Traveltime-based Raytracing for Reflection Waves

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Abstract

A new approach for the raytracing of reflected waves is proposed in this paper. The algorithm is based on the Fermat's principle and reciprocity and it can be adapted with any type of traveltime calculation technique (Vidale(1988), Saito(1989), Moser(1991), Asakawa and Kawanaka(1993)). The method is practical and efficient for complicated subsurface structures. In accordance with Fermat's principle, raypaths with extremum traveltimes are traced, including the minimum traveltimes (first arrivals); multi-valued traveltimes can be handled with this method.

First, using Linear Traveltime Interpolation (Asakawa and Kawanaka (1993)) for traveltime calculations, the effectiveness of this approach is shown. Then we extend it for the calculations of traveltimes/raypaths of normal incident rays. This approach can be also applied to compute traveltimes/raypaths of P-S/S-P converted waves or multiple reflections.

Raytracing of reflected waves

In general, it is difficult to compute a reflected raypath in a cell-based velocity model because of the complexity of the velocity distribution. Matsuoka and Ezaka (1992) proposed a method to estimate the reflection points along a certain reflector on the basis of reciprocal theory and Fermat's Principle. Based on their approach, the reflection points can be obtained by finding the extrema two-way traveltime along the specified reflector. By combining the LTI traveltime computation and their method, the reflection points on the reflector can be efficiently obtained in a cell-based velocity model. Once a reflection point is obtained on the reflector, the raypath from

shot point to the reflection point, and the raypath from the reflection point to the receiver point can both be traced. The two-way reflected raypath is obtained by connecting these two raypaths. We perform numerical experiments to evaluate the accuracy of the raytracing method based on LTI, in a complicated velocity model. We compare the traveltimes calculated by the procedure described above with the finite difference simulation (Sato et al. 1995). The velocity model is shown in Figure 1. Figure 2 shows the simulated seismic trace when the shot point is located at the middle point of the model. The traveltimes of reflection wave from the layer boundary are obtained by the proposed approach, and then the curve is overlaid with a solid line.

Normal incident raypath

The proposed method can be applied to the normal incident rays which is useful for the depth-to-time conversion of the velocity model or for the simulation of stack sections. Since the normal incident raypaths are zero-offset raypaths, the shot point and the receiver point should be coincident. The raypaths from the shot point to the reflection point and the one from the reflection point to the receiver point should be the same. Therefore, the two-way traveltime becomes the twice of the transmission traveltime from the shot point to a reflection point. In this case, the transmission traveltime map from the source and the reflection traveltime map from the reflection point are the same. Employing the method described in the previous section, reflection points can be easily obtained by searching the extremum traveltime points along the target horizon.

We apply this procedure to the simple two-layer model shown in Figure 1. The exploding reflector simulation using the finite difference method is employed to obtain the stack section of the exploding reflector simulation shown in Figure 3. The computed traveltime curves obtained by the proposed method are overlain on Figure 3 with a solid line.

Raytracing of multiple reflection waves

We extend this principle to evaluate traveltimes/raypaths of multiple reflected waves. A typical case of modeling of multiple reflections is shown in Figure 4. There are three reflection points; one surface reflection point (middle reflection point) and two reflection points on subsurface reflector. The multiple reflection waves also satisfy the Fermat's principle. The traveltime from a shot point to a receiver point through three reflection points can be obtained by summing the traveltimes from source to the mid-point and from mid point to the receiver based on the same concept as described in the previous section.

$$T(P_2) = T_1(SP \rightarrow P_2) + T_2(P_2 \rightarrow RP) \quad (1)$$

Here the position of middle reflection point P_2 is unknown. In order to evaluate $T(P_2)$, the position of P_2 should be assumed. Then the traveltimes $T_1(SP \rightarrow P_2)$ and $T_2(P_2 \rightarrow RP)$ can be obtained by the same procedures as single reflection case. The adopted algorithm is as follows; first numbers of the temporary positions of the point (P_2) are assumed along the surface, and then the traveltime $T(P_2)$ is evaluated for each position. Thus the traveltime $T(P_2)$ is obtained as a function of the position of P_2 . By searching the positions of P_2 with extremum traveltime, the true reflection points are estimated. Once the point P_2 is determined, two reflection raypaths from source to the middle point and from the middle point to receiver can be calculated by raytracing of reflection waves using LTI backward process.

