Three-dimensional joint inversion of gravity and magnetic anomalies using fuzzy c-means clustering

Yosuke Teranishi, Hitoshi Mikada, Tada-nori Goto and Junichi Takekawa

¹Dept. of Civil and Earth Res. Eng., Kyoto University

We present a 3D joint inversion method to estimate two physical parameters, density and magnetization of subsurface materials using field intensity measurements. In the method, we introduce the fuzzy c-means (FCM) clustering technique to relate gravity with magnetic data. In the approach, the subsurface structure is discretized to a set of rectangular prisms. For estimating the density and magnetization of each prism, we minimize the quadratic norm of the residuals between the observed and the forward-modeled. Two regularization terms, i.e. the roughness and the similarity of the two physical parameters, are introduced in our joint inversion to control the degree of model roughness and similarity. We determine their regularization parameters using the L-curve criterion. We apply our method to a numerical model which represents submarine massive sulphides (SMS). The joint inversion results, which have the advantages of both gravity and magnetic inversion, show better accuracy and resolution than the individual ones.

1. Introduction

The gravity and magnetic surveys have been widely carried out over the years, especially for the exploration of metallic mineral deposits and geothermal resources. These field intensity data could be acquired in much quicker and simpler way than the other geophysical or geological data. The inversion of potential field data, however, has been known as a non-uniqueness problem expressed in the Green's equivalent layer theory. Because of this problem, gravity and magnetic data have no inherent resolution in depth (Li and Oldenburg, 1996; Li and Oldenburg, 1998). As conventional surveys, the measurement of potential fields is a simple and convenient method before conducting any specific surveys. The utilization of potential field data, however, requires reliable subsurface structure or some other data acquired by the other means for modeled or joint inversion.

Natural resources such as submarine massive sulphides (SMS) are attractive material due to the recent rapid growth of global economical activities, but their deposit locations below deep seafloor restricts the access. This necessitates detailed exploration using potential field data. We, therefore, need to develop a way to make use of high exploration efficiency that takes the advantages of the convenience to conduct gravity and magnetic surveys. Actually, a joint inversion using geostatistical method of gravity and magnetic data sets has been proposed useful for to estimate the

major lithological properties of rocks (Bosch, 1999). Also, joint inversion using a cross-gradient function of two disparate geophysical data sets increases accuracy and resolution of three-dimensional (3D) geophysical models (Fregoso et al., 2009). Several studies have proposed methods such as how to associate gravity with magnetic data, how to define their object functions or regularization parameters, etc. These examples clearly indicate that we could develop a method to utilize both gravity and magnetic data in joint inversion algorithms. In general, petrophysical estimation of the relation between two different physical properties of rocks requires a fitting of empirical functions between the two properties. However, it is not so simple to estimate the empirical equation that may suffer lack of geophysical meaning.

In this paper, we introduce the fuzzy c-means (FCM) clustering technique in our joint inversion algorithm to consider the petrophysical relation between density and magnetization of subsurface materials. The fuzzy c-means clustering technique we introduce does not necessitate any empirical equation but deals with a linear combination of the influence from multiple clusters given a piece of data to belong to plural clusters in the parameter space formed by the petrophysical parameters. We test our inversion method using synthetic numerical experiments for SMS, and compare the results from our joint inversion with those from an inversion scheme applied independently for every parameter. Although several surveys have been conducted for SMS using gravity or magnetic measurements

(Evans, 1996; Tivey et al., 2002), there has been no report that estimate the volume of SMS in a sufficiently accurate way. We think that our joint inversion method demonstrates the accuracy of our method in the estimation of SMS in terms of the gravity and magnetic anomalies.

2. Formulation of 3D inversion

Gravity and magnetic anomaly observed above the surface with N of observations (gravity anomaly vector $\mathbf{d}_g$, magnetic anomaly vector $\mathbf{d}_m$) are related to density $\mathbf{\rho}$ and magnetization $\mathbf{m}$ of M underground prisms linearly:

$$\mathbf{d}_g = \mathbf{A}_g \mathbf{\rho} \quad (1)$$

$$\mathbf{d}_m = \mathbf{A}_m \mathbf{m} \quad (2)$$

The matrices ($\mathbf{A}_g$, $\mathbf{A}_m$) have elements (A_{gij} , A_{mij}), which are the contributions of unit density and magnetization in the j th prism to the i th data. Since (A_{gij} , A_{mij}) consist of constants, our inversion is linear algorithm.

The gravity and magnetic inversion are formulated as optimization problems that minimizes the following each objective functions U_g , U_m

$$U_g = \left\| \mathbf{W}_g (\mathbf{d}_g - \mathbf{A}_g \mathbf{\rho}) \right\|^2 + \alpha_g^2 \left\| \mathbf{C} \mathbf{\rho} \right\|^2 \quad (3)$$

$$U_m = \left\| \mathbf{W}_m (\mathbf{d}_m - \mathbf{A}_m \mathbf{m}) \right\|^2 + \alpha_m^2 \left\| \mathbf{C} \mathbf{m} \right\|^2 \quad (4)$$

($\mathbf{W}_g$, $\mathbf{W}_m$) are weighting matrices. If noise contaminating each data is uncorrelated each other and has a standard deviation of (σ_{gi} , σ_{mi}), the weighting matrices are usually defined as $\mathbf{W}_g = \text{diag}\{1/\sigma_{g1}, \dots, 1/\sigma_{gn}\}$, $\mathbf{W}_m = \text{diag}\{1/\sigma_{m1}, \dots, 1/\sigma_{mn}\}$. The notation $\| \cdot \|$ in the above two equations is the L-2 norm measure. In each equation (3) and (4), to avoid unstableness, we add a smoothing Laplacian operator matrix $\mathbf{C}$ as a smoothness regularization term to our inversion algorithm. Smoothness regularization parameters α_g and α_m control a degree of spatial smoothing in inversion.

