SUMMARY

In order to incorporate the size of reservoir grids into the seismic frequency scale, the standard Backus averaging method is used for upscaling. We propose to extend the Backus averaging method for the low-frequency case and introduce the dispersive Backus model using the Baker-Campbell-Hausdorff (BCH) series. The accurate description of velocity dispersion for effective medium is very important in seismic modelling and inversion of seismic data into effective reservoir properties.
Introduction

An accurate description of a reservoir is crucial to the management of production and efficiency of oil recovery. Reservoir modelling is an important step in a reservoir’s future performance, which is in direct proportion to reservoir management, risk analysis and making key economic decisions. Saturation and pressure changes, and porosity and permeability distributions are the most common parameters to estimate in the oil industry.

Time-lapse (or 4D) seismic consists of two or more 3D seismic surveys shot at different calendar times. Time-lapse seismic surveys produce images at different times in a reservoir’s history. The seismic response of a reservoir may change due to changes in pressure, fluid saturation and temperature. These changes in seismic images due to a variation in saturation and pressure can be used as additional observation data. Time-lapse seismic data are dynamical measurements which have a high resolution in the horizontal direction, and give a significant image of fluid and pressure changes from the entire reservoir. Forward seismic modelling software, based on rock physical formulations (Gassmann equation and Hertz-Mindlin model) and matrix propagating techniques are used to provide 4D seismic amplitudes from saturation and pressure changes.

In order to incorporate the size of reservoir grids into the seismic frequency scale, the standard Backus (1962) averaging method is used for upscaling. In many cases, with strong heterogeneity within the reservoir unit, the Backus averaging is not accurate enough. We propose to extend the Backus averaging method for the low-frequency case and introduce the dispersive Backus model using the Baker-Campbell-Hausdorff (BCH) series (Serre, 1965). We derive the first- and second-order terms of this series, and extend this technique to the medium with arbitrary number of layers in a period. That results in the correction term for velocity dispersion at low frequencies. We show that the dispersion equation in such media is the even function of frequency.

The accurate description of velocity dispersion for effective medium is very important in seismic modelling and inversion of seismic data into effective reservoir properties.

The wave propagation in finely layered media

The vector containing the stress-strain components \( f(z) \) defined for each layer, satisfies the differential equation,

\[
\frac{df(z)}{dz} = \omega M(\omega)f(z),
\]

where \( \omega = \sqrt{-1} \), \( \omega \) is the frequency and matrix \( M(\omega) \) is defined by the horizontal slowness and the type of the medium. We can define the matrix \( \hat{M}(\omega) \) by following equation

\[
\hat{M}(\omega) = \frac{1}{i\omega H} \log P(\omega) = E^{-1} \text{diag}(q_k)E,
\]

where the matrix \( E \) is composed from the eigen-vectors of matrix \( P(\omega) \) and \( H = \sum_{j=1}^{N} z_j \) is the overall layer thickness (reservoir thickness). Let us denote \( A_j = \sum_{j=1}^{N} z_j^j M_j^j \). The BCH series is given by

\[
\hat{H}\hat{M}(\omega) = \hat{M}_0 + i\omega\hat{M}_1 - \omega^2\hat{M}_2 + o(\omega^3),
\]

\[
\hat{M}_0 = A_j, \quad \hat{M}_1 = \frac{1}{2} \sum_{N \neq j \neq k} z_z (M_j M_k - M_k M_j),
\]

\[
\hat{M}_2 = -\frac{1}{6} A_j + \frac{1}{4} (A_{j+k} + A_j A_k) - \frac{1}{3} A_{j+k} + \frac{1}{2} \sum_{N \neq j \neq k \neq l} z_z z_{kl} (M_j M_k M_l + M_k M_l M_j).
\]
For the vertical propagation and finite number of isotropic or transversely isotropic layers, the elements of matrices $\tilde{M}_0$, $\tilde{M}_1$ and $\tilde{M}_2$ can be given by simple equations. Note that matrix $\tilde{M}_0$ corresponds to the standard effective Backus (1962) medium. For a single layer $j$, we have

$$z_j M_j = t_j \begin{pmatrix} 0 & Z_j \\ Z_j & 0 \end{pmatrix},$$

(5)

where $Z_j = \rho_j V_j$ is the impedance, $t_j = z_j / V_j$ is the vertical traveltime in layer $j$, $\rho_j$ is the density and $V_j$ is the vertical velocity. From equations (4) we obtain

$$\begin{pmatrix} a_{0j} \\ b_{0j} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} a_{1j} \\ b_{1j} \end{pmatrix} = \begin{pmatrix} -a_{0j} \\ 0 \end{pmatrix}, \quad \begin{pmatrix} a_{2j} \\ b_{2j} \end{pmatrix} = \begin{pmatrix} 0 \\ b_{0j} \end{pmatrix},$$

(6)

$$a_j = \sum_{i=1}^{N_j} t_{ji} Z_{ji}, \quad b_j = \sum_{i=1}^{N_j} \frac{t_{ji}}{Z_{ji}}, \quad a_j = \frac{1}{2} \sum_{i \neq j, j=1}^{N} t_{ji} \left( \frac{Z_{ji}}{Z_j} - \frac{Z_j}{Z_{ji}} \right),$$

(7)

$$a_2 = -\frac{1}{6} a_1 b_1 + \frac{1}{2} a_1 \sum_{i=1}^{N_j} t_{ji}^2 - \frac{1}{3} \sum_{i=1}^{N_j} t_{ji} Z_{ji} + \sum_{i \neq j, j=1}^{N} t_{ji} t_{kj} \frac{Z_{ji}}{Z_{ij}},$$

$$b_2 = -\frac{1}{6} a_1 b_1^2 + \frac{1}{2} b_1 \sum_{i=1}^{N_j} t_{ji}^2 - \frac{1}{3} \sum_{i=1}^{N_j} t_{ji} Z_{ji} + \sum_{i \neq j, j=1}^{N} t_{ji} t_{kj} \frac{Z_{ji}}{Z_{ij}},$$

(8)

The wave propagation in a finely layered medium was investigated in many papers (see, for example, Stovas and Arntsen (2006) and Stovas (2007)).

