3-D velocity images in near surface

Xu Chang, Yike Liu

Institute of Geology and Geophysics, Chinese Academy of Sciences.

#A11, Datun Road, Chaoyang District, Beijing 100101, China.

E-mail: changxu@mail.c-geos.ac.cn

ABSTRACT

A 3-D tomographic inversion approach for reconstructing 3-D near-surface model is presented in this paper. Direct, reflected and turning waves are used simultaneously to update the near-surface model. We analyze the characteristics of the first-break traveltimes in complicated low-velocity layers. To improve the accuracy for the velocity model, the various first-break times from direct, reflected and refracted waves are considered for model inversion. A fractal algorithm is applied to pick first breaks, which overcomes the error caused by wavelet shape differences, and the leg-jump, of refractions. The method is capable of picking a large volume of first breaks automatically. The ray paths and traveltimes are calculated with a 3-D ray tracer which does not increase computation time for complicated geological models. Our method can determine the ray path associated with minimum traveltimes regardless of the wave mode (direct, refracted, or reflected waves). We use a least-squares approach in conjunction with QR decomposition to reconstruct a 3-D near-surface velocity model from the actual first-break times obtained from 3-D data.

INTRODUCTION

In rock engineering study, investigating of 3-D complicated near-surface structure becomes increasingly important. In recent years, many geophysical methods have been used for solving rock engineering problems. In our study, we therefore attempt to develop a method that utilizes all seismic first arrival times to invert a more detailed 3-D near-surface velocity model.

A unified method for 2-D and 3-D near-surface model based on the seismic tomographic principle (Sheriff, 1991) was presented by Tanner et al. (1998), which solved delay-time equations using weighted least squares and conjugate gradient methods. These seismic tomographic inversion methods modify an initial near-surface model by minimizing the residual traveltimes between the observed refraction arrival times and the times obtained by forward modeling. A turning ray tracing is normally used in a refracted layer in order to obtain forward refraction arrival times (Zhu, et al., 1992; Adams, et al., 1994; Stefani, 1995). Turning ray tracing, however, is restrainedly applied to complex 3-D near-surface model. These generalized linear inversion methods assume that the observed times are first breaks that usually are the arrivals from shallow refractors. This assumption is not able to make full use of the redundancy provided by direct and reflected waves from first arrivals, and it is difficult to resolve irregular shot-geophone patterns that are often used in rock engineering survey, especially for the complex near-surface model.

Our work focuses on 3-D near-surface problems. In 3-D seismic data, we use fractal picker to obtain not only the traveltimes of refraction waves, but also the traveltimes for direct waves and reflected waves. The difference between the observed traveltimes and the theoretical traveltimes obtained from ray tracing through an initial 3-D velocity model is minimized by a least-squares QR inversion method, giving a 3-D near-surface velocity reconstruction.

FIRST BREAK PICKING

The input to 3-D near-surface tomographic method is first break traveltimes. Fig.1 shows a near-surface model with complex surface topography. It shows that first breaks consist of direct waves, refracted, and reflected waves. At receiver point A, the first arrivals are direct waves which propagate directly from the source to the receiver through the weathered and low-velocity layer. The region of the velocity model that can be reconstructed from the direct waves are defined by the direct ray path. At positions B and C, refracted waves arrive earlier than other waves, while at receiver position D, the first-break is a reflected wave. Fig.1 indicates that first breaks may consist of different kinds of seismic

wave modes. Our method is not restricted to refracted waves only, but we also use direct waves and reflected waves to estimate the near-surface velocities.

Fig.1 Types of first break.

In order to find a fast and effective picking method, we used a fractal algorithm (Boschetti et al., 1996) to determine first breaks automatically. The trace amplitudes before the first breaks contain only random noise. But after the first breaks, the amplitudes contain both random noise and seismic waves. Therefore, the fractal dimension of a seismic trace is different before and after the first break. This means that the fractal dimension describes quantitatively the shape of seismic traces. The change of fractal dimension coincides with the change of waveform in a seismic trace (Chang et al., 1999) which may indicate the appearance of the first break. Because the fractal algorithm is not based on the crosscorrelation of adjacent traces and waveforms, it is able to overcome the mismatch of first break wavelet shapes due to different dynamite shots and the leg-jump caused by refraction phase rotations.

