Numerical simulation of hour-scale magma migration using pre-eruptive ground deformation

Soma MINAMI$^{1,3}$, Masato IGUCHI$^2$, Hitoshi MIKADA$^3$, Tada-nori GOTO$^3$, Junichi TAKEKAWA$^3$

$^1$Idemitsu Oil and Gas Co., Ltd.
$^3$Dept. of Civil and Earth Res. Eng., Kyoto Univ.

Analysis of ground deformation around a volcano can provide magma accumulating process of plumbing system and infer invisible magma migration in the system. We developed a coupling scheme of modeling of magma plumbing system and magma flow simulation to quantitatively understand pre-eruptive magma migration in hour scale from observation of ground deformation. Our scheme was applied to ground deformation before an explosive eruption at the Showa Crater of Sakurajima volcano on April 9, 2009. The ground deformation shows that a periodic inflation and deflation event had lasted 30 hours before the eruption. Our model composed of shallower gas and deeper magma reservoirs connected by a volcanic conduit that had been suggested by the past geophysical observations. A pressure difference between the two reservoirs forces the magma to move from the deeper up to the shallower reservoir. We assumed a constant rate of magma supply to the deeper reservoir as an input to the magma plumbing system and a viscous magma flow in a cylindrical volcanic conduit. As a result of parametric inversion to reproduce the observed volumetric behavior, it is estimated that relatively high permeable magma ascends to the shallower in a volcanic conduit with 125 m radius. The result also shows that the gas in magma mainly ascends to the shallower and that the supply rate to the deeper reservoir on April 9 is ca. 16 m$^3$/s.

Finally, we found that our numerical scheme can provide the pre-eruptive magma migration in hour scale as well as constrain some unknown parameters.

1. INTRODUCTION

Pre-eruptive magma migration and accumulating process beneath a volcano are of importance to understand precursory phenomena of an eruption. Characteristic time scale of the phenomena can vary depending on depth of the phenomena. Magma flow during an eruption near the surface in time scale of seconds to minutes has been studied by numerical flow simulations to investigate the mechanism and diversity of volcanic eruptions$^1$. On the other hand, the day- to year-scale magma migration and accumulation at depth of below a few kilo meters have been discussed for estimation of the migration path and pattern and for long-period prediction of a next great eruption$^3$. In spite of the many studies on the deep and the shallow part in the plumbing system, few attempts have been made at the hour-scale magma migration and accumulating process at depth of a hundreds of meters to a few kilo meters. In volcanoes with well-arranged observation system like Sakurajima volcano, the magma plumbing system in the medium depth has been discussed. However, the magma migration in hour scale has not been quantitatively estimated yet. We therefore try to capture the hour-scale migration before an eruption using magma flow simulation.

The magma flow simulation has been carried out for explaining the flow effusing to the surface$^1$, and some observable parameters such as discharge rate, an amount of volcanic product etc. could be useful to evaluate the flow. When the simulation is applied to the invisible magma flow before an eruption, there is a difficulty that the simulation cannot be evaluated by the observables. However, the invisible flow could be estimated by observation of ground deformation because the volumetric or pressure change of both pressure sources at starting and terminal points of the flow can be obtained from the ground deformation. This suggests that the coupling scheme of magma flow simulation and modeling of magma plumbing system composed of a volcanic conduit and magma reservoirs can provide quantitative information of the pre-eruptive magma migration.

We developed the coupling scheme in Sakurajima volcano because of the well-arranged observation system and the understanding of the magma plumbing system. Existence of the two reservoirs beneath the volcano has been estimated from the past geophysical studies$^{3,4}$. The ground
deformation preceding the explosive eruption at the Showa crater on April 9, 2009 suggested that there is a short-term periodical inflation and deflation cycle of the two reservoirs with a few hours of time-lag before the eruption. Since this characteristic behavior would reflect the magma ascent to the shallower, we try to explain the observed volumetric variations of the reservoirs using the coupling scheme to explain the magma migration before an eruption.

For this objective, we first modeled a magma plumbing system including the two reservoirs, and then performed numerical simulations of the magma migration in a volcanic conduit between the reservoirs. The unknown parameters such as conduit radius were determined by a parametric inversion to reproduce the observed volumetric variation. Finally, the hour-scale magma migration prior to the eruption on April 9 could be quantitatively discussed with the optimum parameters to explain the data. It is, therefore, meaningful to employ numerical schemes to discuss such magma migration and the accumulating process before eruptions.

