EFFECT OF FRACTIONAL FLOW HETEROGENEITY ON COMPOSITIONAL AND IMMISCIBLE DISPLACEMENTS

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Abstract

The scale of heterogeneities in reservoirs is often smaller than the grid size used in large scale reservoir simulations. Relative permeabilities have the foremost effect on fluid flow and small scale fractional flow must be upscaled to the grid size in flow simulations. Thus, systems with fractional flow heterogeneities, i.e. relative permeability variations, have to be solved and effective relative permeability functions determined. In this paper, benchmark analytical solutions are developed for a system consisting of two media in series where each medium is characterized with a different set of relative permeabilities, residual saturations and porosities. The analytical solutions show a significant discontinuity in the saturation and concentration profiles at the interface of the two media. Numerical results using several weighting schemes are compared against the analytical solutions for two-phase immiscible and partially miscible systems. It is shown that single-point upstream weighting requires about 100 grids to capture the discontinuity at the interface, whereas third order TVD weighting requires much fewer. Lastly, the validity of the JBN method (Johnson et al. 1959) for determination of effective relative permeabilities is tested for flow through two media in series. The JBN method fails to determine the effective relative permeability correctly in such systems where fractional flow heterogeneity is present.

Introduction

The study of two-phase flow is of fundamental importance to problems related to petroleum and environmental engineering. The first major contribution in this field came from Buckley and Leverett (1942) when they solved for the saturations of one-dimensional, two-phase immiscible displacements with negligible gravity and capillary effects. Buckley and Leverett overcame multivalued solutions in the saturation by introducing shocks determined by mass-balance constraints. The problem of two-phase flow is more complicated when flow is partially miscible. Nevertheless, a fundamental understanding of immiscible and compositional flow in 1-D homogeneous systems is relatively well-understood (e.g. Lake 1989 and Johns and Orr 1996).

An analytical solution to two-phase immiscible flow in composite media was developed by Wu et al. (1996). In their work, they considered the effect of Corey exponent variations on the analytical solution but did not compare their results to numerical simulations. Johnson et al. (1959) presented a technique (JBN method) to determine relative permeabilities from slim tube displacement data of pressure and oil recovery. The JBN method has been successfully applied to flow in parallel layers (Pande et al. 1987). The JBN method has not, however, been applied to determine the effective relative permeability for flow through composite media having different relative permeabilities.

In this paper, we address the problem of calculating convective fluxes at the interface of a composite porous medium where two-phase flow with phase behavior takes place. Benchmark analytical solutions are developed for series flow (i.e. composite flow) where each region is characterized by a different set of fractional-flow functions. Results obtained using single-point upstream and third-order TVD weighting are compared with the analytical solutions for a two-phase immiscible flow and a partially
miscible system. The JBN method is applied to the composite medium and the relative permeabilities attained are analyzed. The effect of capillary number on the solutions is also analyzed to demonstrate that fractional flow variability controls advection dominated displacements.

Mathematical Equations

One dimensional linear two-phase flow is considered in a composite system consisting of two domains in series (Fig. 1). Each domain has different rock properties and fractional-flow curves. Fluids are assumed to be incompressible and ideal mixing is assumed. Gravity, capillary pressure gradients, dispersion and diffusion effects are neglected. Equilibrium compositions are assumed to be insensitive to the pressure change over the displacement length. With these assumptions the flow equations for partially miscible flow can be written as

\[ \phi_D \frac{\partial C_i}{\partial x} + \frac{\partial F_i}{\partial x} = 0 \quad m = 1,2 \]  

where \( m \) refers to the two domains and \( C_i \) and \( F_i \) are defined as

\[ C_i = \sum c_y S_j \quad \text{and} \quad F_i = \sum c_y f_j. \]  

These and other parameters are defined in Table 1 and the nomenclature. The relative permeabilities are taken to be of Corey type (Lake 1989). For an immiscible system Eq. 1 reduces to

\[ \phi_D \frac{\partial A_m}{\partial x} + \frac{\partial F_m}{\partial x} = 0 \quad m = 1,2. \]  

At the boundary of the two domains, the overall fractional flows must be equal to conserve mass. Analytical solutions to these equations are obtained by the method of characteristics (Lake 1989).

Example Immiscible Solutions

Equation 1 is solved for an immiscible system with water displacing oil. The values of the parameters used in the analytical and numerical calculations are: \( S_{w_1} = 0.30, S_{w_2} = 0.15, S_{o_1} = 0.35, S_{o_2} = 0.20, n_w = 3.5, n_o = 2.0, n_a = 3.5, n_o = 2.0, k_{r_{m_1}} = k_{r_{m_2}} = k_{r_{o_1}} = k_{r_{o_2}} = 1.0, \mu_w = 1.0 \text{ cp, } \mu_o = 2.0 \text{ cp.} \]  

The interface of the two domains is taken to be at \( x = 0.50 \). These parameters are also defined in the nomenclature.