Compared with background noise of a trace, the first arrivals, regardless of whether they are refracted, reflected or directed waves, usually have a visual difference in amplitude and frequency. The human eye can distinguish the first arrival waves by their overall shape and the fractal algorithm automatically simulates the way the human brain identifies the first arrival (Boschetti, et al., 1996). Fractal dimension enables the roughness or overall shape of the trace to be measured. Accordingly, the fractal algorithm is appropriate for picking first arrival regardless of whether they are the refracted, reflected or directed waves.

Boschetti used a comparison between the fractal method and three other methods to illustrate the ability of fractal algorithm to pick the correct first arrival time (Boschetti, et al., 1996). In our study, we compared picking results between the common shot gathers and common offset gathers to pick the correct first arrival time.

Fig.2 (a), (b), (c), (d) and (e) show a pairs of seismic trace and their respective fractal dimension curves, respectively.

Fig.2 shows one seismic trace and its fractal dimension curve. We added different amplitude levels of random noise into the real data to detect the maximum amount of noise the fractal method could tolerate. Fig.2 (a) shows the original seismic record and its fractal dimension curve calculated by the algorithm in Appendix A. Fig.2 (b), (c), (d) and (e) show the record after adding random noise with 20%, 40%, 60% and 80% of the maximum amplitude of the signal and their fractal dimension curves, respectively. In each figure, the fractal dimensional number is constant before the first arrival time, and its sudden change coincides with the position of first arrival. The result of Fig.2 shows that the fractal first break picker has a strong resistance to noise. In addition, even though there appears a big difference in the traveltime and the waveform between adjacent traces, the fractal method works because the algorithm runs one trace at a time. This picking method can therefore be applied to a complex topography and heterogeneous low-velocity layers, and the reliability of inversion to the weathered layer and low-velocity layer model is ensured.

THEORETICAL 3-D RAY PATH AND CALCULATED FIRST BREAK TIME

In 3-D near-surface tomographic method, theoretical ray paths and first break times through a velocity model need to be calculated using a 3-D ray tracer. For a complex topography and heterogeneous low-velocity earth, the global query algorithm to the ray paths with minimum traveltime is an effective method for calculating theoretical ray paths and traveltimes. The global query algorithm to seek minimum traveltime ray paths on the basis of network theory (Cao and Greenhalgh, 1993) enumerates all ray paths connecting a source and a receiver and searches the minimum traveltime ray path in the 3-D model. Since the method does not impose any restriction on the wave modes, direct, refracted, or reflected waves, the ray paths can be determined. The ray tracing method is applicable to 3-D complex topography and heterogeneous low-velocity layer models. In our study, we use the global query algorithm based on network theory to solve the 3-D ray-tracing problem and thereby calculate the ray paths for a real data (424 shots, 1200 traces per shot) in a 3-D model.

GENERALIZED LINEAR INVERSION CALCULATION OF THE THICKNESS AND VELOCITY OF LOW-VELOCITY LAYERS

In the reconstruction of the velocity images we generally assume that the near-surface model is isotropic. The travel time from source i to receiver j can be expressed as

$$T_{i,j} = \int_{L_{i,j}} \frac{ds}{v}, \quad (1)$$

where v is the wave velocity, ds is a segment of the ray path and $L_{i,j}$ is the ray path from i to j . We describe the velocity function of the medium by specifying the velocities on the nodes of an uneven 3-D grid in space. The near-surface model is sub-divided into a grid of small hexahedrons in a coordinate system. The relative slowness perturbation of a vertex of a hexahedron is taken as a parameter to be solved, and the velocity at any point within a grid is given by an interpolation function. Velocities of the nodes are specified and denoted as $v_{i,j,k}$, where the subscripts i , j and k are the sequential numbers of a grid. For the sake of convenience we rename $v_{i,j,k}$ as v_a , and thereby the velocity at any point within a hexahedron is given by interpolation with the equation

$$v(x, y, z) = \sum_{a \in H} v_a F_a(x, y, z), \quad (2)$$

where $F_a(x, y, z)$ is the known linear interpolation function, and $a \in H$ means that the velocity at the nodes of hexahedron H should be used for the interpolation.

