Estimation of permeability structure with Self Potential inversion

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In this study, we developed the 2D inversion program that estimate the permeability structure from Self-potential(SP) profile. We applied these inversion programs to synthetic SP profiles. The synthetic SP profiles, used as observed data for the inversion, are evaluated with modeled slopes with 800m length and 40m height including various permeability anomalies. Four models including the permeable anomalies located in the center of the model are used for the estimation of the performance of our inversion. A priori information of the distribution of streaming current co-efficient, electrical conductivity and the flux volume at the discharge and recharge are given for our inversion. The horizontal zone with high permeability and the vertical zone with low permeability can be reconstructed with our inversion properly. However, the horizontal low permeable and the vertical high permeable zone cannot be imaged clearly. The regional groundwater flow pattern around the permeability anomaly has great effect on the SP pattern on the surface. The consideration of flow pattern around the permeability anomaly and effect on the SP profile are necessary for the accurate inversion of SP data.

1. INTRODUCTION

Self-potential or streaming potential (SP) method is one of geophysical methods expected to be useful for the estimation of groundwater flow system because the SP is mainly generated by the groundwater flow. The observed SP profiles are often related with the direction of groundwater flow and have been analyzed qualitatively. Recently, several methods have been developed to transform the SP profile to parameters related to the groundwater flow; for example inversion to water table (Fournier, 1989; Revil et al., 2004), water head (Sheffer, 2007; Jardani et al., 2007), Permeability (Revil and Jardani, 2009; Jardani and Revil, 2010), etc., quantitatively.

Groundwater flow is affected by the subsurface permeability structure. The SP is susceptible to the groundwater flow according to its generation process. Our numerical simulations showed that SP profiles reflect on the regional groundwater flow pattern around the permeability anomaly (Ozaki et al. 2012). From this feature, it could be possible to estimate the distribution of water head or permeability structure with the quantitative analysis of SP profile.

In this study, we developed the inversion code for analyzing water head according to Sheffer (2008). We also developed the inversion code for analyzing permeability from the distribution of water head according to Carrera et al. (2005). We combined these two inversion codes and developed the inversion code for analyzing permeability structure from SP profile. We applied this inversion to the synthetic data affected by subsurface permeability anomalies. We discussed the sensitivity of SP to the permeability anomaly.

2. THEORY AND METHOD

The governing equation of the groundwater flow in saturated zone is derived from the Darcy’s law and the mass conservation equation as:
\[ Q = -\nabla (k \nabla h) \]  \hspace{1cm} (1)

where \( Q \) is the external source of groundwater flow [L³/t], \( k \) is permeability [m/s] and \( h \) is water head [m]. When the source of electrical current is only the groundwater flow, the total subsurface electrical current is described with the conduction current and convection current as:
\[ I = -\sigma \nabla \varphi - L \nabla h \]  \hspace{1cm} (2)

where \( I \) is total electrical current [A/m], \( \sigma \) is electrical conductivity [S/m], \( \varphi \) is electrical potential [V] and \( L \) is streaming current co-efficient [A/m²]. The mass conservation of equation(2) is
satisfied 0. Then, the water head and electrical potential is related as
\[
\nabla(\sigma(\nabla \phi)) = -\nabla(L \nabla h) \quad (3)
\]

The finite element modelling substitutes the differential equation (1) and equation (3) for the matrix and vector forms as:
\[
F_1(k)h = q(k) \quad (4)
\]
\[
F_2(\sigma)\phi = -F_3(L)h \quad (5)
\]

,where \(F_1, F_2 \) and \(F_3 \) are matrices driven from the right side of equation (1), left side and right side of equation (3) with finite element modelling respectively.

We defined the objective function \(U \) according to the Tikhonov regularization. The objective functions \(U \) are minimized by the permeability \(k \) for the inversion. The SP is related with the permeability from equation (4) and (5) as:
\[
\phi = -F_2(\sigma)^{-1}F_3(L)F_1^{-1}(k)q(k) \quad (6)
\]

The objective function is described as:
\[
U = \left\| \phi_{\text{obs}} - (-F_2(\sigma)^{-1}F_3(L)F_1^{-1}(k)q(k) \right\|^2 + \lambda\|m\|^2 \quad (7)
\]

where \(\phi_{\text{obs}} \) is observed SP profile, \(\lambda \) is hyper-parameter, \(C \) is the matrix of smoothness constrain and \(m \) is \(\log_{10}(k) \).

3. SYNTHETIC DATA

In this study, we calculate the SP profile and invert the calculated SP profile as the observed data.

**Figure 1** Simulation model A.

**Figure 2** Simulation model B.

**Figure 3** SP profile on the ground surface with model A. Red line shows the SP profile when the permeability in anomaly is high. Blue line shows the SP profile when the permeability in anomaly is low.

**Figure 4** SP profile on the ground surface with model B. Red and blue line show the SP profile in case of high and low permeability anomaly respectively.

