Study on Simulation Algorithm of Borehole Radar for Well Logging and its Application to Measuring Volumetric Water Content of Formation

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Abstract: Borehole radar becomes useful tool for high resolution probing of both deep oil reservoir and shallow subsurface. In order to improve understanding of the electromagnetic wave propagation in single hole, a pseudo-spectral time domain (PSTD) algorithm that allows us to model the full EM wavefield for rotational symmetric structure was developed. This technique is based on Finite Difference Time Domain (FDTD) method and uses fast Fourier transform (FFT) algorithm for spatial derivatives. The PSTD algorithm can reduce grid number and save computer memory for large-scale forward calculation and inversion. The simulation provides a method for processing of field data. The measured volumetric water content from borehole radar data gave good agreement with other logging curves, such as resistivity logging curve.

1. Introduction

Borehole radar uses high frequency pulsed electromagnetic wave to acquire subsurface information. The advantage of prospecting using borehole radar is its high-resolution, non-destructive measurement and rapid measurement. Because of this advantage, recently borehole radar has been widely used to investigate fractures, cavities and lithologic change. Many researchers studied a single hole reflection mode and a cross-hole tomography mode. In the single hole reflection mode, radar provides an image of discontinuities in the surrounding formation. But it cannot provide the continuous information of surrounding formation. In the cross-hole tomography mode, where the transmitter and receiver are in separate borehole, radar provides an image of the planar section between the boreholes. The investigation of borehole radar depends on the antenna frequency and the electrical conductivity of the surrounding formation. The major limitation of this method is the attenuation of the signal in rocks or soils containing clay or conductive fluids. When distance of two-borehole radar is long, the receivers cannot receive EM signal. In the present, a single hole refraction mode for EM logging that can provide continuous electrical property of surrounding formation was developed. Because high-frequency electromagnetic wave propagation phenomena associated with borehole radar experiments are complex, it is difficult to understand the EM wave propagation in field experiment. To improve the understanding of the governing physical processes, a pseudo-spectral time-domain (PSTD) algorithm to

simulate EM wave propagation in a borehole was developed. This technique is based on FDTD method and uses fast Fourier transform (FFT) algorithm for spatial derivatives. The PSTD algorithm can reduce grid number and save computer memory for large-scale forward calculation and inversion. This model can simulate EM wave propagation in complex formation structure. The simulation provides a method for processing of field data. The measured volumetric water content from borehole radar data gave good agreement with other logging curves, such as resistivity logging curve.

2. Methodology

Here, a simplified 2-D PSTD algorithm for rotationally symmetric coordinate is introduced. The assumption that the angular variation of the electromagnetic fields has either a $\sin(m\phi)$ or $\cos(m\phi)$ variation is set. Here, m is a plus integer, ϕ is the rotation angle with respect to the cylindrical symmetry axis. Using the generalized differential matrix operators, the Maxwell's curl equations are expressed as:

$$\begin{bmatrix} 0 & -\partial_z & \pm m/r \\ \partial_z & 0 & -\partial_r \\ \mp m/r & (\partial_r)/r & 0 \end{bmatrix} \begin{bmatrix} E_r \\ E_\phi \\ E_z \end{bmatrix} = - \begin{bmatrix} (\mu\partial_t)H_r \\ (\mu\partial_t)H_\phi \\ (\mu\partial_t)H_z \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} 0 & -\partial_z & \mp m/r \\ \partial_z & 0 & -\partial_r \\ \pm m/r & (\partial_r)/r & 0 \end{bmatrix} \begin{bmatrix} H_r \\ H_\phi \\ H_z \end{bmatrix} = \begin{bmatrix} (\varepsilon\partial_t + \sigma)E_r \\ (\varepsilon\partial_t + \sigma)E_\phi \\ (\varepsilon\partial_t + \sigma)E_z \end{bmatrix} \quad (2)$$

where, ∂_α denotes $\partial/\partial\alpha$ ($\alpha = r, z, \phi$), μ is the magnetic permeability, ε is the dielectric permittivity, σ is the electric conductivity, t is the time, r is the radial distance, z is the depth, and ϕ is rotation angle with respect to the cylindrical symmetry axis. E and H denote the electric and magnetic field components, respectively. The subscripts r, z , and ϕ refer to the radial, vertical, and tangential components of the electric and magnetic fields. Here we use the transverse magnetic (TM) mode in which there are only E_r, E_z and H_ϕ . As an example, we write out the

E_r formula:

$$E_r^{n+1}(i, j) = \frac{1 - \frac{\sigma \cdot \Delta t}{2\varepsilon}}{1 + \frac{\sigma \cdot \Delta t}{2\varepsilon}} E_r^n(i, j) - \frac{\frac{\Delta t}{\varepsilon}}{1 + \frac{\sigma \cdot \Delta t}{2\varepsilon}} \frac{\partial H_\phi}{\partial Z} \quad (3)$$

The calculation of the spatial derivative can be achieved as:

$$\frac{\partial H_\phi}{\partial Z} = F_z^{-1} \{ i k_z F_z [H_\phi] \} \quad (4)$$

Here, F_z and F_z^{-1} denote the forward and inverse Fourier transforms that are achieved by FFT algorithm, k_z is the wavenumber.

