Numerical Simulation of magma plumbing system associated with the eruption at the Showa crater of Sakurajima inferred from ground deformation

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We studied the ground deformation associated with the eruption at the Showa crater of Sakurajima, which has been active since 2006. Using the Mogi’s spherical pressure model, a volume change of magma chambers can be estimated from the displacement, tilt, or strain observations near the ground surface. The tilt and strain data observed 36 hours before an eruption in April 9, 2009, are analyzed using the Mogi’s spherical model to reveal the behavior of magma leading to eruption. From these data, there seems to be a time lag in the inflation between the two magma chambers at a depth of 4 km and 0.1 km, respectively. Moreover, the volume change of the shallow source is about one order less than that of the deep one.

A system which consists of shallow and deep magma chambers and several vertical conduits connecting them is numerically modeled to investigate the mechanism of the time lag and why the difference in the magnitude of the volumetric changes in the two chambers appears as described above. The initial values of magma properties in the deep magma chamber are assumed from the volcanic ejecta of Sakurajima volcano. We assumed that magma is supplied with a constant rate to the deep magma chamber. The pressure limit that the magma starts to ascend to the shallow chamber is assigned to the deep one. In a one-dimensional steady flow model of a magma conduit, we consider the vesiculation of volatile-bearing magma, gas escape and overpressure in the bubble due to the viscous resistance, which largely influences the physical properties of magma.

We confirmed that our hypothetical model could explain the time lag of the inflation and the difference in the volume change although our simulation results cannot exactly describe the data. We would like to propose that the numerical simulation could be a powerful tool for understanding the behavior of magma before eruption once ground deformations are well observed.

1. INTRODUCTION

Sakurajima volcano, an andesitic stratovolcano located at the southern rim of Aira caldera, has been active since 2006, and causes the many volcanic earthquakes, a great number of minor eruptions, outstanding ground deformation and so on. After the application of the Mogi’s spherical model¹ to data in the past observations, the existence of two magma chambers has been inferred beneath the Sakurajima down to a depth of 5 km. In this study, we focus on the ground deformation a few dozen hours before eruption, which is associated with the movement of volatile-bearing magma between two magma chambers. So far, many scientists have been studied the relevance between ground deformation and an eruption in the long time-scale²³ but little is known about the relevance in the short time-scale. However, it goes without saying that the latter is much more important to predict an eruption.

In order to explain the mechanism of eruption from ground deformation, we introduced the hypothetical model of two magma chambers and some vertical conduits with consideration of the vesiculation of volatile-bearing magma, gas escape and overpressure in the bubble due to the viscous resistance, which largely influences the physical properties of magma. Since there has been few studies that tried to consider magma chambers and the conduits as a whole system, our study is valuable to give a physical explanation of the ground deformation associated with the eruption.

2. DATA

We use volume change data of two magma chambers beneath the Showa Crater of Sakurajima from Sakurajima Volcano Research Center (Fig. 1).
Hourly volume changes of the two sources were estimated to minimize residual of theoretical tilt strain changes based on the dual Mogi’s source from values of following 6 components; two tilt, two radial strain and two tangential strain, which had been observed in underground tunnel sites for 36 hours before an eruption in April 9, 2009. Fig.1 indicates that there seems to be a time lag in the inflation between the two magma chambers at a depth of 4 km and 0.1 km, respectively. In addition, the volume change of the deep source is more than ten times of that of the shallow one.

Figure 1 The volume change of two Mogi’s Sources as magma chambers at a depth (D) of 4 km and 0.1 km.

3. Methodology

(1) Model
We propose the model consisting deep and shallow magma chambers at a depth of 4.0 km and 0.1 km, respectively, and a vertical conduit as shown in Fig.2. It is presumed that there are a surface layer with thick 1000 m and density 2.30 g/cm$^3$, and a second layer with 2.65 g/cm$^3$, from Bouguer anomalies$^4)$. We consider the constant magma supply of 7.0 m$^3$/s derived from the initial gradient of volume change in the deep source (Fig.1), and assume pressure-dependent trigger to ascend at the top of the deep source as shown in Fig.2. The magma flows up until the half of overpressure is released, and then the pressure of the trigger is built up. In addition, the magma in the shallow source can descend to the deep one. In the whole system, the temperature of magma is fixed at 1273 K.

(2) The role of volatile contents
a) Henry’s law
We suppose there are bubbles consisting H$_2$O and CO$_2$ with radius 100 $\mu$m in the deep magma chamber. Although studies have been made on the volatile-bearing magma, most of them are focused on only H$_2$O. If CO$_2$ exists in magma, however, the bubble formation can occur under relatively high pressure condition since the solubility of CO$_2$ is small. Thus we need discuss not only H$_2$O but CO$_2$.