### Periodically layered medium

The periodically layered medium is an important tool to analyze the vertically heterogeneous reservoirs consisting of shale and sand layers. For quasi-vertical propagation, the important parameter is the reflection coefficient computed at single shale-sand interface. The implicit formulas for reflection and transmission responses in a periodically layered medium are derived in Stovas and Ursin (2007). An example of reservoir model and synthetic seismic computed by assuming periodically layered sand-shale sequence within reservoir body is shown in Figure 1.

The dispersion equation for a homogeneous reservoir can be defined in terms of reflection coefficient

$$r = \frac{(Z_j - Z_i)}{(Z_j + Z_i)}$$

at interface between the layers,

$$\frac{1}{V^2(\omega)} = \left( \frac{\alpha_i}{V_i} + \frac{\alpha_j}{V_j} \right)^2 + \frac{4\alpha_i^2 \alpha_j^2}{V_i^2 V_j^2} \left( \frac{1}{1 - r^2} \right)^2 + \frac{4\alpha_i^2 \alpha_j^2 r^2}{3V_i^2 V_j^2 (1 - r^2)} + \omega^2 H^2 \left( \frac{1}{V_i^2 V_j^2 (1 - r^2)} \right) + \omega^4 H^4 \left( \frac{4\alpha_i^2 \alpha_j^2 r^2}{45V_i^4 V_j^2 (1 - r^2)} \right) \left[ \frac{\alpha_i}{V_i} + \frac{\alpha_j}{V_j} \right] + O(\omega^6),$$

(9)

where $Z_j = \rho_j V_j$, $j = 1, 2$, are the elastic impedances from shale and sand layers and $\alpha_j$, $j = 1, 2$ are the fractions for shale and sand.

The first term in (9) corresponds to the time-average (ray) velocity that is the infinite-frequency limit. The sum of the first and the second terms represents the Backus (1962) velocity that is the zero-frequency limit. The terms at $\omega^2$ and $\omega^4$ are the first and the second correction terms, respectively.

### Numerical examples

In order to illustrate the velocity dispersion equation, we consider the periodically layered model consisting of two layers with the following properties, $V_{p1} = 2000 \text{ m/s}$, $V_{p2} = 700 \text{ m/s}$,
$\rho_1 = 2000 \text{kg/m}^3$, $V_{S1} = 3000 \text{m/s}$, $V_{P1} = 900 \text{m/s}$ and $\rho_2 = 2200 \text{kg/m}^3$. In Figure 2, one can see the angle-dependent velocity dispersion contour plot (top-left) indicating the frequency-dependent induced anisotropy. The effective phase velocity, the Backus velocity and the Backus velocity with correction versus horizontal slowness for 2-layer velocity model (top right). The effective phase velocity versus horizontal slowness for 2-layer velocity model computed for frequency 10Hz, 20Hz, 30Hz, 40Hz and 50Hz (bottom left). The Backus and the ray velocities are shown by red and blue solid lines, respectively. Velocity versus frequency taken at different values of horizontal slowness is shown in Figure 2 (bottom right). One can see that the dispersive Backus velocity model results in more accurate velocity description at low frequencies for the finely layered medium. In Figure 3 (top), one can see the time-average velocity, the Backus velocity and the dispersive Backus velocity with the second- and fourth-order terms from equation (9). The error in effective velocity taken at frequency of 20Hz is about 60m/s. This error can result in uncertainties for net-to-gross estimation or fluid saturation estimation if seismic data are recorded with central frequency of 20 Hz (Figure 3, bottom).

Conclusions

We propose the new upscaling technique for a vertically heterogeneous medium that takes into account the velocity dispersion at low frequencies. This is an extension of well known Backus averaging method. The proposed method can be used in the modelling of seismic reservoir and inversion of seismic data into effective reservoir properties.

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References


![Figure 1](image_url). The reservoir model (left) and synthetic seismic data (right) computed by proposed technique.
Figure 2. The effective P-wave phase velocity versus horizontal slowness and frequency for 2-layer velocity model (top left). The effective phase velocity, the Backus velocity and the Backus velocity with correction versus horizontal slowness for 2-layer velocity model (top right). The effective phase velocity versus horizontal slowness for 2-layer velocity model computed for frequency 10Hz, 20Hz, 30Hz, 40Hz and 50Hz (bottom left). The Backus and the ray velocities are shown by red and blue solid lines, respectively. Velocity versus frequency taken at different values of horizontal slowness (bottom right).

Figure 3. The effective velocities computed from the time-average limit, the Backus limit, and dispersive Backus velocities of second- and fourth-order shown against the frequency (left) and net-to-gross (right). The errors due to dispersion neglecting are indicated by arrows.