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Stochastic Analysis of Cross-hole Gpr Data for Subsurface Characterization

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SUMMARY

We analyze the relationship between cross-hole GPR travel times and the spatial distribution of dielectric properties that conditions such data. The most common approach is to translate directly travel times into electromagnetic velocities and these in turn into dielectric constants and soil volumetric moisture content. This approach is known to lead to over-smoothed moisture content profiles that are not necessarily compatible with the true soil moisture content that should be reproduced by water flow models. We used an approach based on full waveform propagation and Monte Carlo simulations to assess, in both synthetic and real case studies, how reliable this direct wave approach is, and to develop an alternative stochastic inversion procedures that proves to provide robust estimates of soil moisture content profiles, also in presence of sharp boundaries.

Introduction

Ground penetrating radar (GPR) is a well-established technique, with over two decades of successful applications. In particular GPR is used to estimate soil volumetric moisture content, both from the surface and in boreholes (e.g. *Binley et al.*, 2001 and 2002; *Cassiani et al.*, 2004). The velocity of electromagnetic waves in a medium, v , is related to the bulk relative dielectric permittivity (or dielectric constant), ϵ_r [-], of the medium via

$$\sqrt{\epsilon_r} = c / v \quad (1)$$

where c is the wave velocity in air (0.3 m/ns). There are relationships that link ϵ_r to volumetric moisture content (θ), such as the empirical *Topp et al.* (1980) equation or the semi-empirical complex refractive index method (CRIM) (*Roth et al.*, 1990). In many studies cross-borehole zero offset profiles (ZOP) are used to infer subsoil moisture content, to be used to calibrate hydrological models and consequently estimate hydraulic characteristics (e.g. *Deiana et al.*, 2008; *Looms et al.*, 2008). Different approaches have been developed to obtain hydraulic parameters from ZOP surveys. The easiest is the direct-wave approach, where point ZOP travel times are converted into velocities and subsequently into ϵ_r estimates using eq. (1). This approach can be misleading as it does not take into account two essential factors: volume averaging (Fresnel zone) and critical wave refractions that can occur between sharp ϵ_r boundary transitions. *Cassiani and Binley* (2005) analyzed a ZOP cross-hole GPR response over a long time series. The 50 MHz GPR-derived moisture content profiles, originally derived using a direct-wave approach, were compared against vertically averaged simulated data that took into account the vertical scale of measurements compatible with the antenna length and the Fresnel zone width. *Rucker and Ferré* (2004) analyzed the impact of critically refracted waves on a first arrival travel time profile, derived from ZOP measurements in a layered system with sharp changes in water content. These results confirm that caution is needed in the interpretation of radar ZOPs. The principal aim of the present work is to have a more complete view of how boreholes GPR ZOP measurements are informative of the subsoil geometry and distribution of relative permittivity, with the ultimate goal of acquiring data that can be more reliably used for the calibration of hydrological models.

Methodology

An electromagnetic (EM) wave simulator has been applied within a stochastic Monte Carlo framework. In this manner both averaging and critically refracted wave effects are taken into account. Results from synthetic and real ZOP datasets are statistically analysed. The simple direct-wave method is compared with the results obtained from the stochastic inversion. The aim of the stochastic approach is to reconstruct the dielectric properties distributions that can produce the measured ZOP signal. Every realization generates a random ϵ_r geometry, used as input for the EM wave propagation simulator GprMax2D (*Giannopoulos*, 2005). For the generation of the ϵ_r distributions, the subsurface is idealized as horizontally layered. The 1D approximation is reasonable, since transmitting and receiving antennae are at close distance (boreholes are usually less than 10 m apart), so the underground materials are not expected to vary significantly. The material properties generated are limited to dielectric permittivity. GprMax2D does not model the physical structure of the GPR antennas that are idealized as Hertz dipoles. For each ZOP shot, real (TT_r) and simulated travel times (TT_i) are statistically compared using chi-squared factor to estimate the goodness of fit:

$$\chi^2 = \frac{1}{n} \frac{(TT_i - TT_r)^2}{\sigma^2} \quad (2)$$

where n are the degrees of freedom, in this case equal to 1, and σ is the measurement error standard deviation. The measurement error depends on many features: operators, environment, media properties; for this reason we employed different values of standard deviation (from 0.5 to 1.5 ns). Only values for which $\chi^2 \leq 1$ are stored, and our interest focused on the ϵ_r values that produce TT_i below this threshold. For these values, the Fresnel volume is calculated in a iterative method, starting with the velocity obtained directly from TT_r plus 50%. At each iteration the Fresnel volume is computed and the mean ϵ_r value inside it is kept. Then the posterior velocity is deduced from the