**Fig. 3** and **Fig. 4** show SP profiles on the ground surface with models shown in **Fig. 1** and **Fig. 2** respectively. When the shape of permeability anomaly is horizontal, peaks in the SP profiles appear at 300m and 500m just above side boundaries of permeability anomaly. These peaks due to the high permeability anomaly are higher than peaks due to the low permeability anomaly (**Fig. 3**). When the vertical low permeability anomaly is present, the SP profile drastically changes from 380m to 420m just above the permeability anomaly. The vertical high permeability anomaly do not change the SP profile so much (**Fig. 4**). We applied our inversion to these data as observed SP data.

4. INVERSION RESULTS

We estimated the permeability structure according to the minimization of objective function \(U \) described as equation (7). This inversion is non-linear problem and the initial model is required.
The uniform structure of permeability, streaming current co-efficient and electrical conductivity with \(1.0 \times 10^{-6}\) m/s, \(1.0 \times 10^{-4}\) A/m\(^2\) and 0.01S/m are used as the initial model. The volume of flow at the recharge and discharge are given for the inversion a priori. The hyper-parameter \(\lambda\) is selected with ABIC minimization method.

**Figure 5** The overview of the inversion result. The top picture shows the inversion result of SP profile affected by the horizontal high permeability anomaly. The bottom picture shows the inversion result of SP profile affected by the low permeability anomaly. The dashed line shows the location of permeability anomaly.

**Fig. 5** shows the inversion results when SP profiles shown in **Fig.3** are input. When we input the SP profile affected by horizontal high permeability anomaly, the permeability anomaly is reconstructed around the exact location of permeability anomaly. False images of low permeability at about 200m and 500m with the depth of about 25m also appear. When we input the SP profile affected by horizontal low permeability anomaly, the image permeability anomaly hardly ever appears. The SP anomalies due to the low permeability are small. Small permeability structure compared with the exact model is enough to reproduce the observed SP anomalies. As a result, this inversion cannot reconstruct the exact permeability anomaly in the estimated model.

**Fig. 6** shows the horizontal variation of permeability with the depth of 57.5m and vertical variation at 400m. In case that permeability in anomaly is high, the estimated permeability around the exact location of permeability anomaly becomes \(3.4 \times 10^{-5}\) m/s and almost the half value of exact model. The depth of maximum permeability in the estimated model is located within the area of exact permeability anomaly. However, the estimated area of high permeability spreads widely owing to the smoothness constrain. In case of the low permeability anomaly, the minimum value of estimated permeability is only \(4.8 \times 10^{-7}\). Both horizontal and vertical variations of estimated permeability do not change so much compared with the case of high permeability anomaly.

**Figure 6** The variations of estimated permeability. The top graphs show the estimated permeability from the SP profile affected by the horizontal high permeability anomaly. The bottom picture shows the result from the SP profile affected horizontal low permeability anomaly. Horizontal variations with the depth of 57.5m and vertical variations at 400m are shown. Areas painted with blue are location of permeability anomaly.

**Figure 7** The overview of the inversion result. Both inversion results of SP profile affected by vertical high and low permeability anomalies are shown.

**Fig. 7** shows the inversion results when SP profiles shown in **Fig. 4** are input. When the SP profile affected by the vertical high permeability anomaly is applied to our inversion, the high permeability anomaly appears only on the location of top boundary of the exact permeability structure. This is due to the lack of sensitivity of SP to the thickness of high permeability anomaly. When the SP profile affected by the vertical low permeability anomaly, which has large SP anomalies compared with the SP profile affected by the high permeability anomaly, is input, the estimated low permeability area spread wider than the exact permeability anomaly. The smoothness constrain is included in our inversion method. Then, the solution that the model has wide and smooth permeability anomaly is selected by our inversion.
The consideration of flow permeability anomaly has great effect on the regional groundwater flow pattern which generates small SP anomalies. Vertical low permeability anomaly result in good evaluation that permeability anomaly. Our inversion program of permeability profile according to the Sheffer (2008) and Carrera et al. (2005). Our inversion of permeability structure needs the a priori information of hydraulic condition. The information of well data is better as a priori information for the practical application of inversion. The improvement of the way to give a priori information of ground flow is required.

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REFERENCES


CONCLUSION

In this study, we developed the inversion code for the estimation of permeability anomaly from SP profile according to the Sheffer (2008) and Carrera et al. (2005). Our inversion program of permeability structure differs from the inversion methods by Jardani and Revil (2009) or Revil and Jardani (2010) in the point of view of the calculation of the Jacobian matrix of SP about permeability structure.

We try to reconstruct the permeability structure model from synthetic SP profiles affected by permeability anomaly. Our inversion results show that the inversion of SP has the high sensitivity and result in good evaluation to the horizontal high permeability anomaly or vertical low permeability anomaly. On the other hand, the horizontal low permeability or vertical high permeability anomaly, which generates small SP anomalies in the profile, cannot be imaged well in our inversion results. The regional groundwater flow pattern around the permeability anomaly has great effect on the generated SP profile. The consideration of flow pattern around the permeability anomaly and effect on the SP profile is necessary for the accuracy of inversion result.

Our inversion of permeability structure needs the information of the flux volume at recharge and discharge a priori. The information of well data is better as a priori information for the practical application of inversion. The improvement of the way to give a priori information of groundwater flow is required.