Delineation of subterranean water tunnels from gravity data

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Abstract: To recognize and to delineate the subterranean water tunnels under the constructional area at institute of geophysics (Tehran), a microgravity survey based on 5 parallel profiles were measured. Euler’s equation, analytic signal and Werner deconvolution are applied to interpret the data. The horizontal place and the depth of the main tunnel are determined accurately and the results are presented.

Key words: subterranean water tunnels, Euler’s equation, analytic signal, Werner deconvolution.

1. Introduction
Subterranean water tunnels have broadly spread in Iran since thousands of years ago. Some of these old subsurface passages make instability in the areas under construction. The gravity method is an effective way to detect these subsurface cavities. The most important stage in this job is the quantitative interpretation of the data. The free noise data has a great effect on the results while it seems that the interpretation is the key.

2. Euler’s equation
Thompson (1982) showed that for \( f(x,z) \) to be a homogenous function of degree \( N \), the Euler’s homogenous equation folds

\[
(x - xo) \frac{\partial f}{\partial x} + (z - zo) \frac{\partial f}{\partial z} = Nf,
\]

where \( x_o \) and \( z_o \) are the coordinates of the source. Klingele et al. (1991) proved that the vertical gradient of gravity field of a point mass is a homogenous function of degree -3. Hence in equation (1) the function can be replaced by the vertical gradient of gravity. So we can write,

\[
x \frac{\partial^2 g}{\partial x \partial z} + z \frac{\partial^2 g}{\partial z^2} = Nf + x_0 \frac{\partial^2 g}{\partial x \partial z} + z_0 \frac{\partial^2 g}{\partial z^2}.
\]

For a source body to be represented by an appropriate distribution of point sources on the body, \( N \) would be the structural index (Thompson 1982). The value of this index has to be determined in practice together with the unknowns \( x_o \) and \( z_o \).

3. Analytic signal
According to Nabighian (1982) the analytic signal of a vertical gradient of gravity produced by a 2-D source can be defined as,

\[
A(x) = \frac{\partial^2 g}{\partial x \partial z} - i \frac{\partial^2 g}{\partial z^2},
\]

where the corresponding amplitude is
\[ A(x) = \sqrt{\left( \frac{\partial^2 g}{\partial x \partial z} \right)^2 + \left( \frac{\partial^2 g}{\partial z^2} \right)^2}, \]  

(4)

Green (1976) discussed the properties of analytic signal for 2-D gravity data analysis. Hansen et al. (1978) demonstrated also the usefulness of this technique to the gravity field.

The location of maximum amplitude of the analytic signal is determinable by the method of Blakely and Simpson (1986).

According to Marson and Klingele (1993) the horizontal position of the corners impose the position of the maximum of the amplitude.

4. Werner deconvolution

Klingele et al. (1991) derived the following equation in analogy with the results of Hartman (1971) and Ku & Sharp (1983),

\[ \frac{\partial g}{\partial z} = \frac{2g\rho sh}{(x-x_0)^2 + h^2} + C_0 + Cx + C_2x^2, \]

(5)

where \( \rho \) is the contrast density, \( S \) and \( h \) are the thickness and depth of the ribbon, \( x_0 \) is the x coordinate of the source and \( C_0, C, C_2 \) are added regarding a perturbation term of polynomial form to the measured gradient.

This equation can be rearranged to give a linear system of equations and using an equally spaced seven-point Werner operator this system of equation will be solved and the coordinates of the source \((x_0, h)\) would be determined.

5. Numerical results

The total field data was measured at 2 meters spacing along 5 profiles over the considered area.

The Bouguer anomalies are shown in fig (1). As it is clear the subsurface tunnel is detected by dark color.

Then the residual anomalies are computed and presented in fig (2). This figure confirms the results of Bouguer anomalies. After computing the gravity gradients, the Euler formula (2) is used to estimate the coordinates \((x, z)\) of the point sources and the results are shown in fig(3).

Analytic signal method is also used for locating edges of the passage (fig (4)).

Finally the Werner deconvolution formula is applied to distinguish the subterranean tunnel (fig (5)). To consider the results of these methods, Euler’s equation gives the maximum clustering of the points and reflects the Average depth to the center of the tunnel.

The clustering of the points computed through Werner deconvolution is less than it obtained by Euler’s equation and the depths are to the top of the tunnel.

The analytical signal shows a clustering less than the other two methods but still moderate.

6. Conclusion

Subsurface cavities could be detected by combination of the different quantitative methods. Three methods namely Euler equation, analytic signal and Werner deconvolution are used and the results of Euler and Werner methods are found highly accurate.
Werner deconvolution which is basically used in aeromagnetic profiles shows a good correlation with the others two methods that are broadly used in magnetic and gravity profile. The similarity and correlation of the results implies the recommendation for using these three methods for detection of the subsurface caves or tunnels in urban areas.

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References

Figures
Fig (1) : Bouguer Anomaly. (mGal).
Fig (2) : Residual Anomaly ( mGal).
Fig (3) : The coordinates of the source points by Euler's equation.
Fig (4) : The coordinates of the source points by Analytic signal
Fig (5) : The coordinates of the source points by Werner deconvolution.