A New Saturation Equation based on Maxwell-Garnet Model

Shigeaki Asakura (Schlumberger), Yuzuru Ashida (Kyoto University),
Hiroshi Eida (Japan National Oil Corporation), Alain Brie (Schlumberger),
Masahiro Kida (Idemitsu Oil & Gas Co., Ltd.), Akinori Imayoshi (Idemitsu Oil & Gas Co., Ltd.)
and Tomoaki Fujikawa (Schlumberger)

Abstract

Thinly laminated sands have been a difficult problem for formation evaluation. It is very easy to miss the pay zone on the logs without additional information such as hydrocarbon shows in the mud logs. This is because each sand lamina is so thin that the petrophysical features of the hydrocarbon are often masked by shale lamina. In Japan, particularly in the Niigata area, the mixture of pyroclastic material with sand (Tuffaceous sand) makes this problem more complicated. The calculated Water saturation (S_w) using the conventional equations, such as the Simandoux and Indonesia equations indicate water even if the zone is producing hydrocarbon.

For the conventional equations, the problems arise when a particular shale distribution type becomes dominant, because those equations disregard the type of shale distribution. However, their responses on logs are distinguishable. In this paper, a new saturation equation has been proposed which can accommodate three different distribution types of shale (Laminated, Structural and Dispersed shale), in order to obtain more meaningful S_w . Also, the methodology to separate the Dispersed shale portion from the total shale volume (V_{sh}) by comparing Density-Neutron porosity and Sonic porosity, has been discussed.

Introduction

Inspection of cores reveals that shale material may be distributed in formations in three possible ways (Fig. 1):

1. Shale may exist in the form of laminae between which are layers of sand. The *Laminar shale* does not affect the porosity or permeability of the sand streaks themselves. However, when the amount of Laminar shale is increased and the amount of porous medium is correspondingly decreased, overall porosity is reduced in proportion.
2. Shale may exist as grains or nodules in the formation matrix. This matrix shale is termed *Structural shale*, and is considered to have properties similar to those of Laminar shale.

3. The shaly material may be dispersed throughout the sand, partially filling the intergranular interstices. The *Dispersed shale* may be in the form of accumulations adhering to or coating the sand grains, or it may partially fill the smaller pore channels. Dispersed shale in the pores markedly reduces the permeability of the formation.

All of these forms of shale may, of course, occur simultaneously in the same formation.

Fig. 1 - Forms of shale classified by manner of distribution in formation. Pictorial representations above, volumetric representations below (Schlumberger).

Regardless of the type of shale distribution, the saturation equations proposed in many different area work fine when the fraction of the shale is relatively small. These equations have a general form:

$$C_t = a S_w^2 + g S_w \quad (\text{Eq-1})$$

Where a denotes a predominant sand term and g denotes a predominant shale term. Those equations often indicate a pessimistic value when a particular type of shale becomes dominant and those are recognized as an abnormal resistivity profile in pay sand.

Traditionally, the Indonesia equation is commonly used in Japan. This is because the equation becomes a simple linear form assuming $m=n=2$ and $V^{1-V/2}$ is close to V .

$$\sqrt{C_t} = [f\sqrt{C_w/a} + V_{cl}\sqrt{C_{cl}}] S_w \quad (\text{Eq-2})$$

This equation also gives a pessimistic result on S_w in Japan, particularly in the Japan Sea where the formation is often thinly laminated (Submarine turbidity) and the pore spaces are contaminated by the Dispersed shale due to poor sorting of the grains.

In this paper, the Maxwell-Garnet model has been used to examine the respective response to the shale distribution types.

(1) Maxwell - Garnet Model

An equivalent approximation of the saturation equation is provided by the Maxwell - Garnet Model by examining its response to the various shale distribution types.

The largest benefit of this technique is that it is based on physics, and therefore does not require the extensive core studies on which conventional saturation equations rely.

