A08

Intrinsic Attenuation in a Periodically Layered Medium

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SUMMARY

In this paper, I introduce the intrinsic attenuation for periodically layered structure to investigate how the intrinsic attenuation interferes with the scattering due to layering. The presence of intrinsic attenuation results in the stretch of reflection and transmission responses in frequency domain. I show that the anomalies in energy loss follow the stop-bands in frequency scale which are wider for higher acoustic contrast between the layers.

Introduction

Many authors are considering the stratigraphic filtering in their studies (O'Doherty and Anstey, 1971; Stovas and Arntsen, 2006). Physical experiments for periodically layered media were performed by Marion and Coudin, (1992) and analyzed by Hovem (1995) and Stovas and Ursin (2007). The standard eigenvalue decomposition method used for analysis of reflection and transmission responses in periodically layered media, does not take into account the intrinsic attenuation.

In our paper we extend this method for an anelastic medium and compute the reflection-transmission responses as well as the energy loss for the waves propagating in periodically layered structures. We consider the effect of layering and different contrasts between the layers. The low frequency limit for the phase velocity is exactly the same as for elastic case while the high frequency limit for anelastic case is different from the one computed for elastic case.

Intrinsic attenuation model

In order to introduce the intrinsic attenuation we choose the standard linear solid model (Carcione, 2001). The velocity dispersion is given by

$$v(\omega) = v_0 \sqrt{\frac{1 + i\omega\tau_\epsilon}{1 + i\omega\tau_\sigma}}, \quad (1)$$

where τ_ϵ and τ_σ are the relaxation times, and $v_0 = v(\omega = 0)$. The high frequency limit is

$$\lim_{\omega \rightarrow \infty} v = v_0 \sqrt{\tau_\epsilon / \tau_\sigma}.$$

The quality factor inverse is defined by

$$Q^{-1}(\omega) = \frac{\omega(\tau_\epsilon - \tau_\sigma)}{1 + \omega^2 \tau_\epsilon \tau_\sigma}. \quad (2)$$

We choose the relaxation times such that the minimum quality factor

$Q_{\min} = 2\sqrt{\tau_\epsilon \tau_\sigma} / (\tau_\epsilon - \tau_\sigma) = 20$ is reached at $f_{\min} = \omega_{\min} / 2\pi = \sqrt{\tau_\epsilon \tau_\sigma} / 2\pi \approx 150 \text{ Hz}$. The graphs for the quality factor inverse and real and imaginary part of the velocity are shown in Figure 1. Considering the anelastic-elastic interface, the complex frequency dependent normal incidence reflection coefficient is given by

$$r(\omega) = \frac{\xi^2 v_2^2 - |v_1|^2 - 2i\xi v_2 \text{Im} v_1(\omega)}{\xi^2 v_2^2 - |v_1|^2 + 2\xi v_2 \text{Re} v_1(\omega)}, \quad (3)$$

where the density ratio $\xi = \rho_2 / \rho_1$.

The propagator matrix

Hovem (1995) and Stovas and Ursin (2007) show that for the wave vertically propagating in a periodically layered medium, the propagator matrix for one period (two layers of thicknesses d_1 and d_2 and velocities v_1 and v_2) is given by

$$\mathbf{S} = \begin{pmatrix} a_1 & b_1 \\ b_2 & a_2 \end{pmatrix}. \quad (4)$$

For elastic case (Stovas and Ursin, 2007) $a_2 = a_1^*$ and $b_2 = b_1^*$, where the star denotes complex conjugate, and functions

$$\begin{aligned} a_{1,2} &= e^{\text{Im}\theta_1} e^{\pm i(\text{Re}\theta_1 + \theta_2)} \frac{1 - r^2 e^{\mp 2i\theta_2}}{1 - r^2} \\ b_{1,2} &= \pm e^{\text{Im}\theta_1} e^{\pm i\text{Re}\theta_1} \frac{2ir \sin \theta_2}{1 - r^2} \end{aligned} \quad (5)$$

with r being the reflection coefficient for the interface between the layers, and $\theta_k = \omega d_k / v_k$ are the corresponding phase factors. For the anelastic media, both $r = r(\omega)$ and $v_k = v_k(\omega)$ are complex functions.