mean ϵ_r value and compared with the prior velocity; this procedure goes on until the error between the two subsequent velocities is less than 5%. In the end, the magnitude of the vertical segment around the position of each transmitter-receiver depth is obtained, i.e. the Fresnel ellipsoid minor axis. The distribution of ϵ_r values, generated at that realization and related to each detected vertical segment, is stored. Where overlapping segments are present, ϵ_r -distributions are intersected to preserve only values present in both ϵ_r sets. In the end, an uncertainty quantification of ϵ_r in function of depth is possible, computing median, mean and data variance.

Fig. 1 – Synthetic dataset. Right panels: distributions of ϵ_r values after the reduced chi-squared analysis on 20000 realizations, with different standard deviation values (0.75 and 1.25 ns). The black line is the true 1-D ϵ_r profile and the red dashed line is the ϵ_r profile obtained from the direct-wave approach. Left panels: the grey lines are the ϵ_r confidence intervals computed as two times the standard deviation around the mean value. The orange line is the median of the data distribution.

Analysis and discussion

Synthetic dataset

We first considered a synthetic case made of an alternating sequence of fine, thin layers (highest ϵ_r values) and coarse, thick layers (lower ϵ_r values). Alternating lithological sequences are common in shallow sediment covers and they are hard to define from ZOPs. The synthetic profile, taken as the *true* field-measured ϵ_r profile, is a one-dimensional ϵ_r distribution with homogeneous values in each layer (Fig.1). Transmitter and receiver antennas of 100 MHz frequency are virtually placed in different boreholes, 5 m apart, and lowered simultaneously of 0.25 m from -1.5 to -8 m depth. The

first shot is located at -1.5 m depth to avoid critically refracted waves in air. The geometry of the system is made to vary in a Monte Carlo manner, albeit with well-defined constraints limiting the ranges of thickness and approximate location of the layers. Permittivity ranges are chosen according to the literature. The methodology described in the previous section is applied in a Monte Carlo simulation with 20000 realizations. Fig.1 (left panels) shows the resulting ϵ_r distributions as a function of depth with two different standard deviation values ($\Delta = 0.75$ and 1.25 ns). The *true* ϵ_r profile and the ϵ_r profile directly derived from first-arrival travel times are also shown. A large discrepancy is present between the curve obtained directly from the *true* first-arrival travel times and the *true* ϵ_r profile (solid black line): the sharp geometry is smoothed. The finer high permittivity layer at about -2.6 m depth is not reproduced in its features and its ϵ_r value is underestimated. The narrow fine layer located at about -4.3 m is completely invisible for the direct-wave profile, in spite of its possible relevance in hydrogeological characterization. The variance of ϵ_r , derived from the stochastic approach, is about centred around the real profile, meaning that the results are close to the real physics of the system. Fig.1 (right panels) shows the mean, median and confidence interval profiles derived from the statistical analysis above. The statistical analysis demonstrates that thin layers can be invisible to investigation, underestimated or misinterpreted, in spite of their high relevance in hydrological processes. Thicker layers have, on the contrary, narrow range boundaries, where permittivity could be determined with a good degree of uncertainty.

Fig. 2 - Gorgonzola site dataset: comparison between the core log (on the right) and stochastic ϵ_r - range boundaries with 1.5 ns as error standard deviation (on the left).

The Gorgonzola field site

The considered experimental dataset comes from a field site located at Gorgonzola, east of Milan, in the Po River valley, northern Italy. The water table at the moment of sounding, on 07 April 2005, was at -20 m depth. The unsaturated zone is composed of Quaternary sediments with a fairly coarse sand-gravel grain size distribution (Deiana et al, 2008). Analysis of a soil core, extracted about 3 m apart the measurement boreholes, has provided direct knowledge of the site stratigraphy (Fig.2). A PulseEKKO 100 system was used with 100 MHz borehole antennas, which were lowered with 0.25 m vertical spacing from -3 to -18 m. Boreholes are 6.65 m apart. We applied to this real dataset the same procedure described above. Similar to the synthetic case, the direct-wave approach is not able to

reproduce the sharp heterogeneities, particularly between -10 to -18 m. At Gorgonzola, a drilled core was extracted very close to the GPR boreholes, so a direct comparison is possible. The core log and the ϵ_r profile from the stochastic approach, below -10 m depth, are in perfect accordance, reproducing the geometry of the system (Fig.2). In particular the cemented layer between -12 and -14.5 m and the gravel-sandy layer above are perfectly defined with a sharp geometry, as evidenced by abrupt changes in core lithology. The comparison between the geophysical inversion and the core log gives us confidence on the validity and applicability of the stochastic approach in this heterogeneous and complex subsoil system. The detection of the cemented layer, between -12 and -14.5 m, as a high permittivity material is a key point for the identification of the subsurface hydrological behaviour.