The primary host is assumed to be uniformly conductive, with a conductivity C_p . The model predicts the effect of spherical pore inclusions on the bulk conductivity. When the spherical inclusions contain conductivity of C_{sh} , the model simulates connected spherical inclusions. The approximation that we use assumes that the primary medium follows Archie's Law, and V is the fractional volume of spherical inclusion (Eq-3).

$$\frac{C_p - C_t}{2C_p + C_t} + V \frac{C_{sh} - C_p}{C_{sh} + 2C_p} = 0 \quad (\text{Eq-3})$$

$$C_p = f_0^n S_w^n C_w / a \quad (\text{Eq-4})$$

The Maxwell-Garnet Model is an ideal model for the prediction of a vuggy formation, for example, since the spherical inclusions take place randomly and inclusions replace grain and pore spaces simultaneously.

However, the application of this model to Dispersed shale and Structural shale is problematic as the spherical inclusions need to be uniformly distributed.

Structural Shale

Fig-2 shows the inclusions of Structural shale (V_s , C_s) against the primary medium. In the case of Structural shale, only the quartz grains are replaced by the shale and the original porosity (f_0) in the primary medium is not reduced. To achieve this condition, an additional pore space needs to be included in the primary medium.

Fig. 2

With the condition mentioned above, (Eq-3) becomes equation (Eq-5),

$$\text{where } C_p = (f_0 + \Delta f)^2 S_w^2 C_w / a$$

$$\text{and } C_{sd} = f_0^2 S_w^2 C_w / a.$$

$$C_t = \frac{C_{sd}}{(1 - V_s)} \cdot \frac{2C_{sd} + C_s(1 + 2V_s)(1 - V_s)}{C_{sd}(2 + V_s) + C_s(1 - V_s)^3} \quad (\text{Eq-5})$$

(Eq-5) is plotted to examine the effect on C_t by increasing the Structural shale (V_s). (Fig.A) and (Fig.B) are the comparisons with the Indonesia and Simandoux equations respectively using typical values in the equations ($f_0=30\%$, $m=n=2$, $a=0.8$, $C_w=10$, $C_s=1$). For the Indonesia equation, the discrepancy increases as V_s does, on the other hand, the Simandoux equation shows a better agreement with Maxwell-Garnet Model. Therefore, the Simandoux equation (Eq-6) was chosen for the saturation equation in the case of Structural shale.

$$C_t = \frac{f_0^2 S_w^2 C_w}{a(1 - V_s)} + V_s C_s S_w \quad (\text{Eq-6})$$

Dispersed Shale

(Fig.3) shows the inclusion of Dispersed shale (V_d , C_d) in the primary medium. In the case of Dispersed shale, only the pore space is replaced by the shale. With the definition of (q) as the fraction of the original porosity (f_0) occupied by Dispersed shale, the Maxwell-Garnet Model can be used to model the change of the apparent fluid conductivity.

$$C_{wa} = C_{wp} \cdot \frac{2C_{wp}(1-q) + C_d(1+2q)}{C_{wp}(2+q) + C_d(1-q)} \quad (\text{Eq-7})$$

C_t is derived from $C_t = f_0^2 C_{wa} / a$ and (Eq-7)

$$C_t = C_{sd} \frac{2C_{sd}(1-q) + C_d f_0^2 (1+2q)/a}{C_{sd}(2+q) + C_d f_0^2 (1-q)/a} \quad (\text{Eq-8})$$

Fig.3

The Indonesia equation shows a large difference from the Maxwell-Garnet Model (Fig.C), particularly when S_w is low, which explains why the low resistivity profile is observed when Dispersed shale is associated with oil bearing sand. After examining various saturation equations, (Eq-9) was confirmed to be the best fit to the given conditions (Fig.D).

$$\left. \begin{aligned} C_t &= f_0^2 [(1-q)C_w S_w^2 + qC_d] / a \\ \text{or} \\ C_t &= (f + V_d)(fC_w S_w^2 + V_d C_d) / a \end{aligned} \right\} (\text{Eq-9})$$

Laminated Shale

The Maxwell-Garnet Model is not valid for Laminated shale as the spherical inclusions take place randomly, i.e. not isotropically. However, the equivalent electrical model is known for Laminated shale (Fig-4).