The solubility ($S$) of the volatile component $i$ is described as

$$S_i = k_i p_i^{\gamma_i}$$  \hspace{1cm} (1)$$

where $k_i$, $p_i$, and $\gamma_i$ are the solubility coefficient and the partial pressure of the component $i$, respectively. From equation (1) and the initial concentration of each volatile component in magma, the proportion of H$_2$O to CO$_2$ in the bubble and the gas fraction of both magma and melt are determined. Since these values are widely changed with the concentrations, needless to say the existence of CO$_2$, the behavior of gas is necessary to be considered.

b) Density
Assuming that the density of melt without water ($\rho_0$) at a depth of 4000 m is 2.89 g/cm$^3$ from volcanic ejecta of Sakurajima, the density of melt with H$_2$O ($\rho_{melt}$) can be described as

$$\frac{1}{\rho_{melt}} = \frac{1 - Y_{H_2O}}{\rho_0} + \frac{Y_{H_2O}}{\rho_{H_2O}}$$  \hspace{1cm} (2)$$

where $Y_{H_2O}$ and $\rho_{H_2O}$ is mass fraction of the dissolved H$_2$O in melt and the density of H$_2$O, respectively. Similarly the density of magma is also
determined from the equation (2) by changing indices \( \text{H}_2\text{O} \) to gas, 0 to melt, and melt to magma.

c) Viscosity

The viscosity of melt \( \eta \) is assumed from the following equation for andesite melts\(^5\)

\[
\log(\eta) = \log(\eta_0) - \frac{6.886w}{w^{1.1} + 2.891} \tag{3}
\]

where \( \eta_0 = 10^{6.1} \) Pa \cdot s is the viscosity of melt without water, and \( w \) is the water content in wt\%. Then the viscosity of magma is determined using Pal’s Equation\(^5\)

\[
\eta_r \left( \frac{1 - 12 \eta_r^2 \text{Ca}^2}{1 - 12 \text{Ca}^2} \right)^{-\frac{4}{5}} = (1 - X)^{-1} \tag{4}
\]

where \( \eta_r, \text{Ca} \) and \( X \) are the ratio of the viscosity of magma to melt, the Capillary number and the gas volume fraction, respectively.

(3) The flow between magma chambers

Since the physical parameters such as density are widely changed with ascent, we calculate all parameters from 3500 m to 100 m by 1 m using equations (1) – (4) and following ones.

a) Mass conservation equation

\[
\rho_{\text{melt}} u (1 - Y_{\text{gas}}) + \rho_{\text{gas}} u Y_{\text{gas}} \pi r^2 = -Q_{\text{esc}} \tag{5}
\]

where \( u \) is the velocity to flow up, \( r \) is the radius of a conduit, and \( Q_{\text{esc}} \) shows the gas escape to surrounding rocks.

b) Momentum conservation equation

\[
dp \over dz = -\rho_{\text{mag}} (g + F_{\text{fric}}) \tag{6}
\]

\[
F_{\text{fric}} = \begin{cases} 
\frac{8 \eta_{\text{mag}} u}{\rho_{\text{mag}} r^2} & : \text{bubbly flow} \\
0.0025 u^2 \over r & : \text{spray flow}
\end{cases} \tag{7}
\]

where \( F_{\text{fric}} \) is the force of friction between the magma and the conduit wall. The inertial term which is omitted from equation (6) can be negligible when considering the magma flow, and the friction term is similar to the viscous resistance of Poiseuille flow under the fragmentation point that the gas volume fraction is the maximum packing fraction of bubbles, i.e. 0.74, and similar to the resistance of the turbulent flow over the fragmentation point.

c) Equation of state for ideal gas

\[
\frac{1}{\rho_{\text{gas}}} = \frac{RT}{M P_{\text{gas}}} \tag{8}
\]

where \( R \) and \( M \) are the gas constant and the molecular weight, respectively.

d) Rayleigh-Lomb Equation\(^7\)

\[
P_{\text{gas}} = P_{\text{melt}} + \frac{4 \eta_{\text{melt}} dr}{r} + \frac{2S}{r} \tag{9}
\]

where \( S \) is the surface tension of the magma, which is assumed to 0.10 N/m. The inertial term is also omitted because it is not important as far as the conduit dynamics is concerned. This equation expresses the gas overpressure in the bubble due to the viscous resistance, and can be numerically solved using forward difference.