Conclusions

We conducted an analysis of how a cross-hole GPR ZOP can be informative of soil dielectric properties. A stochastic Monte Carlo framework is applied, with the aim of inferring knowledge about the subsoil geometry, the permittivity distribution and the related uncertainty on the physical underground system. One synthetic and one experimental field cases are analysed. The synthetic case is relevant in the validation of the stochastic approach, since the material properties, targets of the method, are a priori defined. Both real and synthetic datasets illustrate how the often employed direct-wave approach is not able to take into account the complexity of the system. This simple approach usually reconstructs a smooth profile, underestimating or overestimating the real permittivity values: particularly a high ϵ_r (moist) layer might be underestimated, when it is enclosed between low ϵ_r media. Results show that care must be used when inverting ZOP data for physical parameter estimation, as subsurface stratification could be more complex than that apparent from direct ZOP evidences. Misleading assignment of material properties could make the evaluation significantly diverge from reality. The evidence, from the synthetic and the real datasets, is that thin layers with high permittivity values, e.g. thin clay layers or lens, are not well defined. A large uncertainty is linked to such layers that could be invisible to investigation or misinterpreted. Where the subsurface is characterized by thicker layers, both high or low ϵ_r values are well defined with lower uncertainty. The comparison with core logs, in the case of the Gorgonzola field site, shows a good match of system geometries, allowing more confidence on the validity of the analysis and on the one dimensional underground simplification. The way hydrological state variables, such as moisture content, are derived from geophysical data is a key point, as every misinterpretations can alter the results and move them away from the true hydraulic state of the system. The understanding of uncertainty is essential to improve knowledge on the probable parameter distribution, while ignoring thin finer layers or underestimating media properties leads to inaccurate hydrological assumptions.

References

- Binley A., Winship P., Middleton R., Pokar M., West J.; 2001: High Resolution characterization of vadose zone dynamics using cross-borehole radar. *Water Resources Research*, 37, 2639–2652.
- Binley A., Cassiani G., Middleton R., Winship P.; 2002: Vadose zone flow model parameterisation using cross-borehole radar and resistivity imaging. *Journal of Hydrology*, 267, 147–159.
- Cassiani, G., Strobbia C., Gallotti L.; 2004: Vertical radar profiles for the characterization of deep vadose zones. *Vadose Zone J.*, 3, 1093–1115.
- Cassiani G., Binley A.M.; 2005: Modeling unsaturated flow in a layered formation under quasi-steady state conditions using geophysical data constraints. *Adv. Water Resour.*, 28, 467–477.
- Deiana R., Cassiani G., Villa A., Bagliani A., Bruno V.; 2008: Calibration of a Vadose Zone Model Using Water Injection Monitored by GPR and Electrical Resistance Tomography. *Vadose Zone J.*, 7, 215–226.
- Giannopoulos A.; 2005: Modelling ground penetrating radar by GprMax. [Construction and Building Materials](#), 19, 755–762.
- Looms M.C., Jensen K.H., Binley A., Nielsen L.; 2008: Monitoring unsaturated flow and transport using cross-borehole geophysical methods. *Vadose Zone J.*, 7, 227–237.
- Roth K., Schulin R., Fluhler H., Hattinger W.; 1990: Calibration of time domain reflectometry for water content measurements using a composite dielectric approach. *Water Resources Research*, 26(10), 2267–2273.
- Rucker D.F., Ferré T.P.A.; 2004: Correcting water content measurement errors associated with critically refracted first arrivals on zero offset profiling borehole ground penetrating radar profiles. *Vadose Zone J.*, 3, 278–287.
- Topp, G.C., Davis J.L., Annan A.P.; 1980: Electromagnetic determination of soil water content: Measurements in coaxial transmission lines. *Water Resour. Res.*, 16, 574–582.