$$a \geq 0,$$
yielding $(N,a,a) = (20.9, 1.6, 1.6) \times 10^6 \text{ Sm}^3.$

Graphic illustration of this solution is shown in Fig. 4.

This information enables to confirm or define more precisely *proven*, *probable* and *possible* hydrocarbon reserves obtained earlier by volumetric estimate which makes application of both methods more consistent.

Field Development Planning

Let us now try to estimate uncertainties in production performance evaluation directly, i.e. by using fuzzy approach and, in particular, possibility theory² and fuzzy arithmetics^{3,4}.

Table 4 contains information about cumulative gas production for different field development scenarios evaluated by the Fault Block Model¹¹.

Scenario	No. of	Gas Pr	oduction,	Bsm ³	
No.	wells	PV	MLV	ov	
1	11	74	97	114	
2	15	91	111	130	
3	17	100	118	135	
4	19	102	122	140	
5	21	107	126	145	•
6	23	111	130	149	
7	25	111	132	153	

Table 4. Pessimistic (PV), Most Likely (MLV)Let us assume that uncertainties in cumulative gasand Optimistic (OV) values of cumulative gasproduction can be represented by symmetricalproduction for different field development planstriangular fuzzy numbers (SFN).

It is also assumed that cumulative gas production depends on the total number of wells according to the following model¹¹:

where A_i and A_2 are SFNs and are constant, n_i -number of wells in the *i*-th scenario.

Fuzzy numbers A_1 and A_2 can be found by means of a linear possibility regression model^{2,11}.

Solution to this problem at two levels of confidence is shown below:

$$h=0.5$$
: $G_{p,i} = (161.96, 42.00) - \frac{749.1}{n_i};$ $h=0.67$: $G_{p,i} = (161.96, 63.64) - \frac{749.1}{n_i};$

It should be noted that using most likely values of the cumulative gas production and applying the Least Squares Method we derive at the following deterministic model:

$$G_{p,i} = 164.67 - \frac{811.92}{n_i}$$

with its values being different from most likely values obtained by a possibility regression model.

Fig. 5 represents a fuzzy model for cumulative gas recovery against the total number of wells at 0.5-level of confidence. This simple model can be further utilized in optimization of field deveopment scenario or long-term planning.

Fig. 6 shows how uncertainty in cumulative gas recovery evaluation depends on the total number of wells and the degree of presumption (level of confidence). As the trend indicates, evaluation of field performance developed by few wells creates large uncertainty in output data. By increasing the number of wells we reduce uncertainty in the recoverable reserve estimate.

Uncertainty in Production Forecast and Decision Making

Production forecast and field development planning are the areas where uncertainties associated with the problem (production forecast, future capital investments and sales prices of oil and gas, etc) can be handled by fuzzy methods in an effective way.

Production forecast based on rigorous methods of reservoir simulation with uncertainties included into analysis is still a very challenging problem whose solution is still on the «waiting list». However, at early stages of field development production forecast can be generated by a simple exponential-decline model:

where q_i = initial oil rate in Sm³ per year, and a = fractional decline per year. Introducing uncertainties into q_i and ramp-up a and representing them as triangular fuzzy numbers $(q_{ai}, \delta_i q_i, \delta_2 q_i)$ and $(a, \delta_j a, \delta_2 a)$, resprecively, and rearranging the terms one can obtain a fuzzified exponential decline model:

where uncertainties in the initial oil rate and fractional decline are directly incorporated into the model. Production forecast for a small field based on eq. (15) is illustrated in Fig. 7. Economic evaluations (NPV, Return On Investment, etc.) can easily be obtained by using fuzzy arithmetics. Results of NPV evaluation for a hypothetical offshore field ¹² obtained by using a fuzzified analogue of classical formula

$$NPV = \sum_{i=1}^{n} \frac{Q_i \cdot P_i - (CI + OC)_i}{(1 + \varepsilon)^i} \quad \dots \quad \dots \quad \dots \quad (16)$$

are presented in Fig. 8. One of the important conclusions that can be drawn from these results is that the optimal scenario as well as duration of exploitation essentially depends on the attitude of a decision maker toward risk. For a risk-seeking, agressive decision maker optimistic forecast with 12-year duration of production and 500 million NOK NPV is sound quite attractive while for a risk-averse person «attractiveness» of this scenario is rather negative: possible losses can far exceed feasible gain of 150 million NOK after 5 years of exploitation.

