Estimation of seismic anisotropy with azimuth from sonic data by full waveform inversion

Satoshi FUSE[1], Hitoshi MIKADA[2] and Junichi TAKEKAWA[2]

[1]Dept. of Civil and Earth Res. Eng., Kyoto University (Now at Schlumberger Ltd.)
[2]Dept. of Civil and Earth Res. Eng., Kyoto University

Seismic anisotropy is defined as the directional dependency of seismic phase velocities caused by the anisotropy in the elastic properties of medium. In the past 30 years of research on seismic anisotropy, it has become well known that subsurface materials are more anisotropic than the foreseen. An estimation of anisotropic properties becomes more important for processing seismic data and planning hydraulic fracturing. However, algebraic problems for estimating the anisotropic parameters from seismic data still remain due to the ill-posedness and the instability in the inverse mapping. It is, however, difficult to estimate directly all of 21 independent parameters in the general elastic medium in the 3D Cartesian coordinate system, and a method to deal with seismic anisotropy for complex anisotropic materials has been waited for.

In this study, we conduct numerical simulation for transversely isotropic medium (TI) which has 5 independent stiffness elements in 3-dimensional logging model. We propose a new parameterization strategy that minimizes the number of parameters so as to alleviate the ill-posedness and the instability: 7 parameters to express a general anisotropic medium. Instead of the full rank stiffness matrix for general triclinic materials, we assume a transversely isotropic material with horizontal axis of rotation (HTI) as an elastic medium with orientation and dip angles of the axis that becomes an analogue to general elastic medium. We attempt to estimate these parameters by full waveform strategy because azimuthal anisotropy influences the waveform. Since one of the crucial problems of FWI is the predicted model would be possible to converge to local minimum as the number of parameters increases, the small number of unknowns in the proposed strategy could play a key role to deal with complex anisotropy. As a result, all elements come close to true values by full waveform inversion process. Our results suggest that the proposed parameterization strategy and FWI have an advantage over the conventional methods in terms of accuracy and stability.

1. INTRODUCTION

Seismic anisotropy is defined as the directional dependency of seismic phase velocities caused by the anisotropy in the elastic properties of medium which seismic waves pass through. There are several causes of seismic anisotropy, such as aligned cracks, time-variant sedimentation rate, sea level changes, etc., that lead to inhomogeneous elastic properties of sub-seismic scale, i.e., spatial scale smaller than the wavelength of seismic waves we are looking at. In particular, shale is known as anisotropic medium with phase velocity difference of 30% due to its laminar or flake structure in the sedimentation. The importance of seismic anisotropy in exploration geophysics was re-realized in the oil industry with increased attention to unconventional resources such as shale oil and gas.

In the past three decades, many number of researches associated with seismic anisotropy have conducted in theoretical and experimental ways. Alford developed an innovative theory to detect the azimuthal anisotropy by virtually rotating the recorded S waveforms that generally split into two types of S waves, i.e., one fast and the other slow S waves[1)]. His method, called as Alford rotation, uses the travel time difference between fast and slow S waves to estimate the order of anisotropy in the assumption that the medium is transversely isotropic with the axis of symmetry normal to the propagation direction of S waves. This type of elastic material is called transversely isotropic (TI) medium that could be expressed with 5 independent parameters in the stiffness matrix. When the condition for the axis of symmetry is not satisfied or the number of independent parameters becomes more than 5, the Alford rotation may therefore fail to estimate the order of anisotropy[2)]. It is difficult to estimate directly all of 21 independent parameters in the general elastic medium in the 3D Cartesian coordinate system[3], and a method to deal with seismic anisotropy for complexly anisotropic materials has been waited for.

In this research, we set orientation and dip as model parameters as well as 5 independent parameters instead of estimating 21 parameters directly under a hypothesis that the stable solution could be obtained. We attempt to estimate these parameters by full waveform strategy because azimuthal anisotropy influences the waveform. Although the Alford rotation uses only travel times of the fast and slow S waves, we would like to introduce a method of waveform inversion technique to use both the amplitude and phase in recorded waveform known as full waveform inversion[4]. Our strategy for reducing the number of parameters would surely alleviate the computational difficulties and could become a breakthrough in the estimation of elastic parameters of complexly anisotropic media. We confirm the effectiveness of our strategy using numerical experiments.

