Least Squares Wavepath Migration

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Summary

We present the theory and some numerical examples for least squares wavepath migration (LSWM). This method migrates data by applying iterative conjugate gradients to the wavepath migration operator. Like least squares Kirchhoff migration, LSWM is designed for reducing migration artifacts, improving image resolution and energy focusing. Different from least squares Kirchhoff migration, LSWM is expected to be an inexpensive iterative algorithm because wavepath migration is computationally efficient. Numerical results with both synthetic and field data show that the LSWM image is noticeably better than the wavepath migration image in image resolution, artifacts attenuation, and interface continuity.

Introduction

Standard diffraction stack migration (French, 1974) smears each time sample in a trace along a quasi-ellipsoid in the velocity model. The summation of all such smears for all of the prestack traces yields a prestack migration image. For large 3-D data sets and large velocity models this computational expense is too large, especially for iterative algorithms, e.g., least squares Kirchhoff migration (LSKM). According to Nemeth et al. (1999), LSKM can effectively suppress the acquisition footprint and increase the image resolution. However, a few tens of iterations are normally required by LSKM to reach an acceptable residual level, which is a huge computation burden for many data sets.

To overcome this problem, we use wavepath migration instead of Kirchhoff migration as the inversion operator in the conjugate gradient iterations. The reason is that wavepath migration is much more efficient than Kirchhoff migration, because it smears a trace's energy along wavepaths rather than a volume of quasi-concentric ellipsoids (Sun and Schuster, 2001). We call this new method as least squares wavepath migration (LSWM). According to Sun and Schuster (2000), 3-D wavepath migration can be more than an order-of-magnitude faster than 3-D Kirchhoff migration, suggesting wavepath migration a potentially preferred method for 3-D least squares calculations.

This paper first presents the theory of least squares wavepath migration, followed by a section showing numerical results for both synthetic and field data sets. Finally, we draw some conclusions and describe future plans.

Theory

In general, the seismic forward modeling operator can be represented by:

$$\mathbf{d} = \mathbf{L}\mathbf{m}, \quad (1)$$

where $\mathbf{d}$ is the forward modeled seismic data, $\mathbf{m}$ is the reflectivity model, and $\mathbf{L}$ is a linear forward modeling operator.

Standard migration operator is the adjoint of the forward modeling operator which can be given by:

$$\mathbf{m}' = \mathbf{L}^T \mathbf{d}, \quad (2)$$

where $\mathbf{m}'$ is a rough approximation to the true reflectivity model. In practice, $\mathbf{L}^T$ can be chosen as a Kirchhoff, a wavepath, or a wave equation migration operator, leading to an image with large difference in both quality and computational efficiency.

Substituting equation (1) into (2) gives

$$\mathbf{m} = [\mathbf{L}^T \mathbf{L}]^{-1} \mathbf{L}^T \mathbf{d}. \quad (3)$$

Here, $\mathbf{m}$ is a better approximation to the true reflectivity model as it is produced by simulating the observed data in a least squares sense. In practice, equation (3) is solved using an iterative scheme, and the computational efficiency of the scheme largely depends on the migration operator applied.

Sun and Schuster (2001) showed that the computational cost of a Kirchhoff-type $\mathbf{L}^T$ could be significantly reduced by applying a stationary phase approximation. The resulting method was denoted as wavepath migration (WM). Compared to Kirchhoff migration (KM) which

smears a reflection arrival along a quasi-ellipsoid in the image space, wavepath migration computes a specular ray for the arrival using a predetermined incidence angle, then smears the arrival to a small Fresnel zone centered about the specular reflection point. This suggests that wavepath migration could be used as the inversion operator to speed up the entire least squares algorithm. The WM method was preceded by similar beam-like migration methods such as Gaussian Beam migration (Hill, 1990) and Kirchhoff beam migration (Sun et al., 2000).

Although the computational cost of WM can be up to two orders of magnitude less than KM, the WM section is sometimes plagued by weak reflectivity and poor interface continuity. This is partly because the reflection arrivals are smeared along small Fresnel zones, leading to lower stacking folds for some image points. Similar problem exists when migrating incomplete data, where least squares migration had been verified to be useful in improving the image quality (Nemeth et al., 1999). Therefore, by combining wavepath migration with least squares iteration, we can expect an improved migration method for both image quality and computational efficiency.

