Multivariate Auto-Regressive Model for Groundwater Flow Around Dam Site

Yoshitada Mito\textsuperscript{1)}, Shinya Yamamoto\textsuperscript{1)}, Takashi Kodama\textsuperscript{1)} and Toshifumi Matsuoka\textsuperscript{1)}

\textsuperscript{1)} Dept. of Earth Resources Engineering, Kyoto University, Kyoto, 606-8501, Japan.

Abstract: Time series analysis is carried out using multivariate auto-regressive model for the monitoring of temporal variations of groundwater behavior around the fill dam site. The filtered time series referring atmospheric temperature is suggested as the main manipulated variables in order to take the melting snow, which causes a sudden rise in the level of the river, into account. Its effectiveness is verified through analyzing the power contribution. The results of the regression show that the multivariate auto-regressive model using the proposed variables is very effective tool for the estimation of the groundwater behavior of the dam site.

1. Introduction

Monitoring of temporal variations of groundwater flow behavior around the completed dam site is indispensable to secure the permanent stability of dam and its foundation. The observed data for dam monitoring, which varies in the time domain, is composed of multifarious data such as groundwater pressure, dam and foundation displacement, flow rate of the river, weather condition and so on. In general, the stability of dam and foundation is statistically examined, day-by-day, hour-by-hour or sometimes minute-by-minute, referring the differences between the observed data and the predicted data using multivariate regression model. However, the regression results do not always have a desirable accuracy, since the observed data often includes any of the effect of the past behavior, which is not taken into consideration in the case of multivariate regression model. Therefore, it is essential to carry out multivariate time series analysis in order to reflect the effect of the past data.

In this study, multivariate auto-regressive model (vector auto-regressive model), which is a variation of the time series analysis model, is applied to the regression of temporal variations of groundwater flow head around a dam site, and the factor analysis of variations is carried out by evaluating the relative power contribution.

2. Multivariate auto-regressive model

In the case that stochastic process can be expressed as

\[ x(t) = \sum_{m=1}^{M} a(m)x(t-m) + u(t) \]

\(x(t)\) is called as the \(M\)-th order auto-regressive process where \(t, u(t),\) and \(a(m)\) denote time, white noise and auto-regressive coefficient, respectively. Extending this process to \(k\) dimensional multivariate case, we can obtain a multivariate auto-regressive process;
\[ x(t) = \sum_{m=1}^{M} A(m)x(t-m) + u(t) \]

where \( u(t) \) and \( A(m) \) denote white noise vector (\( k \) dimensional) and auto-regressive coefficient matrix (\( k \times k \) dimensional), respectively.

### 3. Relative power contribution

Assuming that there is no correlation between white noises of different variables, the power spectrum is given by

\[ p_n(f) = \sum_{j=1}^{k} \left| \left( A(f)^{-1} \right)_{ij} \right|^2 \sigma_{ij}^2 \]

where

\[ \sigma_{ij}^2 = A(f)P(f)[A(f)']^T \]

\[ A(f) = \sum_{m=0}^{M} A(m) \exp(-2\pi fm) \]

\[ P(f) = \begin{bmatrix} p_{11}(f) & \cdots & p_{1k}(f) \\ \vdots & \ddots & \vdots \\ p_{k1}(f) & \cdots & p_{kk}(f) \end{bmatrix} \]

This expression implies power spectrum can be divided into effects of \( p \) noise sources. Therefore we can obtain the relative contribution of noise to the variation of \( x_i(t) \) which is related to frequency \( f \), relative power contribution

\[ r_{ij}(f) = \frac{\left| \left( A(f)^{-1} \right)_{ij} \right|^2 \sigma_{ij}^2}{p_n(f)} \]

This value allows us to carry out factor analysis of time series data.

### 4. Time series data of the dam site

This study is applied to a rock fill dam, which is located in northern district of Japan. The height, the crest length, and the volume of the dam are 63.5m, 611m, and 1,387m³, respectively. The dam foundation is composed of 2 types of Quaternary pyroclastic rocks spewed out in different stages. Lower part of the sequence is fresh and highly permeable welded tuff; because the less permeable, poorly welded zone has been eroded and only the highly welded zone remains where open, super conductive columnar joints are distributed. On the other hand, upper part of the sequence shows lower permeability. At the dam site, lower permeable part is distributed on
the right bank of the river, on the other hand, the upper impermeable part exists on the left.

20 boreholes for observing groundwater head (water pressure) are established around the dam site as shown in Figure 1. Most of them are located on the down-streamside of the grout line. Time series data of the groundwater head was automatically observed within a given period of time (from September 1997 to June 1999, occasionally, to December 1998) in order to grasp the groundwater flow behavior around the dam site spatially and temporally. The water level of dam reservoir, the rainfall and the air temperature were also observed as time elapsed. Temporal variations of the observed data (1 datum/day) are shown in Figure 2. The 20 time series data of the groundwater head as the controlled variables (target variables) and the other times series are treated as manipulated variables (explanatory variables).