From equation (1) we know that the travel time is given by an integration along the ray path. On the other hand, the path itself depends upon the velocities in the medium. Therefore the tomographic problem will be highly non-linear if equation (1) is used for velocity inversion. We can linearize the problem in the following way

$$\mathbf{d}T_{i,j} = T_{i,j}^{obs} - T_{i,j}^{cal} = \int_{L_{i,j}} \mathbf{d} \frac{1}{v} ds, \quad (3)$$

where $T_{i,j}^{obs}$ is the observed travel time, $T_{i,j}^{cal}$ is the travel time calculated by ray tracing in a reference velocity structure v_0 , and $\mathbf{d}T_{i,j}$ is the change in the travel time caused by a velocity perturbation $\mathbf{d}v$ relative to v_0 . According to Fermat's principle, the effect of the change in the ray path caused by this perturbation on the travel time is of the second order. Therefore the ray path $L_{i,j}$ can be determined by a ray-tracing equation according to the reference velocity v_0 . Consequently we can write

$$\int_{L_{i,j}} \mathbf{d} \left(\frac{1}{v} \right) ds = \sum_{l=1}^{NL} \mathbf{d}_l, \quad \mathbf{d}_l = \int_{s_{l-1}}^{s_l} \mathbf{d} \left(\frac{1}{v} \right) ds, \quad (4)$$

where l denotes the l th segment of the path $L_{i,j}$. It starts from s_{l-1} and ends at s_l , where the corresponding velocities are $v(s_{l-1})$ and $v(s_l)$ respectively. NL is the number of a segment of the path and is determined by ray tracing. From the mean-value theorem of an integration we obtain

$$\mathbf{d}_l = \frac{\Delta s_l}{2} \left[\mathbf{d} \left(\frac{1}{v_{l-1}} \right) + \mathbf{d} \left(\frac{1}{v_l} \right) \right], \quad \Delta s_l = s_l - s_{l-1} \quad (5)$$

Taking equation (2) for v we have

$$\begin{aligned} \mathbf{d}_l = \frac{1}{2} \left[\frac{\Delta s_l}{v(s_{l-1})} \sum_{a1 \in H} f_{a1} \left(-\frac{\mathbf{d}v_{a1}}{v_{a1}} \right) \right. \\ \left. + \frac{\Delta s_l}{v(s_l)} \sum_{a2 \in H'} f_{a2} \left(-\frac{\mathbf{d}v_{a2}}{v_{a2}} \right) \right] \end{aligned} \quad (6)$$

where H and H' are the hexahedra containing the two ends, and

$$f_{a1} = F_{a1} \frac{v_{a1}}{v(s_{l-1})}, \quad f_{a2} = F_{a2} \frac{v_{a2}}{v(s_l)}. \quad (7)$$

We define the relative slowness perturbation as

$$x_a = -\frac{\mathbf{d}v_a}{v_a} = \frac{\mathbf{d}u_a}{u_a}, \quad (8)$$

where $u_a = 1/v_a$ is the wave slowness. Finally equation (3) becomes

$$\mathbf{d}T_{i,j} = \frac{1}{2} \sum_{l=1}^{NL} \left[\frac{\Delta s_l}{v(s_{l-1})} \sum_{a1 \in H} f_{a1} x_{a1} + \frac{\Delta s_l}{v(s_l)} \sum_{a2 \in H'} f_{a2} x_{a2} \right] \quad (9)$$

The ray paths in Equation (9) will be determined by ray tracing.

