Application of particle method to forward modeling of marine controlled-source electromagnetic survey

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We developed a controlled-source electromagnetic (CSEM) forward modeling scheme based on a particle method to simulate electromagnetic fields. As a particle method, we used the moving particle semi-implicit (MPS) method. In MPS method, the choice of particle distribution has large arbitrariness. Since numerical accuracy and computational load depend highly on the number of particles and their distribution, the choice of particle distribution is important. We select non-staggered and staggered particle distributions to compare the numerical accuracy and computational load. As a result, the numerical accuracy of two kinds of particle distributions is almost the same as each other if the spatial interval is equivalent. Because the number of particles required in the staggered particle distribution becomes half that in the non-staggered particle distribution, we conclude that staggered particle placement is numerically effective than the non-staggered.

1. INTRODUCTION

To date, controlled-source electromagnetic (CSEM) method is widely used for surveying subsurface resistivity structure. This technique has been used for the detection of target such as hydrocarbon reservoirs¹⁾, structures from crust down to uppermost mantle²⁾, gas hydrate³⁾, seafloor mounds⁴⁾, CO₂ monitoring⁵⁾ and hydrothermal vents⁶⁾. In previous studies, many numerical schemes have been developed to improve computational performance and numerical accuracy.

One of the simple and powerful methods is the finite-difference method (FDM). In FDM, a staggered grid technique developed by Yee⁷⁾ is often used. In this method, the electric and magnetic fields are defined at two different sets of grid points staggered to each other. Recently, particle-based methods have been applied to numerical simulations. A moving particle semi-implicit (MPS) method is one of the particle method developed by Koshizuka and Oka⁸⁾. This method is applied to seismic wave propagation by Takekawa⁹⁾ et al..

In this study, we applied MPS method to the simulation of CSEM simulation. We defined two kinds of basic particle distribution to investigate the numerically effective particle distribution. One of the basic particle distributions is a non-staggered particle arrangement, which defines both electric and magnetic fields in each particle. The other is a

staggered particle distribution, which defines the electric and magnetic fields at two different sets of grid points staggered to each other. The reason why we choose these particle distributions is that these particle distributions are basic. We discussed the numerical accuracy and computational load between these particle distributions with the same spatial interval. The numerical accuracy is checked by the analytical solution obtained by EM1D.

2. METHOD

In this section, we limit ourselves to a discussion of the main ideas, because FDTD modeling algorithm except for the discretization is described in detail in Mittet (2010). In the low-frequency limit relevant to CSEM exploration, the physical problem of interest can be modeled by the relations,

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial}{\partial t} \mathbf{H} - \mathbf{K} \quad (1)$$

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \mathbf{J} \quad (2)$$

Here $\mathbf{E}$ and $\mathbf{H}$ are electric and magnetic fields, $\mathbf{J}$ and $\mathbf{K}$ denote electric and magnetic current densities of any external electromagnetic sources, σ is electrical conductivity, and μ_0 is magnetic permeability of free space. The main idea behind FDTD modeling is to transform eq. (1) and (2) mathematically to avoid the wide range of propagation velocities. After transforming these equations, this range of propagation is reduced as suggested by Mittet (2010). Following the derivation in Mittet (2010), we obtain

the Maxwell equation in the fictitious time domain as,

$$\nabla \times \mathbf{E}' = -\mu_0 \frac{\partial}{\partial t'} \mathbf{H}' - \mathbf{K}' \quad (3)$$

$$\nabla \times \mathbf{H}' = \varepsilon' \frac{\partial}{\partial t'} \mathbf{E}' + \mathbf{J}' \quad (4)$$

Here ε' satisfy $\varepsilon' = \sigma/2\omega_0$ with $\omega_0 \in \mathbb{R}^+$ is an arbitrary parameter. In this case, ω_0 is set as $\omega_0 = 2\pi$. $\mathbf{E}'$ and $\mathbf{H}'$ are electromagnetic fields in the fictitious time domain. $\mathbf{J}'$ and $\mathbf{K}'$ denote electromagnetic current densities in the same domain. After calculating electromagnetic fields in the fictitious time domain, the physical electromagnetic fields $\mathbf{E}$ and $\mathbf{H}$ can be recovered from $\mathbf{E}'$, $\mathbf{H}'$, $\mathbf{J}'$ using the modified Fourier transform.