The PSTD algorithm represents the derivative by a trigonometric polynomial

$$\left[\frac{\partial H_\phi}{\partial z} \right]_{z=z_k} = \frac{1}{N} \sum_{n=0}^{N-1} j n k_z \exp\left(\frac{j n k_z}{2}\right) \tilde{H}_\phi \cdot \exp\left(\frac{j 2\pi m n}{N}\right) \quad (5)$$

Here, $\tilde{H}_\phi$ is the forward Fourier transforms of H_ϕ . The new spectral differential operator is purely real at the Nyquist wavenumber, so the numerical computed derivative is also purely real. Using the same manner, we can get the E_z and H_ϕ formulas in the TM mode.

3. Stability Condition in Cylindrical Coordinate

The dispersion analysis and stability condition of the 2-D PSTD algorithm can be formulated for homogeneous, lossless media. For 2-D PSTD algorithm in cylindrical coordinates, the stability condition

$$\Delta t \leq \frac{1}{\alpha \cdot C_{\max} \sqrt{\frac{1}{(\Delta r)^2} + \frac{1}{(\Delta z)^2}}} \quad (6)$$

Where, Δr , Δz , and Δt are discretization intervals with regard to r , z and t , respectively, $C_{\max}$ is the maximum phase velocity in finite-difference grid, $\alpha = \pi$

$$/2 \approx 1.5708$$

4. Numerical Tests

An ideal electric dipole as the radar source is used. The source wavelet is the Ricker wavelet with a peak frequency of 100 MHz. The source function is added to the E_z component as the source function at each time step. Receivers are also aligned in the borehole and record the axial electric field component E_z . The electromagnetic wave propagation in the formation containing two or three layers is simulated.

The formation contains three layers with two horizontal interfaces. The parameters are shown in Table 1. The source and receivers are set close to the boundary of the borehole.

Table 1. The parameters

	Conductivity (s/m)	Relative permittivity
Layer 1	0.005	8
Layer 2	0.03	20
Layer 3	0.01	12
Mud	0.1	30

(a) Model

Fig. 1. Simulated results.

Fig. 1(b) shows that the waveforms have strong reflection wave from the formation interface since both the conductivities and the relative permittivities are different between layer 1 and layer 2. Fig. 1(e) and (f) show the calculated conductivity and relative permittivity curves are sensitive to the boundary of the formation and the conductivity and relative permittivity are very close to the true value of the formation. Usually, in the water saturated layer, there are large conductivity and relative permittivity, so using the single hole borehole radar technique, the water saturated layer can be detected.

5. The Experiment of Borehole Radar for Well Logging and Data Processing

The experiment was done at Tsukuba Technology Center in OYO Company. Two borehole radars in the experiment were used, one is the source and the other is the receiver. The source diameter is 70 mm; the receiver diameter is 37 mm. The diameter of borehole is 75 mm.

The field data from Tsukuba Technology Center in OYO Company are processed. Fig. 1. shows the relative permittivity logging curve calculated from field data.

Fig. 2. Relative permittivity logging curve from field data.

Factors other than water content, such as formation density, texture and temperature, have a negligible influence on the relative permittivity. Using its assumptions, the volumetric water content (θ) is found by substitution of ϵ_r in the empirical relationship:

$$\begin{aligned} \theta = & [-5.3 * 10^{-2}] + [(2.92 * 10^{-2}) * (\epsilon_r)] \\ & - [(5.5 * 10^{-4}) * (\epsilon_r)^2] + [(4.3 * 10^{-6}) * (\epsilon_r)^3] \end{aligned} \quad (4.16)$$

Where, ϵ_r is relative permittivity. Fig. 3. shows the relationship between volumetric water content and relative permittivity.

Fig. 3. Volumetric water content – Relative permittivity curve.

The resistivity of formation has relationship with water content. Their relationship is shown in Fig. 4. Where EC_w is pore-water electric conductivity, mmho/cm.

Fig. 4. Relationship between volumetric water content, θ , and resistivity, ρ_o , for different values of pore-water conductivity, EC_w .

The calculated volumetric water content (θ) curve from relative permittivity logging curve is shown in Fig. 5.

Fig. 5. Volumetr water content logging curve with resistivity logging curve.

Comparing the calculated volumetric water content logging curve with resistivity logging curves shown in Fig. 5, it was found that there are two layers that have high water content at the depth 17-19m and 37-41m, the resistivity is low, the relative permittivity is high. The calculated volumetric water content from borehole radar data

gave good agreement with resistivity logging curve.

4.4 Conclusion

In the present, the PSTD algorithm in symmetric cylindrical coordinate was developed to simulate electromagnetic wave propagation in single hole. The PSTD algorithm can reduce grid number and save computer memory for large-scale forward calculation and inversion. The conductivity and relative permittivity logging curve are obtained after processing of synthetic data. The calculated conductivity and relative permittivity logging curve are close to the true value of surrounding formation.

The field experiment of borehole radar is introduced. The refracted wave from formation is the first arrival in received signal. The relative permittivity logging curve was obtained after processing of field data. The relative permittivity logging curve was used to calculate volumetric water content of formation. The measured volumetric water content from borehole radar data gave good agreement with other logging curves, such as resistivity logging curve. The introduced EM logging using borehole radar is convenient and effective method for detecting water content of formation.

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