Fig.4

$$\begin{aligned} C_t &= (1-V_L) f_0^2 S_w^2 C_w / a + V_L C_L \\ &= \frac{f^2 S_w^2 C_w}{a(1-V_L)} + V_L C_L \end{aligned} \quad (\text{Eq-10})$$

(2) Integrated Saturation Equation

When three types of shale are associated in the formation, the respective saturation equations (Eq-6, 9, 10) are integrated into one formula (Eq-11). The equation cannot be solved unless V_L , V_s , and V_d are known, however, V_s and V_L are merged when $S_w=1$ (C_0) and assuming $C_L=C_s=C_{sh}$.

$$C_t = \frac{(f + V_d)(f S_w^2 C_w + C_d V_d)}{a(1-V_L - V_s)} + V_s C_s S_w + V_L C_L \quad (\text{Eq-11})$$

$$\left. \begin{aligned} C_0 &= \frac{(f + V_d)(f C_w + C_d V_d)}{a(1-V_{LS})} + V_{LS} C_{sh} \\ V_L + V_s &= V_{LS} \end{aligned} \right\} (\text{Eq-12})$$

$$S_w^2 = \frac{C_t - V_{LS} C_{sh}}{C_0 - V_{LS} C_{sh}} \quad (\text{Eq-13})$$

(3) Derivation of V_d from Sonic

Laminated and Structural shale are subject to the same overburden pressure as the bedded shales. On the other hand, Dispersed shale is subject only to hydrostatic, rather than overburden pressure, as it is under suspension in the fluid. For this reason, the Sonic response to the Dispersed shale is assumed to be that of water ($DT_{dis}=DT_f$, Eq-14).

$$DT = (1 - \bar{f} - V_{sh})DT_{ma} + V_{LS}DT_{sh} + (\bar{f} + V_d)DT_f \quad (\text{Eq-14})$$

$$\bar{f}_s = \frac{DT - DT_{ma}}{DT_f - DT_{ma}}, (\bar{f}_s)_{sh} = \frac{DT_{sh} - DT_{ma}}{DT_f - DT_{ma}}$$

$$\bar{f}_s = C_{pF}(\bar{f} + V_d) + (V_{sh} - V_d)(\bar{f}_s)_{sh}$$

$$V_d = \frac{\dot{\bar{f}}_s - \bar{f} - V_{sh}(\dot{\bar{f}}_s)_{sh}}{1 - (\dot{\bar{f}}_s)_{sh}} \quad (\text{Eq-15})$$

- $\dot{\bar{f}}_s$ is the Sonic porosity with the consideration of the Comp action Factor (C_{pF}).
- $\bar{f}$ from the Density-Neutron cross plot.

By comparing the Sonic porosity and Cross plot porosity, V_d can be obtained (Eq-15) as well as $V_{LS}=V_{sh}-V_d$.

(4) Interpretation Example

EXAMPLE-A

(Fig.E, F) show the example of the thinly laminated sand of the Teradomari formation, Japan Sea. GR is flat and Density-Neutron never shows the separation over the interval. Deep resistivity never exceeds 5 ohm-m; the use of KCL mud makes SP flat. The log features make it extremely difficult for the log interpreter to decide where to test with a strong gas/oil showing from the mud logs.

The traditional approach to this problem in Japan is the use of the D-Ct overlay method in combination with the mud log to decide on the test zone. In this example, zones A and B were tested to find that zone A did not flow while zone B produced both oil and gas.

The new model has been attempted with the same set of input parameters to obtain more meaningful S_w in zones A and B. Also the q-value indicated that more than 50% of the pore space is plugged by the Dispersed shale for zone A, which makes fluid flow difficult. On the other hand, part of zone B is relatively free of Dispersed shale, which enabled hydrocarbon flow.