Numerical Examples

The least squares wavepath migration method is now tested on both synthetic and field data sets. The synthetic data are associated with a point scatterer model, and the field data were collected by Mobil from the North Sea.

Point Scatterer Model

A point scatterer model is used to test the effectiveness of the LSWM method. There are 101 shot stations uniformly distributed along the free surface with a shot interval of 40 m. Each shot gather consists of 201 traces with a geophone interval of 20 m. The source is a 50-Hz Ricker wavelet, the medium velocity is homogeneous with $c=5000$ m/s, the time sampling interval is 1.0 ms, and the model is represented by a 201×201 grid with a horizontal and vertical spacing of 20 m. A diffraction stack forward modeling algorithm is used to generate the synthetic seismograms.

Figure 1 shows the forward modeled zero offset data. Figure 2 (top) shows the KM image, where the point scatterer is well resolved, but the migration artifacts are also clearly seen. In contrast, the WM image in Figure 2 (middle) shows much fewer migration artifacts, and the point scatterer is as well resolved as in the KM image. Figure 2 (bottom) shows the LSWM image, which is superior in both image fidelity and artifacts attenuation. Ten iterations were used to generate the LSWM image.

North Sea Field Data

The Mobil data were provided courtesy of Robert Keys (ExxonMobil). It is a marine data set, with sources and receivers deployed along a 2-D line. There are 668 shot gathers, each of which contains 120 traces. Both the source and the receiver intervals are 25 m. The recording length is 1500 samples with a time sampling interval of 4 ms. The model is discretized into a 1594×281 grid with a grid interval of 12.5 m in both the horizontal and vertical directions. The migration velocity is shown in Figure 3, which was obtained by using conventional stacking velocity analysis.

Figure 4 shows the WM image (top) and the LSWM image (bottom) after fifteen iterations. All of the traces were used in the migration. Compared to the WM image, the LSWM image shows better energy focusing, interface continuity, and image resolution. Major layers are more clearly defined in the LSWM image, facilitating geological interpretations. Figure 5 shows a zoom view of the boxed area in Figure 4, where two major interfaces in the LSWM image are resolved noticeably better than in the WM image.

Conclusion

We have presented a potentially efficient form of least squares migration denoted as least squares wavepath migration. Numerical results with both synthetic and field data show that, compared to wavepath migration, LSWM can effectively reduce migration artifacts and improve image resolution. In addition, energy focusing and interface continuity are also noticeably improved in the LSWM image. Typically, ten to fifteen iterations are required by LSWM to achieve an acceptable residual level.

For the 2-D field data, CPU comparison shows that wavepath migration is 1.5 times faster than standard Kirchhoff migration. Thus, compared to a standard KM method, our LSWM scheme is still much more expensive. For 3-D cases, however, LSWM could require similar or even less computational cost compared to KM. The reason is that 3-D wavepath migration is more than an order-of-magnitude faster than 3-D Kirchhoff migration (Sun and Schuster, 2000). On the other hand, for 3-D migration where severe acquisition footprints usually exist, LSWM could be able to improve the image quality more effectively. Future work will thus apply LSWM to 3-D depth imaging.

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Figure 1: Zero offset data associated with the 2-D point scatterer model. 201 traces were generated by a 50-Hz ricker wavelet source. The point scatterer is centered below the geophone configuration at a depth of 2000 m.

Figure 2: Comparison of migrated images for the (top) KM image, (middle) WM image, and (bottom) LSWM image. The LSWM image contains the fewest migration artifacts and shows the best point scatterer resolution.

Figure 3: Migration velocity for the Mobil data.

Figure 4: Comparison of migrated images for the (top) WM image, and (bottom) LSWM image. The LSWM image shows better energy focusing and interface continuity. Fifteen iterations were used to generate the LSWM image.

Figure 5: Zoom views of the boxed area in the previous figure. Interfaces in the LSWM image (bottom) are resolved noticeably better than in the WM image (top).