Reverse time migration for ground penetrating radar using finite difference time domain method

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ABSTRACT
The finite difference time domain (FDTD) method is calculated by stepping finite approximation scheme for two Maxwell's curl equations. Since it allows arbitrary electrical conductivity and permittivity variations within a model, the FDTD method has become one of the powerful forward modeling methods for electromagnetic (EM) phenomena. The reverse time migration, which is performed by inserting the recorded data as a boundary conditions at each recorder position in reverse time order, is one of the imaging algorithms. In this paper, the reverse time migration for the ground penetrating radar (GPR) data is formulated using FDTD scheme. In a lossless media case, the method is successfully demonstrated to synthetic data for two models: steeply dipping structure and point diffractors model. In a lossy media case, the forward scheme includes diffusion term, while the reverse time scheme includes divergence term. In such a case, when the EM wave velocity is regarded as constant, this methodology is applicable successively. We discussed the reverse time migration under the lossless media condition for the lossy media after the amplitude recovery.

INTRODUCTION
Recently, the use of GPR for the near surface environments and engineerings has increased significantly in advantage of it's high resolution. However, since it is difficult to understand the behaviors of EM wave phenomena in inhomogeneous media such as the soil or rock, the imaging algorithm for complicated structure is still in developing stage. The FDTD method (Yee, 1966), which is calculated by the two approximations of Maxwell’s curl simultaneous equations, has been used for the simulation of EM problems (Sanada and Ashida, 1997; Wang and Tripp, 1996). The reverse time migration is a imaging method of the inverse of the forward
modeling of wave equation. The acoustic and elastic wave equation (Baysal, Kosloff and Sherwood, 1983; Chang, and McMechan, 1987) and EM wave equation (Fisher, McMechan, Annan and Cosway, 1992) have been applied. In EM wave equation, the finite difference calculation of the reverse time migration is able to be carried by the FDTD method. Because both the reverse time migration and the FDTD method are calculated by the explicit scheme, we can calculate bigger and more accurate model with the recent rapid progress of computer for a background. In this paper, we formulate the reverse time migration using the FDTD method and discuss the cases of lossless and lossy media.

FDTD METHOD

We consider a two dimensional problem, which has the y-component electric field and the x- and z- component magnetic fields in a linear and isotopic media. The Maxwell's equations are as follows:

\[ \mu \frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial z} \]
\[ \mu \frac{\partial H_x}{\partial t} = -\frac{\partial E_y}{\partial x} \]
\[ \varepsilon \frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial z} - \frac{\partial H_y}{\partial x} - \sigma E_x \]

where \( H, E, \mu, \varepsilon \) and \( \sigma \) are the magnetic field strength, the electric field strength, the magnetic permeability, the electrical permittivity and the electric conductivity (reciprocal of resistivity), respectively. In the FDTD method, the electric and the magnetic field in time and space domain are calculated by the finite difference approximations of Maxwell's equations. A two dimensional staggered grid of Yee (Yee, 1966) is shown in Fig.1. We note function \( F_n(i,j) \) as follows:

\[ F(i\Delta x, j\Delta z, n\Delta t) = F_n(i,j) \]

where \( \Delta x, \Delta z \) and \( \Delta t \) are the grid space and the time increment, respectively. The approximations by central differences for equations 1, 2 and 3 are expressed as follows:

\[ H_x^{n+1/2}(i,j+\frac{1}{2}) = H_x^{n-1/2}(i,j+\frac{1}{2}) + \frac{\Delta t}{\Delta z} \{ E_z^n(i,j+1) - E_z^n(i,j) \} \]
\[ H_z^{n+1/2}(i+\frac{1}{2},j) = H_z^{n-1/2}(i+\frac{1}{2},j) - \frac{\Delta t}{\Delta x} \{ E_x^n(i+1,j) - E_x^n(i,j) \} \]
\[ E'_{r}(i, j) = \alpha_{i} E_{r}(i, j) \]
\[ + \frac{\alpha_{1}}{\Delta \epsilon} \left[ H''_{r}(i, j + \frac{1}{2}) - H''_{r}(i, j - \frac{1}{2}) \right] - \frac{\alpha_{1}}{\Delta \tau} \left[ H''_{r}(i + \frac{1}{2}, j) - H''_{r}(i - \frac{1}{2}, j) \right] \]  \( (7) \)
\[ \alpha_{1} = \frac{\Delta t}{\mu(i, j)} \]
\[ \alpha_{2} = \frac{1 - \Delta \sigma(i, j) / 2 \epsilon(i, j)}{1 + \Delta \sigma(i, j) / 2 \epsilon(i, j)} \]
\[ \alpha_{3} = \frac{\Delta t / \epsilon(i, j)}{1 + \Delta \sigma(i, j) / 2 \epsilon(i, j)} \]  \( (8) \)

By using equations 5, 6 and 7, \( H \) and \( E \) are calculated alternately. Consequently, radar data sets at each receiver are obtained.