2. MAGMA PLUMBING SYSTEM

Geophysical observations have indicated the existence of three pressure sources in the subsurface around Sakurajima volcano. The deepest pressure source at ca. 10 km in the north of the volcano was suggested at a depth of 8-9 km from GPS data \(^5\). Observations of volcanic earthquakes and ground deformation estimated a pressure source at a depth of 3-4 km beneath the volcano \(^3\). After the analyses of explosion earthquakes, Tameguri et al. \(^4\) mentioned a model of a gas pocket was formed near the surface before the eruptions. In addition, a cylindrical volcanic conduit between near-surface and deep sources beneath the volcano was indicated from hypocenter distribution of volcanic earthquakes \(^6\). This implies that a conduit connecting the two pressure sources beneath the main crater could be a path of magma flow. Since the deepest reservoir contributes to long-term magma accumulation, we would like to focus on the plumbing system beneath Sakurajima volcano.

In our numerical simulations, we modeled the magma plumbing system composed of two reservoirs, one for the gas and the other for magma reservoirs, interconnected with a volcanic conduit (Fig. 1). It is assumed that the magma is constantly supplied to the deeper reservoir, and the gas leaks to the surface from the shallower as observed continuous minor eruptions.

3. GROUND DEFORMATION

On April 9, 2009, the Showa crater erupted with pyroclastic flow and a plume rise for over 4000 m. Precursory ground deformation was observed before the eruption at the two observation sites. After assuming the two pressure sources at depths of 0.1 km and 4.0 km, the volumetric variations were estimated by the Mogi’s model \(^7\). The results show a periodic inflation and deflation time sequence of the reservoirs preceding the eruption (Fig. 2). The volumetric variations of the shallower source are about one order less than those of the deeper source. The time lags of 3-6 hours in the inflations between the reservoirs are clearly visible. In the periods to simultaneously show contraction of the magma reservoir and the expansion of the gas reservoir (stages II and IV), there is a high probability that slow magma migration process prior to the eruption occurs. We accordingly try to reproduce the phenomena to quantitatively demonstrate the pre-eruptive magma migration in the stages II and IV. Here, the stages I and V related to the eruption are not considered in this study because we focus on the pre-eruptive magma migration and accumulating process.
4. METHOD

Observed volumetric variation of a reservoir $V_{\text{obs}}$ should not be equivalent with a volume of inflow to and outflow from the reservoir $V_{\text{ch}}$. It is necessary to discuss the bulk modulus of the deeper magma reservoir $K$ and rigidity of the surrounding rock when the reservoir pressure is increased due to magma accumulation. $V_{\text{ch}}$ can be calculated by the gas leakage to the surface, the magma supply to the deeper reservoir and the magma migration in the conduit. The gas leakage was adopted by assuming the permeable zone above the shallower reservoir. The supply rate was estimated by the observed $V_{\text{obs}}$ in the stage III. The magma migration was simulated by the numerical flow simulations.

We performed one-dimensional flow simulation of magma consisting of crystals, melt and volatile contents (H$_2$O and CO$_2$). The flow is assumed to be permeable flow, and can be described by equations of mass and momentum conservation for the unsteady multi-phase flow. The permeable flow is theoretically equivalent with a bundle of the Hagen-Poiseuille flow with the same radii (referred as equivalent radii $r_e$ in this study) on condition that those pressure losses are equal. That is, the permeability of magma can be given by the equivalent radius.

As magma ascends, volatiles in magma are gradually exsolved, and then properties of the magma are greatly changed. Viscosity of melt with crystals can be calculated by the crystal volume content and the melt compositions$^{8,9}$. Densities of the melt and the gas can be derived by the melt compositions and an equation of state for an ideal gas$^{10}$.

The unsteady flow simulation needs initial conditions in the whole conduit. Pressures in the gas reservoir and the conduit were assumed to be lithostatic and magma-static, respectively. The magma head depth is assumed to be 500 m. H$_2$O and CO$_2$ content at the deeper reservoir were fixed to be 3.0 wt.$^{\circ}$ and 100 wt.ppm. If the gas contents in the conduit set the same as those at the deeper reservoir, the gas volume fraction exceeds the unrealistic value of 70 %. To avoid this problem, the initial gas volume fraction was limited to below $\phi_{\text{init}}$. On other words, the magma degassed to some extent is assumed as the initial condition.