2. METHOD

(1) Governing Equation

To express seismic wave propagation, we use a Hamiltonian particle method with staggered particles[5]. HPM is one of the particle methods which has high computational efficiency and can be applied to flexible models. The governing equation is described as follows.

$$\rho_i \Delta B_i \frac{\partial \boldsymbol{v_i}}{\partial t} = -\frac{\partial H}{\partial \boldsymbol{x_i}} \qquad (1)$$

where ρ_i, ΔB_i, $\boldsymbol{v_i}$ is the density, volume and particle velocity of ith particle and H is determined by the sum of kinetic and strain energy.

In an elastic medium, stress and strain are connected by Hooke's law.

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \qquad (2)$$

where the σ is the second rank stress tensor, ε is the second rank strain tensor, and C is the fourth rank tensor of elastic stiffness with 81 elements. Symmetry of the stress and strain tensors leads to the $C_{ijkl} = C_{jikl} = C_{jilk} = C_{ijlk} = C_{klij}$ which reduces the number of the independent elements to 21. The stiffness matrix of TI is represented by the 5 independent elements.

$$\boldsymbol{C} = \begin{pmatrix} C_{11} & C_{13} & C_{13} & & & \\ & C_{33} & C_{33}-2C_{44} & & 0 & \\ & & C_{33} & & & \\ & & & C_{44} & & \\ & Sym. & & & C_{66} & \\ & & & & & C_{66} \end{pmatrix} \qquad (3)$$

(2) Numerical Model

As a three-dimensional acoustic logging model, we assumed a model which contains a set of source and receiver in a uniform anisotropic layer. As for source, a monopole source and a pair of orthogonal dipole sources are set. As the physical properties of the anisotropic layer, we assumed a formation with 15% seismic anisotropy for both P and S wave (Figure 1).

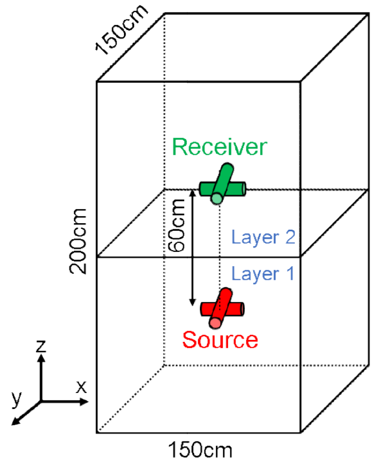

Figure 1 Model Configuration

(3) Full Waveform Inversion

FWI is a method to estimate a true model with high accuracy by updating model parameters so as to minimize the misfit between observed and calculated waveform. The misfit function is defined by the following equation.

$$E = \frac{1}{2}\sum (d_{obs} - d_{cal})^2 \qquad (4)$$

The model parameter is updated by the following.

$$\boldsymbol{m}_{k+1} = \boldsymbol{m}_k - a_k \delta \boldsymbol{m}_k \qquad (5)$$

For the kth iteration, the model parameters, step length, and gradient are defined as $\boldsymbol{m}_k$, a_k and $\delta\boldsymbol{m}_k$, respectively. The model was updated using the conjugate gradient method.

For efficient estimation of the elastic element, the gradient of the elastic element of the misfit function E is theoretically obtained based on the wave equation[6].

$$\frac{\partial E}{\partial C_{ijkl}} = \int \frac{\partial u_i}{\partial x_j}\frac{\partial \psi_k}{\partial x_l} dt \qquad (6)$$

where C_{ijkl} is the elastic element, u_i is the wave field propagated from the source, and ψ_k is the back propagated wave field of residual from receiver.

3. RESULTS

(1) FWI for two layers model

FWI is performed to estimate anisotropic parameters of two-layered model which have different orientation and dip. The physical properties of the model are shown in Table 1. The parameters of initial model are 10% away from the true values.

