Tu SP1 06

Time-frequency Analysis of Noisy Seismogram Using Modified S-transform

A.P. Gahlot* (Indian School of Mines)

SUMMARY

In noisy seismogram, it is very difficult to determine the arrival time of various phases like, P and S. However, as the seismic signals are non-stationary in nature analysing seismogram in time-frequency representation can give the location of P,S-wave with high accuracy. For exact location, higher time and frequency resolution is required and hence, needs a method which can provide enhanced energy concentration in the time-frequency domain. In past few years, S-transform has become a popular tool for analysis of time-frequency distribution of seismic signals. The method provides multi-resolution analysis while retaining the absolute phase information of each frequency component of the signal, and hence is superior to others like Short Time Fourier Transform (STFT), Continuous Wavelet transform (CWT). The S-transform is considered conceptually a hybrid of STFT and CWT. Standard S-transform technique has frequency dependent window,so it has some limitations as frequency resolution will be poor at high frequency and degradable time resolution at low frequency. In this paper, we analyse the noisy seismogram in the time-frequency plane using modified S-transform technique in which window consists of additional parameters which controls the scale and shape of window in such a way as to provide improved energy concentration of spectrum.

Introduction

In noisy seismograms, it is very difficult to determine the arrival time of various phases like, P and S. The exact location of these phases necessitates using a method that provides enhanced energy concentration in the time-frequency domain leading to better resolution. In the past few years, S-transform has become a popular tool for analysis of the time-frequency distribution of seismic signals. Standard S-transform technique has frequency dependent window and hence, it has some limitations as frequency resolution will be poor at high frequency and degradable time resolution at low frequency. In this paper, I have analyzed noisy seismograms in the time-frequency plane using modified S-transform technique where localizing window consists of additional parameters that controls the scale and shape of window leading to improved energy concentration of spectrum.

S-transform

S-transform (Stockwell et al. 1996) is a time-frequency technique consisting of moving and scalable Gaussian window which gives frequency dependent resolution while maintaining direct relationship with the Fourier transform. S-transform of the signal $x(t)$ is defined as

$$ST(\tau, f) = \int_{-\infty}^{\infty} x(t) \frac{|f|}{\sqrt{2\pi}} e^{-\frac{f^2(\tau-t)^2}{2}} e^{-i2\pi ft} dt \quad (1.1)$$

where τ is the time of localization of Gaussian window and f is the frequency. To allow for invertibility of this technique, window must satisfy the following normalization condition

$$\int_{-\infty}^{\infty} \frac{|f|}{\sqrt{2\pi}} e^{-\frac{f^2(\tau-t)^2}{2}} d\tau = 1 \quad (1.2)$$

Pinnegar and Mansinha (2003) took hyperbolic window and generalized the S-transform by taking f to be f/γ , for better control of the window function. For analyzing seismogram, good time resolution at event initiation time and good frequency resolution for determining the frequency content at that time is required. This can be achieved by modifying the S-transform by providing additional parameters controlling the shape and scale of localizing window to enhance time-frequency localization.

Modified S-transform

Assous and Boashash (2012) modified the S-transform by defining the parameter γ to vary linearly with frequency to get progressive control of the window width as follows:

$$\gamma(f) = mf + k \quad (2.1)$$

where m is the slope and k is the intercept for linear change in frequency. They found empirically that depending upon the signal under consideration, the value of k equals $1/N$ and that of m equals four times the variance of the signal, where N is the length of the signal. Using the similar concept, I have introduced additionally, a parameter 'p' which is user defined, for the better control of the window width. With this parameter, γ varies with frequency, though not linearly but gives good time and frequency localization. Using this methodology, MST becomes

$$MST(\tau, f, m, k, p) = \int_{-\infty}^{\infty} x(t) \left\{ \frac{|f|}{\sqrt{2\pi}((mf)^{-p} + k)} e^{-\frac{f^2(\tau-t)^2}{2((mf)^{-p} + k)}} e^{-i2\pi ft} \right\} dt \quad (2.2)$$

Simulation Results and Interpretation

In order to simulate the suitability of the proposed method in determining the location of P-wave arrival time, I carried out tests over a synthetic sinusoidal signal (Fig. 1(a)) of length 4π and having an impulse at 0.9 sec. To make this signal more realistic, I added Gaussian distributed noise and also random noise to the whole signal. Fig. 1(b) shows earthquake seismogram which was recorded on July 26, 2011 near Idukki reservoir, Kerala, India (9.75° N, 76.93° E). The magnitude of earthquake was 3.5, having focal depth 9.5 Km and the origin time was recorded to be 7Hr 39Min 16Sec.

Figure 1 (a) Synthetic Noisy Signal (b) Earthquake Seismogram.

Figure-2 shows S-transform of the full length of synthetic signal (Fig. 2(a)) and of 9 sec. segment of the earthquake signal (Fig. 2(b)). Also, I carried out Modified S-transform of these signals by taking the value of parameter, $p = 0.06$ in case of synthetic signal and $p=0.020$ for the earthquake signal (Figure 3a,b). It can be visualized easily that the time –frequency representation of the signals in Figure-3 have more vertical signature than that of the S-transform in Figure-2.

Figure 2 (a) ST of Synthetic signal (b) ST of Earthquake Seismogram.

Figure 3 (a) MST of Synthetic signal (b) MST of Earthquake Seismogram.

Time-Frequency Filter

For filtering non-stationary signals, filters with time-varying pass-bands and reject-bands need to be applied to the time-frequency representation of the signal. This possibilities of filtering the signal in the invertible S-transform domain have been discussed in literature (Pinnegar and Mansinha 2003, Simon et al. 2007). Inverse S-transform of the filtered region containing impulse in synthetic signal in Fig. 4(a) and region containing P-wave arrival in earthquake seismogram in Fig.4(b) has been carried out which gives the polarity and exact location of both the events. Position of the impulse (peak before this event is due to noise) in Synthetic signal is 0.94 sec. while the positive polarity of the first motion showing P-wave arrival in earthquake signal is at 280.83 sec. as shown in the figure.

Figure 4 (a) Filtered Synthetic signal (b) Filtered Earthquake Seismogram.

Conclusions

In this paper, I have presented a modified version of S-transform in which the localizing window consists of additional parameters to provide enhanced energy concentration in the time-frequency plane. This modified version has been used to locate the P-wave arrival time with high accuracy in a very noisy earthquake data. Using time-frequency filter, I have filtered the signal in S-transform domain to get the polarity of the P-wave (first motion) which is useful in determining the focal mechanism of an earthquake and in interpreting the fault-plane.

References

- Assous, S. and Boashash, B. [2012] Evaluation of the modified S-transform for time-frequency synchrony analysis and source localization. *EURASIP J. Adv. signal Process.*, **49**.
- Pinnegar, C.R., and Mansinha, L. [2003] The S-transform with windows of arbitrary and varying shape. *Geophysics*, **68**, 381-385.
- Simon, C., Ventosa, S. and Schimmel, M. [2007] The S transform and its inverses: side effects of discretizing and filtering. *IEEE Transactions on Signal Processing*, **55**(10), 4928-4937.
- Stockwell, R.G., Mansinha, L. and Lowe, R.P. [1996] Localization of the complex spectrum: the S Transform. *IEEE Trans. Signal Process.*, **44**, 998-1001.