Estimation of pre-eruptive magma ascent using a hydrokinetic model of magma plumbing system

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We have developed a hydrokinetic model of magma plumbing system to illustrate pre-eruptive magma migration and accumulation. The pre-eruptive magma flow is not observable but computable using both a magma flow simulation and ground deformation. For this purpose, ground deformation on April 9, 2009 at the Showa crater of Sakurajima Volcano was used because the pre-eruptive magma flow is implied from the observed data, which shows a periodic inflation and deflation event with time lags of 3-6 hours prior to the explosive eruption. Three-phase Poiseuille and permeable flows were adopted to demonstrate the observed data. In consequence, it was found that the data can be reproduced by the permeable model much better than the Poiseuille model. In addition, compressibility of the reservoir, a magma supply rate to the deeper reservoir and a size of the volcanic conduit can be constrained to be ca. 1 GPa, 35 m³/s and 225 m, respectively. Our scheme will be therefore effective to quantitatively evaluate the invisible magma migration preceding an eruption.

1. INTRODUCTION

Simulations of magma flow during an eruption enable us to quantitatively interpret the eruptive dynamics, and have been studied well for explaining diverse eruptive styles¹⁾. On the contrary, little attempt has been conducted in the low-velocity magma migration into or from a magma reservoir preceding a volcanic eruption to investigate such direct and imminent precursory phenomena. The pre-eruptive magma migration is of importance to understand the mechanism of the accumulation of volcanic source materials before an eruption. It is, therefore, meaningful to study how magma migrates to eruptions in the course of volcanic activity.

There is a problem that the pre-eruptive magma migration is not directly observable. However, the magma migration into or from a magma reservoir can be approximately calculated from volumetric change of the magma reservoir, which can be estimated from observable ground deformation. The pre-eruptive migration can accordingly be captured by the indirect method using a model of magma plumbing system with a simulation of magma flow and implications from the ground deformation.

In this study, we selected the data of ground deformation observed before an explosive eruption on April 9, 2009 at the Showa crater of Sakurajima Volcano. Sakurajima Volcano is one of the most favorable fields to apply our scheme because of the

understanding of the magma plumbing system. The past study suggested that there are a gas reservoir near the surface²⁾ and a magma reservoir at a depth of 3-4 km³⁾ beneath the volcano. The ground deformation on April 9 indicated that there is a short-term periodical inflation and deflation cycle of the two reservoirs with time lags of 3-6 hours. We hypothesize that the time-variant volumetric behavior would reflect the magma ascent to the shallower, and try to explain the observed data in order to evaluate the pre-eruptive magma migration.

For this objective, we first modeled a magma plumbing system composed of gas and magma reservoirs beneath the volcano and the magma supply to the deeper reservoir. We then performed numerical simulations of magma migration in a volcanic conduit between the reservoirs to demonstrate the ground deformation on April 9, 2009. In the simulations, we try to reproduce the magma flow between the two reservoirs before the eruption. As a result, the simulations indicated that the magma is likely to be migrated as a permeable flow before the eruption. Besides, the key factors to control the inflation and deflation behavior are selectively found to be the conduit radius, the magma supply rate to the system and the compressibility of the magma reservoir. It has finally become clear that these three factors could be constrained by our scheme with a least-square error criterion. We would like to propose that our hydrokinetic modeling of magma plumbing system

provide a new interpretation of ground deformation data in terms of understanding the pre-eruptive magma migration.

2. MAGMA PLUMBING SYSTEM

Geophysical observations have indicated the existence of three pressure sources in the subsurface around Sakurajima volcano. The deepest pressure source at ca. 10 km in the north of the volcano was suggested at a depth of 8-9 km using GPS data⁴⁾. Observations of volcanic earthquakes and ground deformation suggested a pressure source at a depth of 3-4 km beneath the volcano³⁾. After the analyses of explosion earthquakes, Tameguri et al. mentioned a model of a gas pocket was formed near the surface before the eruptions²⁾. In addition, a cylindrical volcanic conduit between near-surface and deep sources beneath the volcano was indicated from hypocenter distribution of volcanic earthquakes⁵⁾. This implies that a conduit connecting the two pressure sources beneath the main crater could be a path of magma flow.