Table 1 Initial and true properties

	c_{11} [GPa]	c_{13} [GPa]	c_{33} [GPa]	c_{44} [GPa]	c_{66} [GPa]
Initial	8.44	2.94	8.28	2.77	2.30
True	7.67	3.27	9.20	3.07	2.56
	Ori1 [deg]	Dip1 [deg]	Ori2 [deg]	Dip2 [deg]	
Initial	0	0	0	0	
True	30	30	60	60	

As a result of FWI, the error between true and predicted model are less than 3% in stiffness elements and $\pm 12^{\circ}$ in orientation and dip (Fig. 2 and Fig. 3).

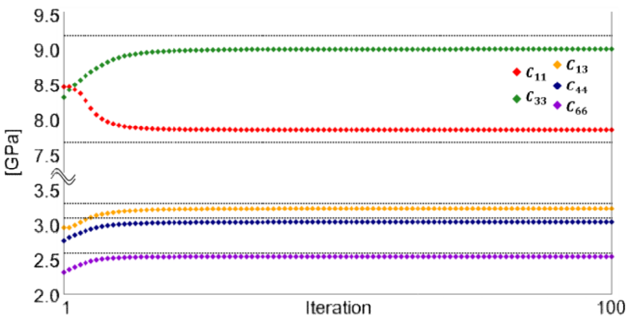

Figure 2 FWI result of stiffness elements

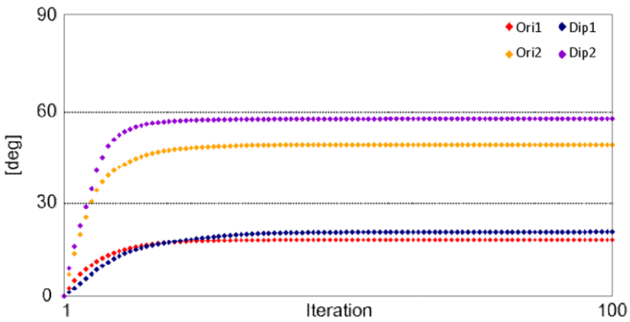

Figure 3 FWI result of orientation and dip

(2) Misfit evaluation

Besides, misfit planes are visualized to reveal the sensitivity (Fig. 4 and Fig.5). The misfit planes indicate that there is no local minimum in solution space and it is apparent the predicted model does not steady down at local minimum. The gradients of Orientation1 - Orientation2 around the true value are too moderate to converge that would result in small misfit at final condition. The contour of misfit plane in Orientation1 - Orientation2 is more extensive than the other parameters. On the other hand, the combinations of Dip1 – Dip2, have the fine gradient around the true value that contributes the well convergence. This trend indicates that the parameters which have wide contour are slowly recovered compared to those which have sharp contour in the misfit plane.

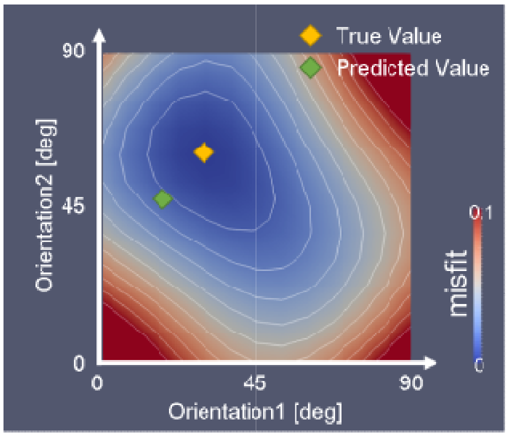

Figure 4 Misfit plane of orientation for 1st and 2nd layers.

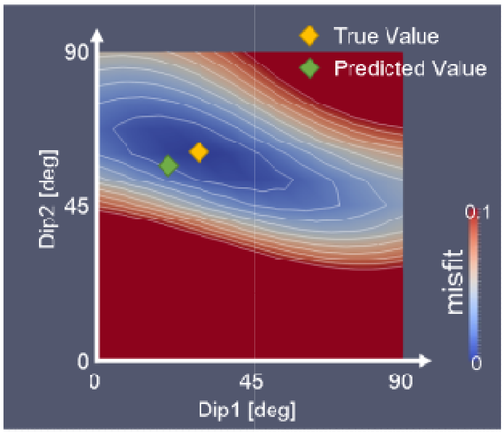

Figure 5 Misfit plane of dip for 1st and 2nd layers.