**REVERSE TIME MIGRATION**

The reverse time migration is performed by inserting the recorded data as boundary condition at each recorder position in reverse time order. In the calculation, the EM wave velocity is the half of that in model. When the time is reversed to zero, the migrated image is obtained. Therefore the reverse time migration is calculated by reverse time scheme of finite approximations. By central differences the reverse time scheme are formulated, these terms are as follows:

\[ \alpha_{1} = \frac{\Delta t}{\mu(i, j)} \]
\[ \alpha_{2} = \frac{1 + \Delta \sigma(i, j) / 2 \epsilon(i, j)}{1 - \Delta \sigma(i, j) / 2 \epsilon(i, j)} \]
\[ \alpha_{3} = \frac{\Delta t / \epsilon(i, j)}{1 - \Delta \sigma(i, j) / 2 \epsilon(i, j)} \]  \( (9) \)

In a lossless media \( (\sigma = 0) \), \( \alpha_{2} = \alpha'_{2} = 1 \). Therefore the forward modeling scheme is able to adapt to the reverse time scheme without alternation. In a lossy media \( (\sigma > 0) \), \( \alpha_{2} < 1 \) and \( \alpha'_{2} > 1 \). The forward time scheme includes divergence term. The reverse time scheme includes divergence term. The reverse time migration has the advantages of no limit of the dip angle in a model and adaptability to the complicated structure.

**NUMERICAL EXSAMPLES**

**Lossless media**

The first case considers lossless media, which has infinite resistivity. The steeply dipping structure and two point diffactors model are shown in Fig. 2 and Fig. 3, respectively. The transmitter and receiver antennas are located on ground surface with zero-offset (monostatic). In these figures they are represented as the triangles. The interval is 10 cm each. Both media have a conductivity of 0.0 S/m, a relative permittivity of 9.0 and a relative permeability of 1.0. After here, all models have the same relative permeabilities. The source function is the ricker wavelet with a central frequency of 100 MHz. The synthetic data are calculated by using of the
FDTD method with the exploding reflector method, which is often used in the seismic reflection modeling. Therefore the zero-offset synthetic data is obtained at each receiver. As for the boundary condition, Mur's second order absorbing boundary condition (Mur, 1981) is adopted. The EM wave velocity is expressed as follows:

\[ V = \frac{c_0}{\sqrt{\varepsilon_r}} \]  

(15)

Where \( \omega \) is the angular frequency. In this case EM wave velocity is simply expressed as follows:

\[ V = \left[ \frac{\varepsilon \mu}{2} \left( \sqrt{1 + \frac{\sigma^2}{\omega^2 \varepsilon^2}} + 1 \right) \right]^{-1/2} \]  

(14)

Next case considers lossy media, which has some resistivity. The three point diffractors model is shown in Fig. 8. The media has a conductivity of 0.01 S/m (100 Ohm m) and a relative permittivity of 9.0. The synthetic data is shown in Fig. 9. The amplitude of reflection wave from deeper diffractor is attenuated by the diffusion effect. Before applying the reverse time migration, we have to consider the EM wave velocity under this case. Fig. 10 shows the graph of the EM velocity in a relative permittivity of 9.0 and the relative permeability of 1.0 with resistivities of 10, 50, 100, 500 and 1000 \( \Omega \) m. From Fig. 10, the EM wave velocity in this case is regarded as equation 15. Therefore, the EM wave equation is approximated by the same
way in a lossless media. Fig. 11 shows the result of the reverse time migration. The reverse time migration scheme with divergence term recovered to the original amplitude. We note that if the EM velocity is not approximated to equation 15, the migrated image is distorted.

DISCUSSION

Generally because of the strong attenuation of amplitude, the gain is recovered during data acquisition in the instrument or by software program before the succeeding data processing is often carried out. In such a case, it is considered that the reverse time migration result is divergence. In this case the reverse time migration is able to be calculated with the assumption of the lossless media. Fig. 12 is the data applied AGC (auto gain control) and band-pass filter to Fig. 9. The result of the reverse time migration under the lossless media condition is shown in Fig. 13. Fig. 13 is migrated same as Fig. 11. Since the field data include some noise, we have to consider this problem. In such case, the later methodology is more stable than the former method.

CONCLUSION

In this paper, the reverse time migration is formulated by the FDTD method. And we show the figure of forward and reverse time scheme. In a lossless and lossy media in which the EM wave velocity is regarded as constant, we confirm that this methodology is successfully demonstrated. In a lossy media, after the amplitude recovery the reverse time migration under the lossless media condition is applicable to the lossy media. For a future work, we will discuss the stability with noisy data and apply this methodology to the field data.

REFERENCES


Fig. 1. 2-D Yee's staggerd grid

Fig. 2. Steeply dipping model

Fig. 3. Point diffractors model

Fig. 4. Synthetic data for steeply dipping model

Fig. 5. Synthetic data for point diffractors model
Fig. 6 Migrated image of steeply dipping model

(a) 100 steps  
(b) 120 steps  
(c) 150 steps

Fig. 7 Snapshots in the reverse time migration to point diffractors data at (a) 100 steps, (b) 120 steps and (c) 150 steps
Fig. 8 Point diffractors model with Lossy media

Fig. 9 Synthetic data for point diffractors model with lossy media

Fig. 10 EM wave velocity and frequency in various resistivity (Relative permittivity = 9.0)
Fig. 11 Migrated image of lossy media model

Fig. 12 Synthetic data for lossy media model after AGC and bandpass filter

Fig. 13 Migrated image of lossy media model under the lossless media condition