5. RESULTS AND CONSIDERATIONS

There are five parameters that had not been well estimated by the past studies; $K$, the conduit radius $r_c$, $r_e$, the permeability of gas in magma $k_g$, and $\phi_{\text{init}}$. Since $k_g$ can be described by the empirical equation$^{12}$:

$$k_g = 10^{-17}(100\gamma)^2, 3.0 \leq \gamma \leq 3.8$$  \hspace{1cm} (1)

we used $\gamma$ as an indicator of $k_g$. The parametric inversion is carried out to estimate the optimum condition to reproduce the inflation and deflation behavior. The optimum result is shown in Fig. 3. Here, when the optimum condition in the stage IV is determined, $K$ and $r_e$ are fixed because the compressibility and the geometry should not significantly be changed during a few hours. Although the shallow volumetric change in the stage IV is not well determined, the others are fitted to the data. Furthermore, the time lags in both stages are well reproduced. It was found from the optimum condition listed in Table 1 that the magma is migrated through the fractures with the radius of 0.3-0.4 m scale in the volcanic conduit with the 125 m radius before the eruption on April 9, and the magma has the relatively high permeability and the low porosity.

Validity of the optimum condition should be investigated whether the subsurface situation is reflected on the simulation. The estimated $K$ is ca. 2.3 GPa, and the supply rate can be calculated to be 16 m$^3$/s. The value is about ten times greater than the long-term accumulating rate of the magma reservoir beneath Aira caldera during 1995-2011$^{13}$. However, it should be reasonable because the large fluctuations of the accumulating rate was observed in the period$^{13}$ and the magma volume is increased with the migration from the deepest reservoir at 10 km depth beneath Aira caldera to the reservoir at 4 km depth beneath Sakurajima volcano. The size of the volcanic conduit is estimated to be 125 m. The value is smaller than 200 m inferred from the hypocenter distribution of volcanic earthquakes in the volcano$^{6}$. Since the distribution has some variations due to the accuracy of determining the hypocenters, the estimated conduit radius seems realistic. There are not enough data to verify the other three parameters. On other words, our

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Stage II</th>
<th>Stage IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>2.3 GPa</td>
<td>2.3 GPa</td>
</tr>
<tr>
<td>$r_c$</td>
<td>125 m</td>
<td>125 m</td>
</tr>
<tr>
<td>$r_e$</td>
<td>0.3 m</td>
<td>0.4 m</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>3.8</td>
<td>3.8</td>
</tr>
<tr>
<td>$\phi_{\text{init}}$</td>
<td>0.2</td>
<td>0.2</td>
</tr>
</tbody>
</table>
simulation could provide some parameters that are difficult to be determined from observations.

One of big advantages of the magma flow simulation is that the dynamic change of magma properties in the conduit can be investigated. The magma flow under the optimum condition indicated that the gas in the magma mainly ascends to the shallower, suggesting the expansion near the surface is controlled by the gas. This would support the supposition that there is the gas reservoir near the surface before an eruption. Accordingly, the magma flow simulation is valuable for understanding the dynamic magma behavior leading to an eruption.

6. CONCLUSIONS

We performed the magma flow simulations in the magma plumbing system model to understand the pre-eruptive magma migration from the ground deformation on April 9 at Sakurajima volcano. As a result, the hour-scale magma migration before the eruption could be quantitatively explained. It was also found that our simulation can estimate the unknown subsurface parameters; the size of the volcanic conduit, the bulk modulus of the magma reservoir, the gas permeability, the characteristic radius to represent the permeability of the conduit and the gas volume fraction before the eruption. In addition, it was found from the magma dynamics that the gas in magma selectively ascends toward the surface before the eruption. Therefore, our scheme would be powerful to quantitatively explain the ground deformation and estimate the unknown parameters in the magma plumbing system.

REFERENCES
7) Mogi, K., 1958, Relations between the eruptions of various volcanoes and the deformations of the ground surface around them, Bull., Earthq. Res. Inst., Univ. of Tokyo, 36, 99-134.