4. CONCLUSION

Our strategy considers totally 7 unknown parameters for a general three-dimensional anisotropic medium instead of estimating 21 unknowns in the stiffness matrix. We confirmed the effectiveness of the proposed strategy using numerical simulation. The proposed strategy can successfully predict properties of surrounding anisotropic medium with well convergence at the close value of true model in the two-layered model. In particular, although the titled angle of TI medium is hard to grasp the image in the conventional strategy, proposed strategy reveals that the waveforms change due to the dip angle provides a clue to estimate it by FWI. Since FWI is an iterative process to find the minimum, there is still a concern that the predicted model would possible to converge to local minima as the number of parameters increases. Therefore, visualized misfit planes analysis provides valuable insights into the obtained results. The misfit planes show there is no local minima in the solution space and predicted model does not converge to local minimum. Some combinations have the moderate gradient and some have the sharp gradient around the true value. These trends contribute to the recovery rate of FWI.

REFERENCES

1) Alford, R. M. 1986, Shear data in the presence of azimuthal anisotropy: Dilley, Texas, 56th Annual Meeting, Society of Exploration Geophysicists, Expanded Abstract, 476-479.
2) Fuse, S., Mikada, H. and Takekawa, J., 2016, Full waveform inversion of cross-dipole acoustic waveforms, International Petroleum Tecnology Conference 2016, IPTC-18726-MS.
3) de Hoop, M. V., Spencer, C. and Burridge, R., 1999, The resolving power of seismic amplitude data: An anisotropic inversion/migration approach. *Geophysics*, **64**(3), 852-873.
4) Tarantola, A., 1986, A strategy for nonlinear elastic inversion of seismic reflection data,*Geophysics*, **51** (10), 1893-1903.
5) Takekawa, J., H. Mikada, and T. Goto, 2014, A Hamiltonian particle method with a staggered particle technique for simulating seismic wave propagation, *Pure and Applied Geophysics*, **171** (8), 1747-1757.
6) Kamath, N., and Tsvankin, I., 2016, Elastic full-waveform inversion for VTI media: Methodology and sensitivity analysis, *Geophysics*, **81** (2), C53-C68.
7) Bergslid, T. S., Birger Raknes, E., and Arntsen, B., 2015, The influence of anisotropy on elastic full-waveform inversion. 85th Annual Meeting, Society of Exploration Geophysicists, Expanded Abstract, 1425-1429.
8) Butzer, S., Kurzmann and A., Bohlen, T., 2013, 3D elastic full-waveform inversion of small-scale heterogeneities in transmission geometry, *Geophysical Prospecting*, **61** (6), 1238-1251.
9) Cerjan, C., Kosloff, D., Kosloff, R., and Reshef, M., 1985, A nonreflecting boundary condition for discrete acoustic and elastic wave equations, *Geophysics*, **50** (4), 705-708.
10) Ishikura, K., H. Mikada, T. Goto and J. Takekawa, 2014, Estimation of fluid contact in terms of attenuation, 84th Annual Meeting, Society of Exploration Geophysicists, Expanded Abstracts, 623-627.
11) Kimball, C. V., and Marzetta, T. L., 1984, Semblance processing of borehole acoustic array data, *Geophysics*, **49** (3), 274-281
12) Kitsunezaki, C., 1981, A new method for shear-wave logging, *Geophysics*, **45** (5), 1489-1506.
13) Saenjer, E. H., and Bohlen, T., 2004, Finite-difference modeling of viscoelastic and anisotropic wave propagation using the rotated staggered grid, *Geophysics*, **69** (8), 583-591.
14) Thomsen, L., 1986, Weak elastic anisotropy, *Geophysics*, **51** (10), 1954-1966.
15) Virieux, J. and Operto, S., 2009, An overview of full-waveform inversion in exploration geophysics. Geophysics, **74**(6), WCC1-WCC26.