In our numerical simulations, we modeled the magma plumbing system composed of two reservoirs, one for the gas and the other for magma reservoirs, interconnected with a volcanic conduit (**Fig. 1**). It is assumed that the magma is constantly supplied to the deeper reservoir, and the gas leaks to the surface from the shallower as observed continuous minor eruptions. We also set a pressure valve to control the magma ascent so that the overpressure induced by the magma supply could trigger the magma ascent. The threshold is fixed 2.0 MPa following the condition of fracture formation in rock⁶⁾. Then, the valve is closed when the following equation is satisfied at the conduit bottom.

$$\sqrt{\frac{18\mu u}{\Delta\rho g}} < d = 0.1m \quad (1)$$

Fig 1 Model of the magma plumbing system beneath Sakurajima Volcano.

where μ is the viscosity, u is the melt velocity, $\Delta\rho$ is the density difference between crystals and melt in magma and g is the gravitational acceleration. This equation is theoretical, and describes the diameter limit d that spheres in ascending fluid start to sediment. On other words, the magma ascent is stopped when u is decreased enough that the sphere-like fragments with the diameter of 0.1 m can descend. The fragments are regarded as the products from a collapse when the magma ascent is initiated. The valve is then re-formed again, and the pressure is reaccumulated to another 2.0 MPa.

3. GROUND DEFORMATION

On April 9, 2009, the Showa crater erupted with a plume rise for over 4000 m which is the highest in the eruptions since the Showa crater had become active in April, 2006. An associated pyroclastic flow, which had not been observed since February 6, 2008, reached an area ca. 1 km east of the crater. Precursory ground deformation was observed before the eruption at the two observation sites. After assuming the two pressure sources at depths of 0.1 km and 4.0 km, the volumetric variations were estimated by the Mogi's model⁷⁾. The results show a periodic inflation and deflation time sequence of the reservoirs preceding the eruption (**Fig. 2**). The volumetric variations of the shallower source are about one order less than those of the deeper source. The time lags of 3-6 hours in the inflations between the reservoirs are clearly visible. The observed phenomena of the phase delays could be a good example of slow magma migration process prior to the eruption. We accordingly try to reproduce the phenomena to quantitatively demonstrate the pre-eruptive magma migration.

Fig 2 Volumetric variations ΔV of the two reservoirs before the explosive eruption at 15:29, April 9, 2009. The “analyzed zone” is set to evaluate our calculated results.

4. METHOD

Observed volumetric variation of a reservoir V_{obs} should not be equivalent with a volume of inflow to and outflow from the reservoir V_{ch} . It would be necessary to discuss magma compression when the reservoir pressure is increased due to magma accumulation. According to Johnson et al., V_{obs} can be calculated from V_{ch} besides bulk modulus of magma and rigidity of the surrounding rock⁸⁾. For the evaluation of V_{ch} , the volume transfer from the deeper to the shallower have to be calculated by numerical flow simulations in the conduit.

The magma migration in the cylindrical conduit was simulated as a one-dimensional unsteady flow of magma consisting of crystals, melt and volatile contents (H_2O and CO_2). Two models for magma ascent are adopted in our simulations; permeable and Poiseuille flows. In both models, the flow can basically be described by equations of mass and momentum conservation for the unsteady multi-phase flow.

As magma ascends, volatiles in magma are gradually exsolved, and then properties of the magma are greatly changed. Viscosity of melt with crystals can be calculated by the crystal volume content and the melt compositions^{9,10)}. Densities of the melt and the gas can be derived by the melt compositions¹¹⁾ and an equation of state for an ideal gas.