Analytical Solution

The analytical saturation profile for \( t = 0.3 \), i.e. after the injection of 0.3 pore volumes water, is shown in Fig. 2. The solution shows a large saturation jump from 0.537 to 0.659 at the interface of the two domains and a leading shock that is inside the second domain. The saturation jump is the result of specifying equal fractional flow rates at the interface of the two domains as demonstrated by Fig. 3. Table 2 gives more details of the solution.

Numerical Solution

Figure 2 also shows numerical solutions at 0.3 pore volumes injected (PVI), using the University of Texas Chemical Flooding Simulator (UTCHEM). The solution with 10 grid blocks and single-point upstream weighting does not accurately capture the discontinuity at the interface. The solutions with 100 and 1000 grid blocks capture the discontinuity at the interface reasonably well. The solution with 10 grids and third-order TVD (Liu et al. 1995) captures the discontinuity and the leading shock much more accurately than with single-point upstream weighting.

Example Solutions for Partially Miscible Binary System

Equation 1 is solved both analytically and numerically for a partially miscible displacement of decane by methane at a temperature of 160°F and pressure of 3500 psia. The values of the parameters used in the analytical and numerical calculations are: \( S_{g_1} = 0, S_{g_2} = 0, S_{h_1} = 0.35, S_{h_2} = 0.2, n_h = 3.5, n_i = 2.0, n_k = 3.5, n_k = 2.0, k_{r_{g_1}} = k_{r_{g_2}} = k_{r_{h_1}} = k_{r_{h_2}} = 1.0, \mu_g = 0.02 \text{ cp, } \mu_i = 0.14 \text{ cp, } c_{(CH_4)_g} = 0.963, c_{(CH_4)_i} = 0.645. \]  

The interface of the two domains is again at \( x = 0.50 \).
Analytical Solution

The analytical solution at \( t_0 = 0.5 \) PVI is shown in Fig. 4. An additional trailing shock exists as compared to the two-phase immiscible case. The trailing shock is very slow and hence at \( t_0 = 0.5 \), the trailing shock is still in the first domain. The analytical concentration and saturation profiles again show a significant jump at the interface of the two domains. Details of the solution are given in Table 3.

Numerical Solution

Numerical solutions using single-point upstream weighting for different grid sizes are plotted in Fig. 4. Similar to the immiscible system, single-point upstream weighting requires about 100 grids to capture the concentration and saturation discontinuities at the interface. The binary solution with 10 grid blocks is much less accurate than the solution with 10 grid blocks for the immiscible system.

Upscaling Relative Permeability

The JBN method offers an approach to determine the effective relative permeability functions. This method uses the overall pressure drop and oil production rate data to determine the fractional flow function and the effective relative permeabilities (Johnson et al. 1959, Jones and Roszelle 1978). The JBN technique is applied to the immiscible two-phase flow system. The input oil recovery and the overall pressure drop curves used in the JBN method are given in Fig. 5. The calculated effective oil relative permeability and water fractional flow for an equivalent homogeneous medium determined from those curves are shown in Fig. 6. This fractional flow gives the same oil recovery as Fig. 5 for all PVI. The oil relative permeability curve, however, is multivalued at the average connate water saturation. Thus, the pressure drop calculated is not unique and cannot reproduce Fig. 5. Although not shown, the effective relative permeabilities calculated with a harmonic average of the relative permeabilities (Lee et al. 1995) of the two domains also give a poor match to the curves of Fig. 5. A better fit could be obtained using power law averaging with variable exponents (Saad et al. 1995).

Effect of Capillary Pressure

The effect of capillary pressure on the immiscible system was analyzed for different capillary numbers. Both the fractional flow and capillary pressure must be continuous across the boundary of the two domains. Numerical solutions at \( t_0 = 0.3 \) with 500 grids using third-order TVD for three capillary numbers are plotted in Fig. 7. At low capillary number \((\sim 10^{-7})\), capillary forces dominate and the saturation jump at the interface is reduced. The shock is diffused and a capillary end effect is present at the inlet and outlet. At a higher capillary number \((\sim 10^{-6})\), which may be more representative of waterflood rates in the field, viscous forces dominate and the saturation profile is nearly the same as that with no capillary pressure. Although not shown, a thin boundary layer must exist at the discontinuity between the two domains where capillary pressure continuity is satisfied.