In this study, we employ MPS method to discretize eq. (3) and (4). In MPS method, each particle denotes electromagnetic fields in its position. Partial derivatives of electromagnetic fields on a particle can be calculated with the neighboring particles, which have certain distance from it (Figure 1). We call this domain as the influence domain. The radius of the influence domain controls the number of neighboring particles. A particle interacts with its covered with a weight function $w(r)$, where r is a distance between two particles. A weight function employed in this study is as follows,

$$w(r) = \begin{cases} r_e/r - 1 & (0 \leq r < r_e) \\ 0 & (r_e \leq r) \end{cases} \quad (5)$$

Here r_e restricts interaction to a finite distance. Thus, finite number of neighboring particle is related to the interactions. This weighting value at the considered particle is infinite, and becomes zero outside the influence domain. The gradient vector at $\mathbf{r}_i$ is calculated by the following equation,

$$\langle \nabla \phi \rangle_i = \frac{d}{\langle n \rangle_i} \sum_{j \neq i} \left[\frac{\phi_j - \phi_i}{|\mathbf{r}_j - \mathbf{r}_i|^2} (\mathbf{r}_j - \mathbf{r}_i) w(|\mathbf{r}_j - \mathbf{r}_i|) \right] \quad (6)$$

Here, ϕ is an electromagnetic field on each particle. d is the number of space dimensions. $\mathbf{r}$ denotes the position vector on each particle. $\langle n \rangle$ is a particle number density defined as,

$$\langle n \rangle_i = \sum_{j \neq i} w(|\mathbf{r}_j - \mathbf{r}_i|) \quad (7)$$

The Maxwell's equations are discretized with this gradient vector. The convolutional perfectly matched layer (Roden and Gedney, 2000) is used for absorbing boundary condition.

Figure 1 The particles in the influence domain. The shaded particles are selected ones.

Figure 2 Two kinds of arrangement of particles in MPS discretization. a) Particle distribution is non-staggered. Each particle has the degree of freedom of 3-components of electric and magnetic fields. The particles denote three components of electromagnetic fields. b) Particle distribution is staggered. Each particle has the degree of freedom of 3-components of electric or magnetic fields. The shaded particles denote three components of electric field. The lightened particles denote three components of magnetic fields.

Figure 3: Accuracy of forward simulation changing the arrangement of particles and used frequencies. Upper three figures a), b) and c) are calculated by the model A. Lower three figures d), e) and f) are calculated by the model B. Each of the used frequencies is written in the figures. In each figure, the black dots show the forward solutions against the offset. The black line shows the analytical solution. The shaded dots show the normalized ratio between numerical solution and analytical solution.

3. RESULTS & DISCUSSION

We defined two kinds of basic particle distribution to investigate the numerically effective particle distribution. In our MPS method, the discretized particles are arranged using a unit hexahedron (Figure 2). Figure 2 (a) shows the non-staggered distribution. We call this distribution as model A. Figure 2 (b) shows the staggered distribution and is called model B. From these figures, it is obvious that the degree of freedom in staggered distribution is half compared with the non-staggered distribution. This means that the computational load of model B is almost half compared with that of model A.

To check the numerical accuracy, we used these two kinds of the particle distribution. Conductivity in the model is set to 3.3 S/m. The influence domain is set to 20 m. The particle spacing is 20 m. The number of unit hexahedrons used in this simulation is $61 \times 61 \times 61$. Boundary condition is set 5 unit hexahedrons from each edge. The results are shown in figure 3. The normalized ratio means the noise level of forward simulation. From results, the character of normalized ratio is almost same regardless of the used frequencies. The normalized ratio becomes larger when the offset becomes

smaller. This is because the detail shape of transmitter dipole is practically difficult to include model. The normalized ratio in the model A is from 5×10^{-3} to 6×10^{-2} . The normalized ratio in the model B is from 8×10^{-3} to 5×10^{-2} . From this, the numerical accuracy of model A and B is the almost same, although the numerical load of model B is half of model A. The advantage of model B is that it requires half computational load if the lower spatial interval of electromagnetic fields is sufficient.

4. CONCLUSION

We presented MPS CSEM forward simulation using two kinds of particle distribution. One of the particle distributions is non-staggered placement and the other staggered. In this study, we confirmed that the numerical accuracy is almost the same as each other if the particle spatial interval is equivalent. The staggered placement of particles requires half the number of particles to obtain similar spatial interval compared with the non-staggered. We conclude that the staggered particle placement is numerically effective than the non-staggered.

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