Density estimation strategy in full waveform well-to-well tomography

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Seismic full-waveform inversion (FWI) is a method developed for estimating velocity distribution in a medium in which seismic waves travel through. Seismic velocities are expressed by the combination of two elastic constants and density in a homogeneous elastic medium, but density is usually estimated using an empirical formula or is fixed to a constant value in practice. The majority of elastic FWI studies have ignored the influence of density that could be an important parameter. Inverting for three elastic parameters is difficult problem even in FWI for a homogeneous elastic medium when solving for velocities that are composed of the three parameters coupled with each other, and the coupling effects prevent one from the appropriate estimation of the elastic parameters. Since, density is the useful parameter for hydrocarbon characterization, we investigate the best strategy to estimate density structure and reveal the difficulty of the estimation of density structure. We conduct several numerical experiments to estimate density structure. Our inversion results and grid analysis of misfit function show that density is affected by other parameter in the inversion and sequential inversion procedure is the most suitable strategy to estimate density.

1. INTRODUCTION

The importance to use waveform is widely recognized in seismic explorations. Especially, seismic full waveform inversion (FWI) can detect the high-resolution physical properties in the subsurface. The waveform analysis is a powerful tool to investigate the physical properties in the areas of interest. On the other hand, since the wave propagation is influenced by elastic parameters V_p , V_s and density, it is necessary to include these parameters in the modeling and in the inversion (Virieux and Operto 2009). However, multi-parameter FWI is a challenging problem because parameters are coupled with each other, and the coupling effects prevent the appropriate estimation of the elastic parameters. Especially, the estimation of density is a very difficult exercise because plural elastic parameters including density increases the dimension of the solution space so that any minimization could be trapped in local minima. Therefore, density is typically estimated using an empirical formula such as Gardner's relationship (Gardner et al., 1974) or is fixed to a constant value. Since the density parameter is directly included in the elastic wave equation, it is necessary to see if it is possible to estimate density value exactly or not. Gardner's relationship is an empirical equation and could not always show the exact relation between V_p and density, in particular in media such as salt, coal, etc. Pre-salt exploration

conducted in recent decades would accordingly be influenced. It is, therefore, necessary to investigate how the inclusion of density as an estimating parameter in FWI influences the inversion procedure.

The objective of this study is to investigate the feasibility of the estimation of density structure when the density is inverted with the other elastic parameters and to assess whether density is separable from the other parameters or not. We perform 2D numerical simulations in order to investigate the most important factor in the inversion of density structure as well as V_p and V_s . We conducted four numerical experiments with different inversion strategies; i.e. we invert 1) V_p and density simultaneously, 2) V_s and density simultaneously, 3) V_p , V_s and density simultaneously, and 4) only density. We show the effect of the inversion strategies with different parameterization on the estimation of density structures, and discuss the coupling effect of elastic parameters.

Figure 1 Simulation model. Inversion results are compared at black dashed line

2. THEORY AND METHODOLOGY

Seismic FWI updates the properties of the subsurface parameter from observed data. In waveform inversions, the algorithm minimizes the residuals between the waveforms obtained from forward simulation and the observed seismic data. The residual vector is defined as the difference between the modeled seismic wavefields and observed seismic data. As misfit definition we use the L2 norm of the normalized wavefields. It uses the time domain adjoint method (Tarantola, 1984; Mora, 1987). The gradient of the misfit function can be obtained by either taking partial derivatives or using the backpropagation method. We use the backpropagation method and get the gradient direction $E(x)$ taking the following form:

$$E(x) = \int_T dt \lambda^T(x, t) \frac{\partial N^{-1}(m(x))}{\partial m} \partial_i u(x, t) \quad (1)$$

where $u(x, t)$ is the forward wavefield formed by particle velocity and stress components, $N(m)$ depends on physical properties and $\lambda(x, t)$ is the backpropagation wavefield which could calculate same equation with forward calculation.

We perform 2D numerical simulation in time domain by using standard staggered grid (Virieux, 1986) and use CPML boundary condition on all edges (Komatitsch et al., 2007). We use multiscale method (Banks et al., 1995) and filtered 5Hz, 10Hz, 15Hz with Hamming low pass filter on 18Hz ricker wavelet, with changing the appropriate grid size. We iterate 100 times in each frequency band. Figure 1 shows a numerical model which includes a velocity anomaly. The starting models are homogeneous and have constant values of "background". Each parameter has same structure. In the velocity anomaly, P wave velocity is 3200m/s, S wave velocity is 2500m/s and density is 2300kg/m³, respectively. In background structure, P wave velocity is 3000m/s, S wave velocity is 2300m/s and density is 2000kg/m³.

To investigate the appropriate strategies for the estimation of reliable density structure, we performed the following four strategies (Table 1). Strategy 1 inverted P wave velocity and density, Strategy 2 inverted S wave velocity and density, Strategy 3 is conventional one, it inverts P wave and S wave velocity and density simultaneously. Finally strategy 4 inverted density solely.

Table 1: Each inversion strategies shows this table. The sign of "✓" indicates inversion parameter in the strategy and the sign of "—" means this parameter is not estimated in the strategy.

