Experiments of P-wave propagation in imperfectly saturated sand -Effect of pore-water pressure -

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Abstract A series of P-wave propagation tests were conducted for imperfectly saturated sand specimens in a chamber under confining pressure in the range between 0 and 10 a.t.m. High- as well as low- frequency waves were observed to propagate with velocities and attenuations varying with the confining pressure. The natures of these waves are discussed by comparing the observed results with theories including those by Biot (1956). It is concluded that these waves are very likely various frequency components of Biot waves of the 1st kind.

1 Introduction

High- and low- frequency waves are frequently observed in laboratory experiments of wave propagation in imperfectly saturated sand under low confining pressure (see Fig.2). The natures of these waves, however, have not been so far fully discussed except Paterson (1956) who referred to these as frame wave and airwave or liquid wave propagated through the frame and interstitial fluid. Paterson published this study in the same year as a well-known paper on waves in porous media by Biot (1956), and naturally he did not discuss about the relationship of these waves with the Biot's 1st and 2nd kind waves.

Kitsunozaki (2003) discussed about waves in a pipe containing water to help understanding the Biot's waves. According to him, "(if it is simply expressed) one of the waves propagated in a pipe containing water is transmitted mainly through the pipe and another through water, and they correspond to Biot's 1st kind and 2nd kind waves, respectively." If we rely on this concept, do the frame wave and liquid wave by Paterson correspond to Biot's 1st kind and 2nd kind waves?

Bearing this question in mind we have made a series of experiments of wave propagation in sand specimens containing fluid under variable confining pressure.

2 Experiment Procedure

We have made a series of experiments in a pressurized chamber (Fig.1). The sand specimen has a cylindrical shape of 16.5 cm in diameter and 13 cm in height and is covered by a rubber envelope. A hard polyester disk closes each end of the specimen. The top disk is attached with a solenoid hammer and an accelerometer, and the bottom one with an accelerometer. The disks and the rubber envelope are bound together by wire after the inner space is filled with sand. A tube penetrated through the bottom disk to the specimen is connected with the pressure tank. Another tube through the top disk of the specimen is open to the atmosphere. The third tube at the bottom of the chamber also connected with the pressure tank. Three valves are equipped around the chamber for controlling or measuring pressure: The first is at the outlet to the pressure tank, the

2nd at the inlet of the specimen, and the 3rd is at the outlet of the specimen. Two pressure gauges (or regulators) also are equipped: The first pressure gauge is for measuring pressure in the chamber and the second for pressure within the specimen. A rod inserted in the chamber supports the specimen. With the rod we can measure the axial deformation of the specimen from outside of the chamber. However, no load was given on the specimen through the rod but exclusively by fluid pressure in chamber.

The experiments were conducted for three kinds of specimens: 1) naturally dried sand containing air, i.e. interstitial fluid is air; 2) interstitial fluid is simply replaced by water for modeling a sand submerged in water of high air content; and 3) some air is removed from the former sample for modeling a sand submerged in water of low air content.

Preparation procedure for every kind of specimens was as follows: 1) we first filled dry sample sand ($U_c=2.40$, $G_s=2.68$, with 60% fine contents) in a cylindrical rubber envelope supported by a rigid vessel and compacted slightly by tamping with a glass rod (at this stage the porosity is estimated at about 44%); 2) then water is supplied from below to saturate the sample with water (for a high S_g sample), and 3) we stir the submerged sand by a glass rod for mobilizing and facilitating air bubbles to leave from sand particles (for a low S_g sample). For definition of S_g , see a note below.

Experiments for wave propagation were conducted for specimens of every kind under 1) un-drained condition (valves 1 is open, 2 is closed and 3 is closed. See Fig.1), 2) drained condition (valves 1,2 and 3 are open, closed and open, respectively), and 3) pore-water pressure that was equal to the chamber pressure (valves 1, 2 and 3 are open, open and closed, respectively). In all experiments the chamber pressure was varied between 1 and 10 a.t.m. whenever possible.

Note: Fractional gas saturation S_g is defined as the volume ratio of gas volume to total volume of void of a specimen, i.e.

$$S_g = (W_s - W_i) / \rho_w V \phi$$

where W_s is the weight of the specimen in saturated state, W_i that in imperfectly saturated state under investigation, ρ_w the specific weight of water, and $V \phi$ the total void in the specimen. Thus, S_g is related to the degree of saturation S_r by $S_g = 1 - S_r$.