EXAMPLE-B

(Fig.G) shows the recent example of a well close to the previous example in the same depositional environment. For this well, a CMR* measurement was done, which indicated a high capillary water content in the porosity system.

Irreducible water saturation (S_{wir}) is calculated from the CMR measurement. Then, S_{wir} shows a good match with the q-value, suggesting a petrophysical relation between two parameters, as well as the validity of the q-value itself.

(5) Conclusions

- The Maxwell-Garnet Model was employed to examine its response to the various types of shales, proving it is effective and follows the scientific method, rather than an empirical approach.
- A new saturation equation is proposed which enables a more realistic result for (S_w) in thinly laminated formations.
- The q-value is also proposed, which is effective for the evaluation of producibility.
- The q-value was confirmed to have a petrophysical relation with the irreducible water saturation (S_{wir}) derived from CMR measurements.

CMR* Mark of Schlumberger

Acknowledgements

The author would like to thank Japan National Oil Corporation (JNOC) and Teikoku Oil Co., Ltd. (TOC) for the support and permission to publish this work. Also the author would like to express his gratitude to Mr. Masahiko Kida and Mr. Akinori Imayoshi, Idemitsu Oil & Gas Co., Ltd., for their invaluable contribution to the study. Special thanks are due to Dr. Yuzuru Ashida, Kyoto University and Alain Brie, Schlumberger for their continuous encouragements and scientific advice.

About the Author

Shigeaki Asakura is a manager of Schlumberger GeoQuest in charge of Far East operations (Japan, Korea, Taiwan and Sakhalin Russia). He holds BS degree in Earth Resources Engineering from Kyoto University. He joined Schlumberger in 1976 as a wireline field engineer, then Log Analyst, marketing and various management positions.

He is a member of SEG and SPWLA.

References

1. Poupon, A., Clavier, C., Dumanoir, J., Gaymard, R., and Misk, A. : "Log Analysis of Sand Shale Sequences - A systematic Approach", *Jour. Pet. Tech.*, July, 1970.
2. Granberry, R. J., Jenkins, R. E., and Bush, D. C. : "Grain Density Values of Cores from Some Gulf Coast Formations and their Importance in Formation Evaluations", *SPWLA Symposium*, 1968.
3. Poupon, A., Loy, M. E., and Tixier, M. P. : "A Contribution to Electrical Log Interpretation in Shaly sands", *Jour. Pet. Tech.*, June, 1954.
4. deWitte, L. : "Relations Between Resistivities and Fluid Contents of Porous Rocks", *Oil and Gas Journal*, August 24, 1950.
5. Alger, R. P., Raymer, L. L., Hoyle, W. R., and Tixier, M. P. : "Formation Density Log Applications in Liquid-Filled Holes", *Jour. Pet. Tech.*, March, 1963.
6. Tixier, M. P., Morris, R. L., and Connel, J. G. : "Log Evaluation of Low-Resistivity Pay Sands in the Gulf Coast", *The Log Analyst*, Nov.-Dec., 1968.
7. Simandoux, P. : "Mesures Dielectriques en Milieu Poreux, Application a Mesure des Saturation en Eau, Etude du Comportement des Massifs Argileux", (Dielectric Measurements in Porous Media and Application to Shaly Formations), *Revue de l'institut Francais du Petrole*, Supplementary Issue, 1963.
8. Poupon, A., Strecker, I., and Gartner, J. : "A Review of Log Interpretation Methods Used in the Niger Delta", *SPWLA Symposium*, 1963.
9. Sen, P. N., and Tanner, D. B. : "Far-infrared absorption by fine-metal-particle composites", *The American Physical Society*, 1982.
10. Brie, A., and Johnson D. L., and Nurmi, R. D. : "Effect of Spherical Pores on Sonic and Resistivity Measurements", *SPWLA Twenty-Sixth Annual Logging Symposium*, June 17-20, 1985.

STRUCTURAL SHALE

DISPERSED SHALE

(Fig.E)

(Fig.F)

(Fig.G)