We know that the correct determination of an initial near-surface model is important to obtain a 3-D detailed model in tomographic method. From low-velocity layer (LVL) surveys (Cox, 1999) we can determine a few control points containing the velocity and depth of the weathered layer. Based on these control points, we can determine an initial model with irregular interfaces and heterogeneous velocity structure by interpolation among these control points. Equation (9) indicates that perturbation of travel time is only caused by the velocity perturbation. In reality, the travel time perturbation is both the velocity function and the depth of the low velocity interface, so we should also correct for the depth of interface. However, the perturbation of travel time corresponds to the coupled relationship between the perturbation of velocity and the perturbation of depth of interface. There are a few methods for inverting traveltimes data to estimate parameters that concurrently define both the velocity structure and depth of discontinuities in 2-D reflected interface (Bishop, et al., 1985; Madrid and Traslosheros, 1993). For simplicity in 3-D near-surface model

reconstructed calculation, in this paper, we first determine the 3-D velocity structure and then update the depths of interfaces only at the control points. From refraction theory, the inverse of the slope of a line associated with refracted wave arrivals in a time-distance plot is equal to the bedrock velocity, and also the inverse of the slope of a line associated with the direct wave arrivals is equal to the velocity of the weathering layer. The depth of the weathering layer is given by (Yilmaz, 1987)

$$z_w = \frac{v_b v_w t_i}{2(v_b^2 - v_w^2)^{1/2}}, \quad (10)$$

where, v_w, v_b are the velocities of weathering and bedrock, t_i is the intercept time of a refracted arrival time at zero offset and is obtained from LVL survey data used in this paper, and z_w is the depth of weathering layer. By solving equation (9), we can obtain the new velocities of each node of a grid, and new v_w, v_b can be derived by averaging these node velocities vertically beneath a control point. It is also assumed that a small segment of an interface beneath the control point region is a horizontal layer. From these new depths of interfaces in control points, we can interpolate a new irregular interface in the model. These steps are repeated until a detailed near-surface model is achieved.

To define the thickness and velocities of weathered and low-velocity layers, we calculate the differences between the observed traveltimes and theoretical first breaks in equation (9) by using the generalized linear inversion methods. Our methods are summarized as follows:

- 1) Determination of initial model from LVL survey data
- 2) The initial model is divided into a grid of small hexahedron cells, each of which contains a single velocity.
- 3) Determination of first breaks using fractal automatic algorithm.
- 4) Minimum traveltimes ray paths using a 3-D ray tracer.
- 5) Computing the traveltimes residuals between the observed and the theoretical traveltimes for each ray.
- 6) The iterative correction of the velocity structure using LSQR (Paige and Saunders, 1982)
- 7) Correction of the depth of weathering layer by using equation (10)

Fig.3 3-D seismic survey geometries. Star show the position of sources.

To obtain a detailed velocity structure, we have studied a few iterative inversion algorithms. Conjugate-gradient and Gauss-Seidel iterative algorithms (Wang and Treitel, 1973; Koehler and Taner, 1985) were widely used in the past, but now the algebraic reconstruction technique (ART), simultaneous iterative reconstruct technique (SIRT) and iterative least square technique (ILST) algorithms are preferred for determining a velocity perturbation. These methods are based on least squares estimation and are simple and easy to use. But these methods need many iterations to obtain an accurate solution. Usually it is difficult for these methods to obtain a convergent solution because of the unstable behaviors of these algorithms. The singular value decomposition (SVD) algorithm is stable and can give a resolution matrix to appraise the reliability of the solution. But the main advantage of the numerical stability of the SVD over

other algorithms is sacrificed by the large amount of computer memory and the number of calculations required for a large 3-D seismic survey. SVD cannot utilize the characteristics of a sparse matrix, loading all the matrix coefficients into computer memory. In this paper, the velocity model space is divided into a grid ($29 \times 410 \times 160$). There are 424 sources and 1200 receivers for each source in the 3-D data. The computer memory required by SVD is $29 \times 410 \times 160 \times 424 \times 1200 \times 4 = 3606$ Gigabytes. Therefore, the hard disk capacity and computer memory required are not enough to allow us to consider SVD. We use a least-squares approach in conjunction with QR decomposition (LSQR) to solve the huge linear equation system. LSQR only loads non-zero values of matrix coefficients into the memory, which requires a relatively small computer memory, calculates solutions quickly and is also more stable than those methods mentioned above (Nolet, 1987).