To formulate our joint inversion method, we incorporate equation (3) with (4) and consider a coupling measure based on the fuzzy c-means (FCM) clustering (Jiajia Sun et al., 2012) as a similarity regularization term which combines two model parameters and we obtain the following equation;

$$U = \beta^2 \left\| \mathbf{W}_g (\mathbf{d}_g - \mathbf{A}_g \mathbf{\rho}) \right\|^2 + \left\| \mathbf{W}_m (\mathbf{d}_m - \mathbf{A}_m \mathbf{m}) \right\|^2 + \alpha_g^2 \left\| \mathbf{C} \mathbf{\rho} \right\|^2 + \alpha_m^2 \left\| \mathbf{C} \mathbf{m} \right\|^2 + \lambda^2 \left\{ \sum_{k=1}^M \sum_{i=1}^C (u_{ki})^q \left\| \mathbf{x}_k - \mathbf{v}_i \right\|^2 + \eta^2 \sum_{i=1}^C \left\| \mathbf{v}_i - \mathbf{t}_i \right\|^2 \right\} \quad (5)$$

In the equation (5), M is the number of model prisms, C is the number of clusters, $\mathbf{x}_k$ represents the density and magnetization in the k th prism ($\mathbf{x}_k = [\rho_k \ m_k]^T$), $\mathbf{v}_i$ is the i th cluster center estimated by FCM algorithm ($\mathbf{v}_i = [\rho_i^* \ m_i^*]^T$), u_{ki} is the membership function that measures the degree to which the two model parameters of k th prism belong to the i th cluster, $\mathbf{t}_i$ is a possible pair of density and magnetization which we give as a priori information from petrophysical measurements ($\mathbf{t}_i = [\rho_i^{true} \ m_i^{true}]^T$). The last term measures the distance between cluster centers updated by FCM algorithm, $\mathbf{v}_i$, and target cluster centers determined from a priori information, $\mathbf{t}_i$. The parameter q controls the degree of fuzziness of the resulting membership functions. In this study, we set $q = 2.0$, which is widely accepted as a good choice (Hathaway and Bezdek, 2001).

The regularization parameters λ and η determine the weight of the imposed FCM clustering against the data misfit and the model smoothness. The smoothness regularization parameters α_g and α_m control the weight of smoothness constraint against the data misfit and the model similarity. We use the L-curve criterion (Arai, 2003) to determine α_g and α_m in equations (5). At last, parameter β controls the weighting ratio of gravity and magnetic in data misfit is 1 to 1.

3. Synthetic test model

In order to test our joint inversion algorithm, we consider the SMS model shown in **Figure 1**. The model consists of $20 \times 13 \times 12$ (3120) prisms in the analysis model ($400 \times 260 \times 96 \text{ m}^3$). Based on Evans (1996); Tivey et al., (2002), we supposed the difference in density between SMS and surrounding volcanic rock is 2.5 g/cm^3 and that of magnetization is $\pm 4.0 \text{ A/m}$. We calculate anomalous data on the surface over a 21×14 grid with 20m spacing in both directions. Since we assume that the measurement is conducted by remotely operated vehicle (ROV), 0.1 mGal and 5 nT Gaussian noise is added to the synthetic data. The observation level is assumed to be 1 m above the seabed, and the background magnetic field has constant inclination and declinations; $I = 45^\circ$, $D = 0^\circ$. The same inclination and declination are assigned for the magnetization vectors of rocks.

Figure 1 The parallel and resupinate pyramid SMS models. Density and magnetization of seawater, surrounding volcanic rock and SMS are described in the Figure 1.

4. Synthetic examples

Figure 2 shows the results of four cross-sections of density and magnetization structure obtained from separate inversion using gravity data, separate inversion using magnetic data and joint inversion. Now, the similarity regularization parameter λ is fixed to 10. Throughout this paper, we assume that the total number of clusters, C , is known ($C=3$).

In the both separated gravity and magnetic inversion results, the estimated models do not clearly indicate the locations and the stepwise shapes of assumed anomalous blocks. On the other

hand, those from our joint inversion results show good agreement with the true model, especially around boundary of stepwise structure. Joint inversion results determine density and magnetization amplitude values better than the separated cases, and the shape of anomalies inferred from the synthetic data is close to the true shapes which are outlined in each cross-section. **Figure 3** (a) shows a distribution of density and magnetization values obtained by the gravity and magnetic inversions. **Figure 3** (b) shows the similar distribution but obtained by joint inversions and membership value at each cluster.

Figure 2 (a) Density and (b) magnetization models obtained by separate inversion; (c) density and (d) magnetization models obtained by joint inversion. The vertical axis indicates depth below the seabed and the cross axis indicates horizontal distance. The true position of the model is outlined in each cross-section.

Figure 3 (a) The distribution of density and magnetization values obtained by the gravity and magnetic inversions. The white stars show the distribution of true density and magnetization values, (b) that of joint inversions and membership value at each cluster. The white stars show the distribution of true density and magnetization values. The block stars show the distribution of density and magnetization values of each cluster center.

5. Conclusion

We developed an algorithm to jointly invert two geophysical data sets using a constraint of FCM clustering between the density and magnetization. We calculated the noise-contaminated synthetic data expected on the SMS model. In the individual inversion results, the resolution and accuracy of inverted results were not good though the results could detect underground anomalies of density and magnetization. On the other hand, in our joint inversion results, the inverted results show better resolution and accuracy, and might partially overcome the inherent lack of depth resolution in gravity and magnetic data.

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