3 Results of experiments

High- as well as low-frequency waves are visible in an example record in Fig.2, where the high-frequency range is from 24 to 44 KHz while the low-frequency range from 10 to 12 KHz. It is noted that speeds of both high- and low-frequency components increase with the increase in chamber pressure (= confining pressure). The results are plotted in Fig.3 to 5.

It is evident that the velocity depends on the kind of interstitial fluid and also on confining pressure. We consider that the cause of the latter fact is related to the change in the matrix (framework) velocity and in fractional gas saturation S_g with the change in confining pressure.

When all the interstitial fluid is air, i.e. $S_g=1.0$, velocities of both high- and low-frequency waves increase in parallel with the increase in confining pressure (Fig.3a and b). We have made use of these data for estimating the matrix velocity as a function of confining pressure, because the effect of air on wave propagation seemed to be negligible. The velocity vs. confining pressure mainly for high-frequency components may be approximated by the following relationship:

$$v_m = \sqrt{(p + 3.91)/28.4}, \quad (v \text{ in km/s; } p \text{ in a.t.m.})$$

where p is additional confining pressure to ambient atmospheric pressure in a.t.m.

In contrast, results with water as interstitial fluid show that the velocity of the high-frequency wave under un-drained condition increases with confining pressure between 0 and 10 a.t.m. from 700 to 1000m/s, while that under drained condition remained almost steady at a value around 700 to 800m/s (Fig. 4 a and b). It suggests that the high-frequency wave is the wave propagated mainly in interstitial water: Under un-drained condition the speed in water is increased with confining pressure because air bubbles are compressed or dissolved in water with increasing pressure, while under drained condition such does not occur. The low-frequency wave increases its velocity with confining pressure under both drained and un-drained conditions probably because of the increase in matrix velocity. This suggests that the low-frequency wave is not affected much by air bubbles contained in water but mainly by consolidation of matrix.

Next, we have made experiments for lower fractional gas saturation S_g . Under un-drained condition the speeds of high- and low-frequency component waves approached with each other gradually with the increase in confining pressure and merge to a same value at 6 a.t.m., then the speed attained the highest value of 1700 m/s at 10 a.t.m. (Fig.5a). We have left the specimen after the before-mentioned experiments as it was for one day. During the period S_g increased slightly, and similar experiments under un-drained condition were repeated again on the next day: High- and low-frequency components merged at 9 a.t.m. and both took a maximum value of 1000 m/s at 10 a.t.m. (Figure not shown).

High-frequency component wave steadily took a value of about 800 m/s under drained condition. The speed in water does not change with confining pressure because air bubbles are not affected by confining pressure under drained condition. In contrast to this, the speed of low-frequency component increased from 250 m/s to 800 m/s with the increase in confining pressure (Fig.4b, 5b).

We gave up experiments of the third condition that the pore pressure is equal to the confining pressure for specimens for high S_g because the specimens yielded at a confining pressure at about 2 a.t.m. where actually the pore-water pressure was about 80% of confining pressure. Pore-water pressure and confining pressure was nearly equal for low S_g case and the state of the specimens was similar to that for high S_g and we gave up the experiment at confining pressure of 7 a.t.m. The reason for yielding of specimens should be that the effective pressure became very low when pore-water pressure was nearly equal to confining pressure. and the specimen could not preserve its shape.

4 Discussion

Under a low confining pressure the high-frequency wave seems to be propagated mainly through interstitial fluid while the low-frequency wave does through sand matrix. However, it is evident that both waves are propagated not exclusively in one of the media but in another too suggesting that there is an interaction between the fluid and the frame. Concerning to the effect of confining pressure it is considered that a compaction of matrix with increasing confining pressure contributes to increase of frame-wave speed, and in case of un-drained condition a reduction in gas content or a lowering of S_g with increasing pressure also will contribute to the increase of wave speed in water because air bubbles in water are compressed or dissolved in water. It seems that the interaction becomes stronger with decreasing S_g . It is natural that an interaction is easier to occur if two component velocities are nearer and a lower S_g will help making this situation

Case 1: When fluid is air; although under both drained and un-drained conditions high- and low-frequency waves are observable, the high-frequency wave when the fluid is air is not as remarkable as that when fluid is water. The effect of confining pressure is shown in Fig.3a and b. The speed of the low-frequency component increases from 350 m/s at 0.5 a.t.m. up to 650- 750 m/s at 10 a.t.m. For interpreting these waves we have calculated the speed of the Biot's 1st and 2nd kind waves when the fluid is air by using expressions by Geertsma and Smit (1961). It has become clear from the calculation that the speed of the 1st kind wave was near that of matrix P wave and exhibited little dispersion, while that of the 2nd kind wave was about 190 m/s and stayed almost steady for change in the assumed matrix speed. The difference of calculated speeds of 1st and 2nd type are too large compared with the difference of observed speeds of high- and low-frequency waves. Therefore, the high- and low-frequency waves in experiments cannot be considered as the 1st and 2nd kind Biot waves (Figures not shown).