Figure 1  Layout of the boreholes

Figure 2(1) Temporal data observed at the dam site
5. Filtering of air temperature data

Since the study dam site is located in the northern part of Japan, a vast snow lays upstream area of the dam site in every winter season and they melts in every spring season. This atmospheric cycle causes a sudden rise in the water level of the river every spring and raises the groundwater head rapidly. However there is no manipulated variable to be able to explain such a sudden change. Thus, air temperature data, which seems to have possibility to explain the snow-melting phenomenon, is filtered with

\[
F(t) = \max \left\{ \sin \left( \frac{\pi}{2} \cdot \frac{T(t)}{T_m} \right) \right\} \quad \text{for the period from March to August}
\]

\[
F(t) = 0 \quad \text{for the period from September to February}
\]

where \( T(t) \) is the air temperature as the function of time \( t \), and \( T_m \) is the mean air temperature from the end of the March to the beginning of August. That filtering function considers the fact that the air temperature in the dam site exceeds 0 degree centigrade in the end of the March and the snow continues to lay until in the beginning of the August. Figure 3 shows the filtered time series of the air temperature.
6. Regression

2 types of regressions are carried out in this study. Firstly, a multivariate auto-regressive model is defined as

\[
\begin{bmatrix}
    \mathbf{P}(t + 1) \\
    \vdots \\
    \mathbf{P}(t) \\
    \vdots \\
    \mathbf{P}(t - 1) \\
    \vdots \\
    \mathbf{P}(t - m + 1)
\end{bmatrix}
= \mathbf{A}(1) \begin{bmatrix} 
    \mathbf{H}(t) \\
    \vdots \\
    \mathbf{R}(t) \\
    \vdots \\
    \mathbf{T}(t)
\end{bmatrix}
+ \mathbf{A}(2) \begin{bmatrix} 
    \mathbf{H}(t - 1) \\
    \vdots \\
    \mathbf{R}(t - 1) \\
    \vdots \\
    \mathbf{T}(t - 1)
\end{bmatrix}
+ \cdots \mathbf{A}(m) \begin{bmatrix} 
    \mathbf{H}(t - m + 1) \\
    \vdots \\
    \mathbf{R}(t - m + 1) \\
    \vdots \\
    \mathbf{T}(t - m + 1)
\end{bmatrix}
+ \mathbf{u}(t)
\]

where \( \mathbf{P}(t) \): the groundwater flow head, \( \mathbf{H}(t) \): the water level of the reservoir, \( \mathbf{R}(t) \): the rainfall, \( \mathbf{T}(t) \): the air temperature, \( \mathbf{A}(t) \): coefficient matrix, and \( \mathbf{u}(t) \): white noise vector. Secondly, the model using \( F(t) \) is defined as

\[
\begin{bmatrix}
    \mathbf{P}(t + 1) \\
    \vdots \\
    \mathbf{P}(t) \\
    \vdots \\
    \mathbf{P}(t - 1) \\
    \vdots \\
    \mathbf{P}(t - m + 1)
\end{bmatrix}
= \mathbf{A}(1) \begin{bmatrix} 
    \mathbf{H}(t) \\
    \vdots \\
    \mathbf{R}(t) \\
    \vdots \\
    \mathbf{T}(t)
\end{bmatrix}
+ \mathbf{A}(2) \begin{bmatrix} 
    \mathbf{H}(t - 1) \\
    \vdots \\
    \mathbf{R}(t - 1) \\
    \vdots \\
    \mathbf{T}(t - 1)
\end{bmatrix}
+ \cdots \mathbf{A}(m) \begin{bmatrix} 
    \mathbf{H}(t - m + 1) \\
    \vdots \\
    \mathbf{R}(t - m + 1) \\
    \vdots \\
    \mathbf{T}(t - m + 1)
\end{bmatrix}
+ \mathbf{u}(t).
\]

Figure 4 shows 2 illustrations of the regression results. The fitness of the regression curve of the model with \( F(t) \) is better than that without \( F(t) \).

7. Factor analysis

Relative power contribution of each measuring point is calculated as shown in Figure 5. Table 1 shows the qualitative categorical estimations of contributions and regression accuracy of each measuring point. The contribution of itself (auto-correlation) is higher at every measuring point. The points where the contribution of the water level of the reservoir is remarkable are located on the right bank, which is more permeable. The points where the contribution of the rainfall or the melting snow is remarkable are located in certain regions. The points where the notable contribution of \( F(t) \) is perceived are located at the ridge of the lower formation of pyroclastic rocks as shown in Figure 6. Such geo-morphologically characterized points are generally apt to be unsaturated in the dry season (non melting-snow season) because of highly permeable hydraulic property and topological disadvantage to be cultivated. Therefore the hydraulic conductivity of those parts varies annually considering permeability change under saturated/unsaturated condition, and the stationarity of the time series of those parts seemed to be lost. Thus the contribution of \( F(t) \) would be remarkable at those points.
Figure 4 Illustrations of the regression using Multivariate auto-regressive model
Figure 5  Illustrations of the relative power contribution

Table 1. Qualitative estimation of relative power contribution and regression accuracy

<table>
<thead>
<tr>
<th>Point</th>
<th>$H(t)$</th>
<th>$R(t)$</th>
<th>$T(t)$</th>
<th>$F(t)$</th>
<th>$P(t)$</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>GW05</td>
<td>***</td>
<td>*</td>
<td>*</td>
<td>***</td>
<td>***</td>
<td>***</td>
</tr>
<tr>
<td>GW09</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GW10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GW11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GW12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GW13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GW14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GW15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NO4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NO5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L04</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L06</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L07</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L08</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L09</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R03</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R04</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) GW10

(b) GW12
8. Conclusion

In this study, time series analysis is carried out using multivariate auto-regressive model for the monitoring of temporal variations of the groundwater behavior around the fill dam site. The proposed filtered time series referring air temperature is verified to be effective as one of the manipulated variables in order to take the melting snow into account. As the results of examinations, multivariate auto-regressive model using the proposed variables is very effective tool for the estimation of temporal groundwater behavior of the dam site, and relative power contribution is also effective to grasp the factor of the mechanism of temporal variation.

References