FIELD DATA RESULTS

We selected one group of lines from a 3-D seismic survey to perform 3-D near-surface velocity model reconstruction. There are 8 parallel lines with 200-meter line intervals, and 50-meter receiver intervals along the line. There are 424 shots with 1200 receivers per shot. The minimum and maximum offsets are 25 meters and 3725 meters, respectively. Fig.3 shows the survey geometry.

Fig.4 The contour for the result of picks, (a) from common shot gathers, (b) from common offset gathers.

The elevations change rapidly, and the weathered and low-velocity layers are non-homogeneous. We used the 3-D near-surface tomographic method developed by this study. The number of grid points for the velocity model are 29 in the x direction, 410 in the y direction (lines direction), and 160 in the z direction (depth direction). The size of one grid is 50 meters in x , 50 meters in y and 2 meters in z . The initial velocity model consists of a 480 m/s weathered layer and a 1060 m/s low-velocity layer. The velocity of base rock is 1940m/s. We applied the 3-D near-surface tomographic method to these 8 survey lines. For quality control purposes in examining the result of the first break picking, we checked the results of the first break picking in common shot gather and common offset gather. Fig.4 (a) and (b) show the picks of 1200 traces from one common shot gather and the same traces from common offset gathers, respectively. Table 1 shows the deviations of first arrival times for $424 \times 1200 = 508800$ traces between the common shot gather and common offset gathers. The deviation unit is the number of samples. We removed the picks with deviation greater than 3 samples, resulting in 495836 picks being used in our study.

Table 1 The deviation of first arrival times

Deviation (dt)	dt= 0	0<dt<= 1	1<dt<= 2	2<dt<= 3	dt>3
Trace number	493536	849	938	513	12964
Percent of total traces	97.00%	0.17%	0.18%	0.10%	2.55%

RESULT ANALYSIS TO 3-D NEAR-SURFACE TOMOGRAPHY

Fig.5 shows the 3-D topography of the data obtained by seismic survey, and the bottom of the low-velocity layer obtained by the LSQR inversion method. Fig.6 shows a part of the reconstructed velocity model. The reconstructed velocity model describes global and local heterogeneous change in the weathered layer and the low-velocity layer.

Fig.5 (a) and (b) show the 3-D topography of the data obtained from seismic survey, and the 3-D bottom of low-velocity layer obtained from LSQR inversion.

Fig.6 a part of reconstructed velocity model. (a) is a part of 3-D velocity view, the display region is 0 to 1250 meters in x direction, 0 to 5000 meters in y direction and 30 to 320 meters in depth; (b) is one of the horizontal slice at 200 meters depth, the display region is 0 to 1450 meters in x direction and 0 to 5000 meters in y direction; (c) is one of the vertical section in y direction at x=1000 meters, the display region is 0 to 5000 meters in y direction and 30 to 320 meters in depth; (d) is one of the vertical section in x direction at y=500 meters, the display region is 0 to 1450 meters in x direction and 30 to 320 meters in depth.

CONCLUSIONS

This paper proposes a 3-D near-surface tomography generalized linear inversion method in which thickness and velocity structure in weathered and low-velocity layers are reconstructed. Aimed at the leg-jump error in picking first breaks caused by the differences among the source wavelet shapes in observed traveltimes and a refractor phase rotation difference, this study used a method to pick first breaks based on fractal theory that overcomes the shortcoming of conventional methods and provides accurate first-break traveltimes. A 3-D ray tracer is used to calculate the minimum traveltimes and associated ray paths for either the refracted waves, the direct arrivals or the reflected waves. A least-squares QR algorithm is adopted to invert for the thickness and velocities in the low-velocity layers, which speeds up the computation and enhances the accuracy of reconstructed near-surface velocity model.

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