Case 2: when fluid is water; high-frequency components are more remarkable than when fluid is air as seen in Fig.2. We have made a similar calculation of the Biot's 1st and 2nd kind waves by replacing the fluid of the case 1 model by water with the speed of 1500 m/s. The speed of the 1st kind wave was 1730 m/s for the matrix P-wave velocity at 650 m/s and 1795 m/s for the matrix velocity at 900 m/s. The speed of the 2nd kind wave was 280 m/s and 380 m/s, respectively. The velocities for the 1st kind are too high and those for 2nd kind are too low to fit to the values observed in the experiments. It means that the experimental results cannot be explained by Biot (Geertsma and Smit) model on assuming water wave velocity at 1500 m/s.

We then considered a possibility that the assumed velocity for water was too high, and recalculated the 1st and 2nd type Bio's waves by assuming lower velocities for water, because the velocity can be drastically reduced if water contains air bubbles. The results were as shown in Fig.6. The sound velocity of 500m/s in water corresponds to water of fractional gas saturation S_g at 0.95 under 3.3 a.t.m. and/or S_g at 0.9 under 5.7 a.t.m.

In this condition the velocity of the 1st kind wave was 710 or 980 m/s if the matrix velocity was

assumed at 500 or 900 m/s, revealing a reasonable agreement with the experimental results. That is to say, if we assumed a low velocity for water considering the degree of saturation was not so high, the experimental results may possibly be explained by the Bio's 1st kind wave.

White (1975) discussed about waves in porous material with partial gas saturation. We have tried to calculate velocity and attenuation assuming a White model. The results are as shown in Fig.7 and 8. If we assumed matrix velocities for low and high confining pressure cases appropriately, we could explain the experimental results on the whole. The value of attenuation was also sufficiently low to let waves observable. We have here for simplicity ignored the frequency effect on attenuation. The conclusion thus obtained will not be changed by this simplified assumption. A difficult problem in applying White model is the choice of average bubble distance $2b'$. White estimated it at 13.3 cm while Kitsunozaki at 5.0 cm. These values have never been observed directly but were chosen to fit the observed results. In the present calculation we assumed $2b'$ at 6 cm. This value is important since it shifts the frequency characteristics of velocity and attenuation horizontally.

5 Conclusions

P-wave velocity in imperfectly saturated sand was measured when confining pressure was changed under drained and un-drained conditions. In most cases high-frequency waves at 24 to 44 KHz as well as low-frequency waves at 10 to 12 KHz were observed. These waves were distinct while the confining pressure is low, and the high-frequency component is interpreted as the wave transmitted mainly through water and the low-frequency component that transmitted mainly through matrix, although water and matrix waves are not perfectly independent but interact with each other. When the confining pressure is raised, the interaction becomes stronger and both waves merge into one wave at a confining pressure. After a series of trial calculations, the high- and low-frequency waves are considered to be distinct frequency components of the Biot's 1st kind wave.

Literature

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Fig.1 Pressurized chamber for experiments. It is provided with three valves for controlling pore-water pressure in the sand specimen.

Fig.2 An example seismogram under various confining pressure when the fluid is water of higher air content: under un-drained condition.

Fig.3 P-wave velocity versus confining pressure when fluid is air. upper: under un-drained condition and lower: drained condition.

Fig.4 The same as Fig.3 except that the fluid is replaced by water of high air saturation S_g . upper: under un-drained condition and lower: drained condition.

Fig.5 The same as Fig.3 except that the fluid is replaced by water of low air saturation S_g. upper: under un-drained condition and lower: drained condition.

Fig.6 P-wave velocity and attenuation of Biot waves when the interstitial fluid is water of sound velocity at 500m/s.

Fig.7 Velocity versus frequency of White model under a lower confining pressure

Fig.8 Velocity versus frequency of White model under a higher confining pressure