Figure 5 shows traveltimes of the multiple reflections for the model in Figure 1. The results show a close agreement between the proposed method and the finite difference simulation. In this approach, we can calculate the multiple waves for any shape of the interfaces.

Raytracing of converted wave

For the raytracing of converted waves, we modified the scheme described in the previous section. In simple reflection case, we calculate traveltimes from shot point and traveltime from receiver point (virtual shot point) using P-wave velocity model. However, for converted waves, P-S converted wave, a raypath travels to the reflection point with P-velocity and then travels from the reflection point to the receiver point with S-velocity. Therefore, traveltimes from shot point is calculated with P-velocity and traveltimes from receiver point is calculated with S-velocity. Then the two traveltimes are summed to obtain the two-way traveltime of P-S wave. Similar to the reflection, the extremum traveltimes on the target reflector are searched to obtain the reflection points with the P-S traveltime map. Once a reflection point is obtained, the raypaths are traced with P-velocity and S-velocity respectively and then the both raypaths are connected. In case of S-P converted waves, the algorithm is the same and the P and S velocities are swapped.

We use the velocity model shown in Figure 1 again, and the result of shot records at SP128 with elastic wave simulation is shown in Figure 6. In this simulation, V_p/V_s is set to 2. The traveltime computed with the proposed approach is indicated with solid lines. The magenta lines are P-S wave and the other yellow lines are S-P wave. This result shows that both P-S and S-P reflection are coincident with the results of elastic finite difference simulation. The traveltimes of P-S and S-P are almost the same since the reflector is relatively flat on the left-hand side of the model, but there is big difference for the dipping layer on the right-hand side. This leads to the splitting of PS and SP converted waves on the dipping reflector.

Conclusions

In this paper, practical and efficient approaches to compute the traveltimes and raypaths of reflection waves are proposed. These approaches can handle single and multi-valued traveltimes by searching extremum values in accordance with Fermat's principle. This scheme can handle not

only simple reflection but also multiple reflection or converted waves with small modifications. For each case, the computed traveltimes are compared with the seismic traces obtained by the acoustic or elastic full wave simulation using finite difference method. Both results show a close agreement and the validity of this approach is confirmed.

The proposed approach can employ any type of traveltime calculation method. However, it is noted that it requires the accurate traveltimes on grids because they affect the reflection positions directly. Through the numerical experiments, LTI gives accurate traveltimes enough for this approach. The traveltime-based raytracing were originally developed for the raytracing of transmission waves, especially for the crosswell tomography application, but they also give the useful information for reflection or multiple reflection waves.

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Fig. 1: A simple two-layer velocity model. The velocities of upper and lower layers are 1500 and 2250 m/sec respectively.

Fig. 2: The shot gather simulated by finite-difference method. The shot point is located at the middle (SP=128) of the velocity model shown in Figure 1. The traveltime curve is superposed on the trace with a solid line.

Fig. 3: The stack section simulated with exploding reflector modeling using finite difference method. The traveltime of the reflected waves from the boundary is superposed with a solid line.

Fig. 4: The first multiple reflection wave. The wave propagates from SP and reflects at point P_1 on subsurface reflector. It reflects at point P_2 on the surface and at point P_3 on subsurface reflector and then reaches RP . The traveltime is calculated by summing T_1 and T_2 . If P_2 is considered as a parameter, the traveltime is a function of the position P_2 . Based on Fermat's Principle, P_2 can be obtained by searching for the extremum traveltime.

Fig. 5: The same simulated shot gather as Figure 2. The traveltime curve of the primary multiple reflection is also superposed with a solid line.

Fig. 6: The shot gather simulated with elastic modeling using the finite difference method. The model is the same as Figure 1, but the value of V_p/V_s is assumed as 2. The traveltime curves of P-S and S-P converted waves are superposed with solid lines respectively.