	P wave velocity	S wave velocity	density
Strategy 1	✓	—	✓
Strategy 2	—	✓	✓
Strategy 3	✓	✓	✓
Strategy 4	—	—	✓

3. INVERSION RESULTS

Figure 2(a) and figure 4(a) show the inversion results of Vp and density in Strategy 1. Figure 3(a) and figure 4(b) show the inversion results of Vs and density in Strategy 2. Figure 2(b), figure 3(b) and figure 4(c) show the inversion results of Vp, Vs and density in Strategy 3. Finally, figure 4 (d) shows the inversion result in Strategy 4. Figure 5 shows the vertical profile of density inversion results. Focus on the density inversion results, these inversion results show that strategy 4 is the most effective to estimate the density structure. Density structure is affected by the other parameters and inversion results are worse than solely inversion result. So our inversion results suggest that sequential inversion strategy that density is estimated finally is the most efficient for estimating the reliable density structure.

On the other hand, Vp and Vs inversion results are shown in figure 2 and 3. These results indicate Vp inversion results are more influenced by other parameter. Vs is more robust against local minima for the inversion. It is estimated reliable S wave velocity structure in figure 3.

4. GRID ANALYSIS OF MISFIT FUNCTION

We perform the grid analysis of misfit function to investigate the complexity of coupling effect of each parameter. This method assesses factors contributing to the misfit function (Gholami et al., 2013). Figure 6 shows the results of the grid analysis. In figure 6, red circles in the each figures indicate the true values of each parameter. Figure 6(a) shows that Vp and Vs have the same degree of contribution to the misfit function. But in figure 6(b) and 6(c), Vp and Vs have a greater influence on the misfit function than density. These results indicate that it is difficult for simultaneous strategies to reflect the contribution of density to

the misfit function. On the contrary, **figure 6(d)** shows density with low frequency data has greater influence on the misfit function than high frequency results.

Considering these results, density inversion results are affected by the other parameters and inversion results are worse than the result of inverting solely density. Compared with the other parameters, density has less influence on the waveform so it is likely to be ignored in the inversion. So the inversion strategy that inverts density separately is the effective to estimate appropriate density structure as indicating our inversion results.

Figure 6 (d) indicates that low frequency data mitigates the nonlinearity of inversion. It is the key of estimating reliable density structure. So we have to consider the way to deal with low frequency data in the inversion process.

5. CONCLUSION

In this study, we focus on the inversion of density structure using FWI based on elastic equations. We performed a series of numerical experiments to investigate appropriate strategy for inverting reliable density structure. These results suggest that solely inversion of density is the most effective to estimate the reliable value. So when we invert three elastic parameter, sequential inversion procedure gets more reliable inversion result especially for density due to its small effect on other parameters.

REFERENCES

- 1)Bunks, C., F. Saleck, S. Zaleski, and G. Chavent, 1995, Multiscale seismic waveform inversion: Geophysics, 60, 1457–1473.
- 2)Gardner, G., L. Gardner, and A. Gregory, 1974, Formation velocity and density — The diagnostics basics for stratigraphic traps: Geophysics, 39, 770–780
- 3)Gholami, Y., R. Brossier, S. Operto, A Ribodetti, and J. Virieux, 2013, Which parameterization is suitable for acoustic vertical transverse isotropic full waveform inversion? Part 1: Sensitivity and trade-off analysis, Geophysics, 78,81-105
- 4)Komatitsch, D., and R. Martin, 2007, An unsplit convolutional perfectly matched layer improved at grazing incidence for the seismic wave equation, Geophysics, 72, 155-167
- 5)Mora, P. [1987] Nonlinear two-dimensional elastic inversion of multi offset seismic data. Geophysics, 52(9),1211–1228.
- 6)Tarantola, A. [1984] Inversion of seismic reflection data in acoustic approximation, Geophysics., 49, 1259-1266.
- 7)Virieux, J., 1986, P-SV wave propagation in heterogeneous media, Velocity-stress finite-difference method, Geophysics, 51, 889-901.
- 8)Virieux, J., Operto, S., 2009. An overview of full-waveform inversion in exploration geophysics. Geophysics 74 (6), WCC1–WCC26.

Figure 2: Inversion results of P wave velocity (V_p) in strategy 1 and 3. Right number with each figures means the unit of V_p [m/s] (a): displays V_p result in Strategy 1. (b): displays V_p result in Strategy 3.

Figure 3: Inversion results of S wave velocity (V_s) in strategy 2 and 3. Right number with each figures means the unit of V_s [m/s] (a) : displays V_s result in Strategy 2. (b) : displays V_s result in Strategy 3.

Figure 4: Inversion results of density in strategy 1, 2, 3 and 4. Right number with each figures means the unit of density [kg/m^3] (a): displays density result in Strategy 1. (b): displays density result in Strategy 2. (c) : displays density result in Strategy 3. (d) : displays density result in Strategy 4.

Figure 5 : Vertical profile of density inversion results. These are compared in model center shown in Figure 1.

Figure 6: Grid analysis of misfit function. Red circles indicate the true value of each parameter. (a): V_p and V_s parameterization. Source peak frequency has 18Hz (b): V_p and density parameterization. Source peak frequency has 18Hz. (c) : V_s and density parameterization. Source peak frequency has 18 Hz. (d) : V_p and density parameterization. Source cut off frequency is 5Hz.