Time-dependent change in magma properties in the conduit can be determined from the above discussion in addition to initial conditions in the conduit. Pressures in the gas reservoir and the conduit were assumed to be lithostatic and magma-static, respectively. H_2O and CO_2 content at the deeper reservoir were fixed to be 3.0 wt. %¹²⁾ and 0.3 wt. %. If the gas content in the conduit sets the same as that at the deeper reservoir, the gas volume fraction will exceed the unrealistic value of 70 %. To avoid this problem, the initial gas volume fraction was limited to below 20 % by means of decreasing the gas content in the conduit.

There are some parameters that had not been well estimated by the past studies; the bulk modulus of the deeper reservoir, the magma supply rate to the

Table 1 The optimum parameters and the range in case of the permeable model.

*Porosity=0.3	Opt. value	Range
Bulk modulus [GPa]	1	0.5-30
Supply rate [m^3/s]	35	2-100
Conduit radius [m]	225	50-250

deeper and the conduit radius. We tested the combinations within wide ranges of the parameters listed in **Tables 1 and 2**, and evaluated the results using a least-square error criterion. RMS residuals between calculated and observed volumetric variations were calculated in the analyzed zone shown in **Fig. 2**. The zone was selected to avoid the perturbation induced by an eruption on the day before the eruption on April 9.

5. RESULTS AND CONSIDERATIONS

After the parametric study, the optimum result to reproduce the inflation and deflation behavior could be determined by our scheme. **Figs. 3 and 4** show the calculated volumetric variations of the two reservoirs under the optimum parameters in **Tables 1 and 2**. Here, in the permeable model, porosity of the volcanic conduit is fixed to be 0.3. Comparing with the observed data, both models could well explain the volumetric difference of the two reservoirs in one order of magnitude. In contrast, the time lags of 3-6 hours could be described by only the permeable model. The reason is that the volume flow rate at the conduit bottom in the Poiseuille model is decreased more drastically than that in the permeable model.

The optimum bulk moduli of the deeper reservoir for the permeable and Poiseuille models are 1 and 15 GPa, which are consistent with the typical values of magma with and without gas, respectively^{6,8)}. Since there should be gas in the deeper reservoir under the pressure condition in our simulations, the permeable model seems more plausible. Moreover, the optimum conduit radius in the permeable case corresponds to the distribution of volcanic earthquakes beneath the volcano⁵⁾.

Judging from the discussions on the time lags, the estimated bulk modulus and the conduit radius, the magma is likely to be migrated as permeable flow before the eruption on April 9. It was also found from the time-dependent velocity profile in the conduit that the accumulation of the gas reservoir is almost caused by the ascent of gas in magma. Eventually, the results revealed that our scheme could not only reproduce the observed data but provide quantitative evaluation of the pre-eruptive magma dynamics and the constraint of

Table 2 The optimum parameters and the range in case of the Poiseuille model.

	Opt. value	Range
Bulk modulus [GPa]	15	3-40
Supply rate [m^3/s]	2.5	1-10
Conduit radius [m]	14	8-20

Fig 3 The optimum result of the permeable model

Fig 4 The optimum result of the Poiseuille model

the key parameters of the magma accumulation process.

6. CONCLUSIONS

We performed numerical simulations of magma flow in the magma plumbing system model for the purpose to understand the pre-eruptive magma migration on April 9, 2009. Our numerical analyses lead to the following conclusions.

- (1) The pre-eruptive ground deformation could be demonstrated by our hydrokinetic model of the magma plumbing system.
- (2) The pre-eruptive magma migration on April 9 was supposed to be permeable flow.
- (3) The accumulation of the gas reservoir is likely to result from the gas ascent separately from the magma.
- (4) The key factors to reproduce the observed data can be constrained by RMS residuals between calculated and observed volumetric variations of the reservoirs. In this study, the magma supply rate to the deeper reservoir, the compressibility of the deeper reservoir and the conduit radius was estimated to be 35 m³/s, 1.0 GPa, 225 m, respectively.

Accordingly, the pre-eruptive magma migration and accumulation process could quantitatively be evaluated from ground deformation using our

scheme.

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