(4) The trigger in the deep magma chamber

We suppose the deep magma chamber is a sphere with radius 500m. Owing to the magma supply to the deep chamber with the pressure trigger, the overpressure \( \Delta P \) is increased according to the following equation\(^n\)

\[
\Delta P = \frac{4}{3} \mu_c \frac{DA}{V} \tag{10}
\]

where \( \mu_c \) is the rigidity of surrounding rocks, and \( V \) is regarded as a constant even though magma is supplied.

(5) Volume changes of two chambers

We suppose the rate to flow up \( Q_{\text{up}} \) is twice of the supply rate \( Q_{\text{in}} \), and the rate to flow down \( Q_{\text{down}} \) is one fifth. The density of the magma to descend \( \rho_{\text{mag}} \) at a depth of 100 m is assumed to the density of the surface layer, otherwise the magma in the chamber never stay at the same depth.

The volume changes of the deep chamber \( \text{v1} \) and the melt in the shallow chamber \( \text{v2} \) are described as

\[
\text{v1} = \frac{Q_{\text{in}} \Delta t}{\rho_{\text{mag}}(4000) - \rho_{\text{mag}}(3500) + \rho_{\text{down}}(3500)} - \frac{Q_{\text{up}} \Delta t}{\rho_{\text{mag}}(3500) - \rho_{\text{down}}(100)} \tag{11}
\]

\[
\text{v2} = \frac{Q_{\text{up}} \Delta t}{\rho_{\text{mag}}(3500) - \rho_{\text{down}}(100)} + \frac{Q_{\text{down}} \Delta t}{\rho_{\text{down}}(100)} \tag{12}
\]

where the number after each index indicates a depth, and the \( \rho_{\text{down}} \) at a depth of 3500 m is derived from equation (1), (2), (5) and (8) without considering gas escape nor friction loss through the
conduit. From here, we regard the volume change of the shallow chamber as only the volume change of the melt because the volume change of the gas phase is more than ten times of that of the deep chamber. Considering this trend is opposite to the data in Fig.1, we should consider the gas in the shallow chamber escapes to the air with a minor eruption, for example.

4. RESULT & CONSIDERATION

(1) The initial H_2O concentration of the magma

Assuming that the initial CO_2 concentration of the magma is 0.30 wt%, the initial H_2O concentration is calculated to 2.50 wt% using the condition that the magma stay at a depth of 4000 m. The parameters which are calculated at a depth of 4000m are shown in Table 1. We can confirm the density of the magma is the same as the one of the surrounding layer.

Table 1 Physical parameters at a depth of 4000m.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>100 MPa</td>
</tr>
<tr>
<td>Density of magma</td>
<td>2.65 g/cm³</td>
</tr>
<tr>
<td>Viscosity of magma</td>
<td>1220 Pa·s</td>
</tr>
<tr>
<td>Gas volume fraction</td>
<td>2.54 %</td>
</tr>
<tr>
<td>Gas mass fraction</td>
<td>0.324 %</td>
</tr>
<tr>
<td>H_2O fraction in bubble</td>
<td>65.7 %</td>
</tr>
</tbody>
</table>

(2) Volume changes of the magma chambers

From our model with the 5 m-radius conduit, we calculate the volume changes of magma chambers, and then get the result that it takes more than 1 day for the magma to reach the shallow chamber because the flow velocity is less than 0.01 m/s.

In order to shorten the time, let us take another model to explain the time lag correctly. Although decreasing the radius of conduit with the same flow rate is effective in increasing the velocity, the geometry that there is only one thin conduit underground is not geologically real. For this reason, we consider 10 dispersed conduits with radius 5/10 m as shown in Fig.3, and obtain the reasonable time lag which is 3.7 hours as shown in Fig.4.

With respect to the shallow chamber, since the unit of vertical axis is different from the data in Fig.1, the volume change could not completely explained by our model. The unique trends of both chambers, however, are seemed to be evoked, especially for the case of the deep chamber. This indicates that there is a pressure limit whether the magma starts to ascend, and the magma flows up and down before eruption. Thus, the system in our model is reasonable, and helps to analyze the mechanism of the eruption.

5. CONCLUSION

Through the result from the new model with the two chambers and the 10 conduits, following things could be explained: (1) it is necessary to focus on the volume change of the melt in the shallow chamber because of the greatly high volume of gas phase near the surface, (2) there is a pressure limit that the magma start to ascend, (3) the magma flow up and down before eruption, and (4) the magma can ascend near surface from the deep chamber with a few hours.

6. DISCUSSION

In this study, the consideration of the trigger to erupt and the gas of the shallow magma chamber is not enough, and might be the key to solving the
mechanism of eruption. Further study for them would help to explain the mechanism of volcanic eruption.

In addition, if we can propose the more general model which can apply to the other eruptions, the accuracy of the prediction of volcanic eruption would be greatly improved.

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