Conclusions

One-dimensional dispersion-free analytical solutions for composite media with fractional-flow heterogeneity for both immiscible and partially miscible flows were developed. A comparison was made between the numerical solutions using single and third-order upstream weighting and the analytical solutions. The validity of the JBN technique to determine the effective relative permeability and fractional flow functions was tested for the composite system. The effect of capillary pressure was also analyzed for the immiscible system. The following conclusions are made:

1. A significant concentration and saturation discontinuity can result from fractional flow heterogeneity at the interface of two domains.
2. Single-point upstream weighting requires about 100 grid blocks to capture the discontinuity at the interface, whereas a third order TVD method requires much fewer.
3. The JBN technique does not give correct effective relative permeability functions for composite systems with fractional-flow heterogeneity. The oil recovery can be matched but not the overall pressure drop.
4. Capillary pressure has negligible effect on 1-D solutions at high capillary numbers. Such solutions are controlled by fractional flow variations.
Nomenclature

\( A \) Cross-sectional area of the two domains (m^2)

\( c_{ij} \) Equilibrium volume fraction of component \( i \) in phase \( j \) (dimensionless)

\( C_i \) Overall volume fraction of component \( i \) (dimensionless)

\( f_{im} \) Fractional flow of phase \( j \) in domain \( m \) (dimensionless)

\( F_{im} \) Overall fractional flow of component \( i \) in domain \( m \) (dimensionless)

\( k_{ijm}^0 \) End-point value of relative permeability for phase \( j \) in domain \( m \) (dimensionless)

\( L_m \) Length of domain \( m \) (m)

\( n_{im} \) Corey exponent of relative permeability for phase \( j \) in domain \( m \) (dimensionless)

\( q \) Total volumetric flow rate of all phases (m^3/sec)

\( S_j \) Saturation of the phase \( j \) (dimensionless)

\( S_{r_m} \) Residual saturation of phase \( j \) in domain \( m \) (dimensionless)

\( t \) Total injection time (sec)

\( x \) Distance from the inlet (m)

\( \phi_m \) Porosity of domain \( m \) (dimensionless)

\( \mu_j \) Viscosity of phase \( j \) (cp)

\( \Delta P \) Overall pressure drop (psia)

\( (\_)_D \) Indicates dimensionless parameter

References

### Table 1: Definition of Dimensionless Groups Used.

<table>
<thead>
<tr>
<th>Dimensionless groups</th>
<th>Symbol</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>Porosity</td>
<td>( \phi_{D_m} )</td>
<td>( \frac{\phi_m}{\sum_{m=1}^{2} L_m \phi_m} )</td>
</tr>
<tr>
<td>Time (pore volumes injected)</td>
<td>( t_D )</td>
<td>( \frac{q t}{A \sum_{m=1}^{2} L_m \phi_m} )</td>
</tr>
<tr>
<td>Distance</td>
<td>( x_D )</td>
<td>( \frac{x}{\sum_{m=1}^{2} L_m \phi_m} )</td>
</tr>
<tr>
<td>Pressure</td>
<td>( \Delta P_D )</td>
<td>( \frac{\Delta P}{(\Delta P)_{\text{initial}}} )</td>
</tr>
</tbody>
</table>

### Table 2: Saturation Route for the Immiscible System.

<table>
<thead>
<tr>
<th>Domain 1</th>
<th>Domain 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_w )</td>
<td>Velocity (( dx_D / dt_D ))</td>
</tr>
<tr>
<td>Injectant</td>
<td>1.000</td>
</tr>
<tr>
<td>Initial fluid</td>
<td>0.300</td>
</tr>
<tr>
<td>Shock</td>
<td>0.508 to 0.300</td>
</tr>
</tbody>
</table>

### Table 3: Composition and Saturation Route for the Partially Miscible Binary System.

<table>
<thead>
<tr>
<th>Domain 1</th>
<th>Domain 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{CH_4} )</td>
<td>( S_g )</td>
</tr>
<tr>
<td>Injectant</td>
<td>1.000</td>
</tr>
<tr>
<td>Initial fluid</td>
<td>0.000</td>
</tr>
<tr>
<td>Leading shock</td>
<td>0.761 to 0.000</td>
</tr>
<tr>
<td>Trailing shock</td>
<td>1.000 to 0.791</td>
</tr>
</tbody>
</table>

![Flow diagram](image)

**Fig. 1** Flow through a composite media consisting of two domains with different fractional flows.

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Fig. 2 Immiscible system: Analytical and numerical solutions for various grid sizes at $t_D = 0.3$ PVI. SU: Single-point upstream, TVD: Third-order TVD.

Fig. 3 Illustration of how fractional flow continuity across the domains results in a saturation jump.

Fig. 4 Partially miscible binary system: Analytical and numerical solutions for various grid sizes at $t_D = 0.5$ PVI. Only single-point upstream weighting is used.
Fig. 5 Immiscible system: Pressure drop and oil recovery curves for two domains in series.

Fig. 6 Effective fractional flow and oil relative permeability curves for a homogeneous domain determined by JBN method.

Fig. 7 Effect of capillary number on the displacement for the immiscible example of Fig. 2 at $t_D = 0.3$ PVI.