Conclusion

Considered above examples are just few illustrations of numerous applications of fuzzy approach to petroleum engineering problems. Logistics of a problem solving as well as solutions themselves testify that this approach can compete successfully with methods of stochastic modeling. However, a closer look at the problems proves that there should be no competition between different methods but rather an appropriate use of correct methods in the corresponding environment. In other words, fuzzy approach does not substitute deterministic and stochastic methods but complements them spreading mathematical tools and augmenting methods of analysis on scattered environment where statistics is not available and probabilistic methods can fail. As demonstrated probabilistic and fuzzy approaches give *quantitatively different estimates* emphasizing the difference between the *probability* and *possibility*.

Looking at the perspectives, it should be pointed out that a very promising and still untouched area where fuzzy methods of analysis can be applied is *reservoir characterization* and *reservoir simulation*. It is easy to foresee that in the near future reservoir characterisation and simulation will be performed using fuzzy methods at early stages of exploration and field appraisal. When reservoir statistics becomes available these methods can be substituted by stochastic and, in some cases, even by deterministic methods. However, in the area of production forecast and a long-term field development planning fuzzy methods of analysis have no rivals.

References

- 1. Zadeh L.A.: Fuzzy Sets, Information and Control, 1965, v.8.
- 2. D.Dubois, H. Prade: Possibility Theory, Plenum Press, New York, 1988.
- 3. Jones, A. Kaufmann, and H.-J. Zimmerman: Fuzzy Sets Theory and Applications, D. Reidel Publishing Company, Dordrecht, 1985.
- 4. D.Dubois, H. Prade: Operations on Fuzzy Numbers, Int. J. Systems SCI, 1978, v.9.
- 5. Fang, and H.C. Chen: Uncertainties are Better Handled by Fuzzy Arithmetic, AAPG Bull., v. 74, No. 8, 1990.
- 6. Øvreberg, E. Damsleth, and H. Haldorsen: Putting Error Bars on Reservoir Engineering Forecast, JPT, June 1992.
- 7. DeSorcy, G.A. Warne, B.R. Ashton et al., Definitions and Guidelines for Classification of Oil and Gas Reserves, The Journal of Canadian Petroleum Technology, May 1993, v. 32, No. 5.
- 8. Ya. I. Khurgin: Fuzzy Equations in Petroleum Geophysics, Techn. Cybernetics Journal, No. 5, 1993.
- 9. Ya. I. Khurgin: Fuzzy Methods in Petroleum Industry, Moscow, State Gubkin Academy of Oil and Gas, 1995.
- 10. Eremin: Modelling Hydrocarbon Fields by Methods of Fuzzy Logic, Moscow, «Nedra», 1994.
- 11. J.-R. Ursin, A.B. Zolotukhin: Optimization of Gas Condensate Field Production Performance by Fault Block Modelling and Decision Under Uncertainty Technique, Proceedings of the 5th European Conference on the Mathematics of Oil Recovery, Leoben, Austria, Sep. 1996.
- A.B. Zolotukhin, and Ya.I. Khurgin: Fuzzy Approach and Its Applications in Petroleum Sciences, Proceedings of the Conference on Fundamental Problems of Petroleum Industry, Moscow, State Academy of Oil and Gas, Jan. 1997.
- 13. J.A. Murtha: Monte Carlo Simulation: Its Status and Future, JPT, April 1997.
- 14. Havlena, D. and Odeh, A.S. The Material Balance as an Equation of a Straight Line, JPT, August 1963, p. 896-900, Part II - Field Cases, JPT, July 1964.



Fig. 1. Example of triangular fuzzy number (TFN)



Fig 2. Estimation of reserves using TFN



Fig. 3. Fuzzy reserve estimate







Fig. 7. Fuzzy field development forecast model



Fig. 4. Crisp and fuzzy reserve estimates on a basis of MBE



Fig. 6. Uncertainty in gas production vs. the level of confidence



Fig. 8. Fuzzy evaluation of NPV