The real part of function $Tr\mathbf{S}/2$ is given by

$$\frac{1}{2} \text{Re}(Tr\mathbf{S}) = \frac{1}{2} \text{Re}(a_1 + a_2) = \frac{e^{-\text{Im}\theta_1}}{1 - 2|r|^2 \cos 2\varphi + |r|^4} \left[\cos(\text{Re}\theta_1 + \theta_2) - 2|r|^2 \cos(\text{Re}\theta_1) \cos \theta_2 \cos 2\varphi + |r|^4 \cos(\text{Re}\theta_1 - \theta_2) \right] \quad (6)$$

with

$$\begin{aligned} \text{Re}\theta_1 &= \omega d_1 \text{Re} v_1^{-1} = \omega d_1 \text{Re} v_1 / |v_1|^2 \\ \text{Im}\theta_1 &= \omega d_1 \text{Im} v_1^{-1} = -\omega d_1 \text{Im} v_1 / |v_1|^2 \end{aligned} \quad (7)$$

and $\varphi = \varphi(\omega)$ being the phase of reflection coefficient (3). Note, that for elastic case,

$$Tr\mathbf{S}/2 = \text{Re} a_1.$$

The transmission and reflection responses for down-going wave are

$$\begin{aligned} t_D &= Q_{22}^{-1} = \frac{e^{i\alpha}}{\sqrt{1+C^2}} \\ r_D &= Q_{12} Q_{22}^{-1} = \frac{C e^{i\left(\alpha + \text{Re}\theta_1 + \frac{\pi}{2}\right)}}{\sqrt{1+C^2}} \end{aligned} \quad (8)$$

where Q_{ij} , $i, j = 1, 2$ being the elements of propagator matrix $\mathbf{Q} = \mathbf{S}^M$ from equation (9), α and C are the phase and amplitude factors, respectively. These factors are controlled by the conditions: $|Tr\mathbf{S}/2| < 1$ - propagating regime and $|Tr\mathbf{S}/2| > 1$ - blocking regime.

The energy loss for the wave propagating through the medium can be computed as

$$E = 1 - |t_D|^2 - |r_D|^2. \quad (9)$$

Numerical example

To test the intrinsic attenuation versus scattering for periodically layered medium we consider a similar model as in Marion and Coudin (1992) and with different reflection coefficients: the original $r(0) = 0.87$, $r(0) = 0.48$ and $r(0) = 0.16$. Change in reflection coefficient is obtained by changing the densities in the layers (Stovas and Ursin, 2007). The layering is representing by the index of the model, M_j , $j = 1, 2, \dots, 64$, where j denotes the number of periods. The total thickness $D = j(d_1 + d_2) = 51m$ is the same for all the models. The model parameters are taken from Stovas and Ursin (2007). The phase velocities for models with $r(0) = 0.48$ are shown in Figure 1, to the right. Note, the high-frequency limits are different for elastic and anelastic cases.

In Figure 2 we show the reflection and transmission responses for anelastic and elastic cases from equations (10) as well as the energy loss for anelastic case from equation (11) for the models M_1 (time-average medium), M_8 (resonance medium) and M_{64} (effective medium) for the reflection coefficient $r(0) = 0.48$. One can see that, regardless to the model, both reflection and transmission responses are smoother for the anelastic case, and their magnitudes decrease with frequency increasing. The model M_8 captures the transition zone (or stop-band) which is clearly pronounced for both elastic and anelastic cases. The graph for

energy loss has a distinctive anomaly coinciding with the stop-band position. One can also see the stretch effect of the reflection-transmission responses due to presence of attenuation. The model M_{64} represents the effective medium. The reflection-transmission responses and the energy loss curve are very similar to the ones from the model M_1 . The overall magnitude level for the reflection response is smaller for the effective medium. With reflection coefficient increases the stop-bands become wider (Figure 2).

The reflection and transmission response in time domain are shown in Figure 3 for all the models M_j and reflection coefficient $r(0) = 0.48$. Here we considered three cases: elastic-elastic, elastic-anelastic and anelastic-anelastic. The most pronounced difference in reflection response is observed for the multiples, while the primary reflections in effective medium are can not be discriminated. The propagation through the medium is blocked in transition zone regardless to the presence of attenuation. The difference between the phase velocities for time-average and effective medium is clearly seen on transmission responses.

Conclusions

The intrinsic attenuation results in smooth and stretched reflection-transmission responses from a periodically layered medium. The anomalies in the energy loss follow the stop-bands which become wider with reflection coefficient increasing. The high-frequency limit for the phase velocity is bigger for anelastic case.

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Figure 1. Intrinsic attenuation and velocity dispersion model (left) and phase velocities for the elastic (red line) and anelastic (blue lines) models with $r(0) = 0.48$.

Figure 2. The reflection (dash line) and transmission (solid line) responses and energy loss (yellow area) for elastic (red line) and anelastic (blue line) cases for models M_1 (top left), M_{64} (top right) and M_8 (in the middle) with reflection coefficient $r(0) = 0.48$, and model M_8 with reflection coefficients: $r(0) = 0.16$ (bottom left) and $r(0) = 0.87$ (bottom right).

Figure 3. The reflection (left) and transmission (right) responses for elastic-elastic (red line), anelastic-elastic (black line) and anelastic-anelastic (blue line) cases for all the models with reflection coefficient $r(0) = 0.48$.